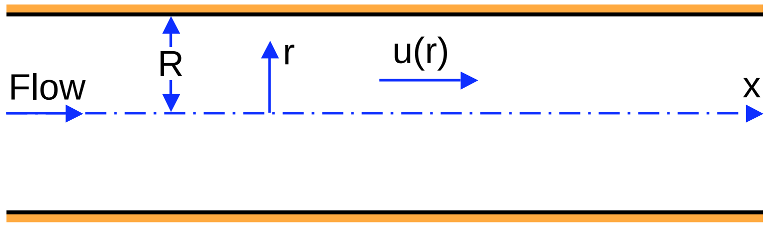
ME19b. **HOMEWORK.** **Jan. 28, 2010. Due Feb. 4**

PROBLEM B12

Consider the fully developed laminar pipe flow of an incompressible, non-Newtonian fluid :



This fluid is such that the normal stress in the x direction is equal to $-p$ where p is the pressure and the shear stress, τ, is related to the velocity gradient by

$$\tau = C\left(-\frac{du}{dr}\right)^2$$

where C is a known constant. Find the friction factor, f , for this pipe flow in terms of C, ρ (the fluid density) and R (the radius of the pipe).

[Note: Remember the definition of the friction factor, f, namely

$$f = \frac{4R}{\rho V^2}\left(-\frac{dp}{dx}\right)$$

where V is the volumetric average velocity of flow in the pipe (the volume flow rate divided by the cross-sectional area).]

PROBLEM B13

Consider a flow in which the density, ρ, of a particular fluid element remains unchanged as it moves along in the flow but in which the density may vary from one fluid element to another. The fluid will be assumed to be inviscid and the body forces are assumed to be conservative.

(a) What are the forms of the equation of continuity and the equation of motion which are appropriate for such a flow?

(b) Now consider such a flow which is in the process of accelerating from rest at time, $t = 0$. The velocity vector $\vec{u}$ is zero but the acceleration, $\partial\vec{u}/\partial t$, is not zero. Show that the rate of increase of vorticity, $\vec{\omega}$, in the flow at time $t = 0$ is directly related to the density gradient and the acceleration by

$$\left(\frac{\partial\vec{\omega}}{\partial t}\right)_{t=0} = -\frac{1}{\rho}\left[(\nabla\rho)\times\frac{\partial\vec{u}}{\partial t}\right]_{t=0}$$

Note the following vector identities:

$$\nabla\times(\rho\vec{a}) = (\nabla\rho)\times\vec{a} + \rho\nabla\times\vec{a} \qquad \text{where } \rho \text{ is a scalar and } \vec{a} \text{ is a vector.}$$

$$\nabla\times(\nabla y) = 0 \qquad \text{where } y \text{ is a scalar.}$$

PROBLEM B14

Consider the long thin racing boats used in competitive rowing events. Assume that the major component of resistance to motion is the skin friction drag on the hull of the boat. Estimate this drag force for any velocity, U, by assuming that

1. the hull/water interface is like a flat plate with a length of 10 m and a width of 1 m.
2. the boundary layer on the hull remains laminar (even though in practice this would not be the case) and unseparated.

The water density, ρ, and kinematic viscosity, $\nu = \mu/\rho$, are respectively 1000 kg/m^3 and 10^{-6} m^2/s.

If the boat is propelled by eight humans *each* capable of a rate of output of work of 0.1 HP (1 $HP = 746$ $kg{\cdot}m^2/s^3$) and if half of this energy is uselessly dissipated in the rowing process what would be the top speed of the boat?

PROBLEM B15

Air enters a long horizontal ventilation duct of circular cross-section (radius 0.25 m) with a velocity of 1.0 m/s. At the entrance it is assumed that this velocity is uniform over the entire cross-section. However as the flow proceeds down the duct a thin laminar boundary develops on the inside wall of the duct. If we first assume that this is like the boundary layer on a flat plate and that the velocity away from the boundary layer remains at 1.0 m/s find the displacement thickness, δ_D (in m), at a distance x (in m) from the entrance. Assume the kinematic viscosity of the air is 2.5×10^{-6} m^2/s.

Having calculated this displacement thickness we recognize that the velocity outside the boundary layer cannot remain precisely constant at 1 m/s. Using the above calculated displacement thickness find the uniform velocity outside the boundary layer at a point 200 m from the entrance. What is the pressure difference between the entrance and this point 200 m from the entrance?