ME19b. **HOMEWORK.** **Jan. 21, 2010. Due Jan. 28**

PROBLEM B8

Consider the laminar, viscous, planar flow of an incompressible fluid contained between two parallel plates a distance H apart. The coordinates x and y are measured parallel to and perpendicular to these plates, respectively. We shall take $y = 0$ at the static plate and $y = H$ at the moving plate for convenience. The plate at $y = H$ moves with a steady velocity, U, in the x direction. However, unlike simple Couette flow, a pressure gradient, dp/dx, is also applied to the fluid. Find:

[1] The velocity distribution, $u(y)$, in the flow as a function of y, U, H, dp/dx and the viscosity of the fluid, μ.

[2] The magnitude and direction of the particular pressure gradient for which there would be zero net volume flow in the x direction.

PROBLEM B9

In cylindrical coordinates, (r, θ, z), the Navier-Stokes equations of motion for an incompressible fluid of constant dynamic viscosity, μ, and density, ρ, are

$$\rho\left[\frac{Du_r}{Dt} - \frac{u_\theta^2}{r}\right] = -\frac{\partial p}{\partial r} + f_r + \mu\left[\nabla^2 u_r - \frac{u_r}{r^2} - \frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta}\right]$$

$$\rho\left[\frac{Du_\theta}{Dt} + \frac{u_\theta u_r}{r}\right] = -\frac{1}{r}\frac{\partial p}{\partial \theta} + f_\theta + \mu\left[\nabla^2 u_\theta - \frac{u_\theta}{r^2} + \frac{2}{r^2}\frac{\partial u_r}{\partial \theta}\right]$$

$$\rho\frac{Du_z}{Dt} = -\frac{\partial p}{\partial z} + f_z + \mu\nabla^2 u_z$$

where u_r, u_θ, u_z are the velocities in the r, θ, z cylindrical coordinate directions, p is the pressure, f_r, f_θ, f_z are the body force components in the r, θ, z directions and the operators D/Dt and ∇^2 are

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u_r\frac{\partial}{\partial r} + \frac{u_\theta}{r}\frac{\partial}{\partial \theta} + u_z\frac{\partial}{\partial z}$$

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}$$

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Now consider the steady, planar, incompressible, viscous flow between two concentric cylinders. The inner cylinder has radius, a, and is rotating with angular velocity, Ω (radians/second). The outer cylinder has radius, b, and is static. There is no flow in the direction parallel to the axis of the cylinders so only the velocity, u_θ, is non-zero. Body forces are to be neglected. The density of the fluid is denoted by ρ. Find:

(a) The velocity distribution, $u_\theta(r)$, in the gap between the two cylinders.

(b) The difference between the pressure on the outer surface of the inner cylinder and the pressure on the inner surface of the outer cylinder.

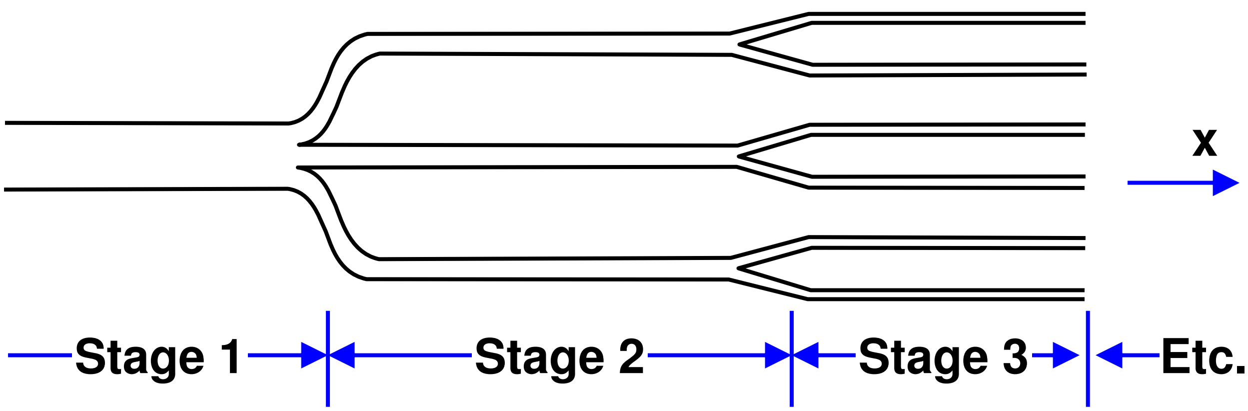
Note: The solution of the ordinary differential equation

$$\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} - \frac{y}{x^2} = 0 \qquad \text{is} \qquad y = A/x + Bx$$

where A and B are constants.

PROBLEM B10

Both the mammalian respiration system and the mammalian blood circulation system are networks of tubes in which the flow from one large tube (respectively the trachea and the aorta) branches into parallel flows in tubes of smaller size. This branching continues through a number of stages:



If, for each stage, the number of tubes is denoted by n and the cross-sectional area for each and every tube in that stage is denoted by A_n, find the relation between A_n and n such that the pressure gradient, dp/dx, is the same for each stage. How does the average velocity depend on n? Assume steady, fully-developed Poiseuille flow in all tubes even though this may not be the case in the actual systems.

If the diameter of the aorta is 3 cm and the diameter of the microcirculation (the smallest tubes) is 8×10^{-6} m, calculate the number of tubes at the microcirculation stage which would be present if the above property were to exist. The actual number is much smaller than this. Where, then, does most of the pressure drop occur in the blood circulation system?

PROBLEM B11

A semi-infinite domain of fluid is bounded only by a single infinite flat plate. The fluid is incompressible with a constant and uniform viscosity, μ, and density, ρ. The plate is then set in accelerating motion, moving in its own plane with an accelerating velocity, Ue^{kt}, where U and k are constants and t is time. If the fluid only reacts by moving parallel with the plate with a velocity, $u(y,t)$, where y is the distance from the plate and if the velocities in the other directions are zero, write down the simplified form of the Navier-Stokes equation that govern this flow and must be solved to find $u(y,t)$. Note that p is uniform; that the velocity far from the plate is zero; and neglect gravitational effects. The result is a partial differential equation for $u(y,t)$ that only includes u, y, t and μ/ρ.

Using separation of variables (or otherwise) solve this equation to find $u(y,t)$ and the vorticity, $\omega(y,t)$, in terms of y, t, U, k, and the fluid properties. If we define a boundary layer next to the plate as the region within which the velocity is at least 10% of the plate velocity, derive an expression for the thickness of the boundary layer as a function of time.