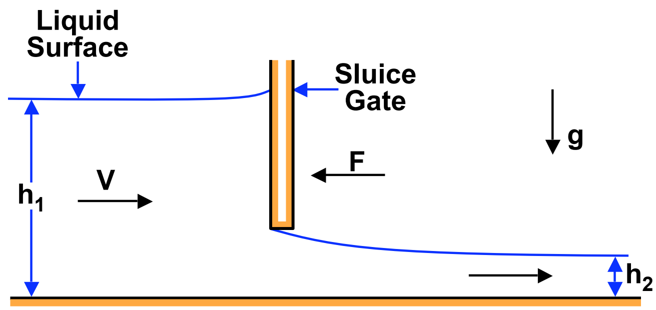
PROBLEM B1

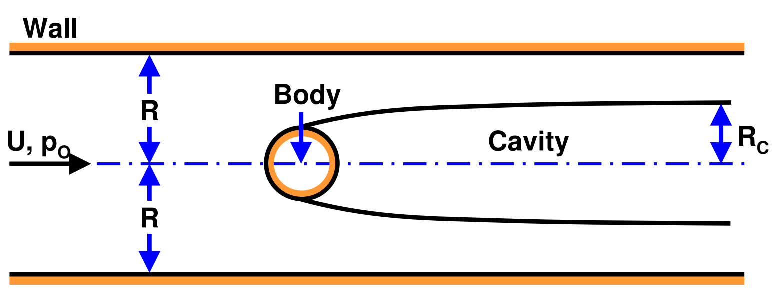
An incompressible, inviscid liquid flow (density, ρ) of depth, h_1, and velocity, V, flows under the action of gravity through a sluice gate:



The depth downstream of the sluice gate is denoted by h_2. Determine the force per unit width (normal to sketch), F, necessary to hold the plate in place in terms of ρ, g, h_1 and h_2.

PROBLEM B2

An axisymmetric body (a sphere if you wish) is mounted in a water tunnel which has a circular cross-section of radius, R. The velocity far upstream, U, is fixed. When the pressure far upstream, p_o, is lowered sufficiently a large vapor-filled wake or cavity forms behind the body:



The pressure in the cavity is simply given by the vapor pressure, p_v, of the water at the operating water temperature ($p_o > p_v$). It is to be assumed that the effect of friction, the effect of gravity, the density of the vapor and the amount of water vaporized at the free surface are all negligible. A parameter called the cavitation number, σ, is defined as

$$\sigma = \frac{p_o - p_v}{\frac{1}{2}\rho U^2}$$

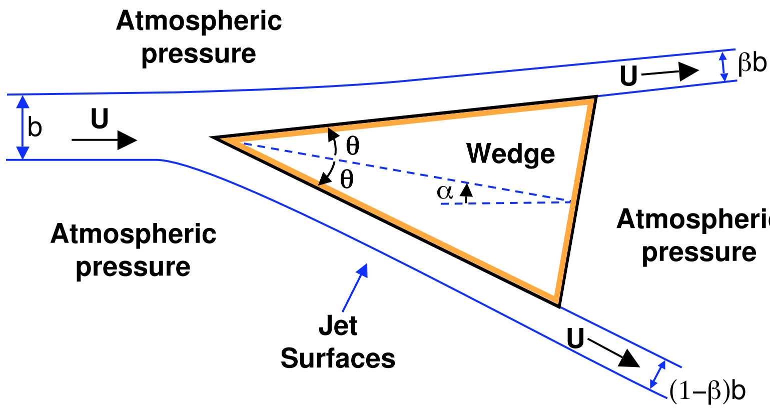
where ρ is the water density.

(a) Find the relation between R_c/R and σ for very long cavities whose asymptotic radius is R_c.

(b) Find the drag on the body in terms of U, R, σ and ρ.

PROBLEM B3

A wedge with a vertex angle of 2θ is inserted into a jet of water (density, ρ) of width, b, and velocity, U, as shown in the sketch below.



The angle of attack, α, of the wedge is also defined in the sketch. The result is that the single incident jet is divided into two jets both of which leave the back edges of the wedge with the velocity, U. The widths of the two departing jets are βb and $(1-\beta)b$ as indicated in the figure. It is assumed that the flow is planar and that the pressure in surrounding air is everywhere atmospheric.

1. Find the lift and drag on the wedge per unit length normal to the sketch as functions of ρ, U, b, β, α and θ. Note that drag and lift are defined as the forces on the wedge which are respectively parallel to and perpendicular to the direction of the incident jet.
2. If the angle of attack, α, is varied while ρ, U, b, β and θ remain fixed, find the angle of attack at which the lift is zero.
3. If, on the other hand, the wedge is moved in a direction perpendicular to the incident jet while ρ, b, U, θ and α remain fixed then β will change. There is one such position at which the lift is zero; what is the value of β at this position in terms of θ and α ? If the wedge were free to move in such a way would this position represent a position of stable or unstable equilibrium?

PROBLEM B4

A turbojet engine in a wind tunnel ingests air at a velocity of 100 m/s and a density of 1 kg/m^3. The velocity is uniform and the cross-sectional area of the approaching stream which enters the engine is 0.1 m^2. The velocity of the exhaust jet from the engine, however, is not uniform but has a velocity which varies over the cross-section according to

$$u(r) = 2U\left(1 - \frac{r^2}{r_0^2}\right)$$

where the constant $U = 600$ m/s and r_0 is the radius of the jet cross-section. The radial position within the axisymmetric jet is denoted by r and the density of the exhaust jet is assumed to be uniform. It is readily shown that the above formula implies that U is the average velocity of the exhaust jet (the volume flow divided by the cross-sectional area of the jet).

(a) Find the thrust of the turbojet engine.

(b) Find what the thrust would be if the exhaust jet had a uniform velocity, U.

Assume the pressures in both the inlet and exhaust jets are the same as the surrounding air and that mass is conserved in the flow through the engine (roughly true in practice).