ME19b. **FINAL REVIEW.** **Mar. 11, 2010.**

EXAMPLE PROBLEM 1

A laboratory wind tunnel has a square test section with side length L. Boundary-layer velocity profiles are measured at two cross-sections and displacement thicknesses are evaluated from the measured profiles. At section 1, where the free-stream speed is U_1, the displacement thickness is δ_{D1}. At section 2, located downstream from 1, the displacement thickness is δ_{D2}.

(a) Calculate the free-stream velocity ratio U_2/U_1 as a function of the known parameters.

(b) Calculate the change in static pressure between sections 1 and 2. Express the result as a fraction of the free-stream dynamic pressure at section 1, i.e., find $(p_1 - p_2)/\frac{1}{2}\rho U_1^2$.

(c) ...and just for fun, write down the displacement thickness, δ_D, and the momentum thickness, δ_M in integral form. What are the results for these from the laminar Blasius solution? What is the physical significance of the disturbance thickness (or boundary layer height), δ, and the other two thicknesses δ_D and δ_M?

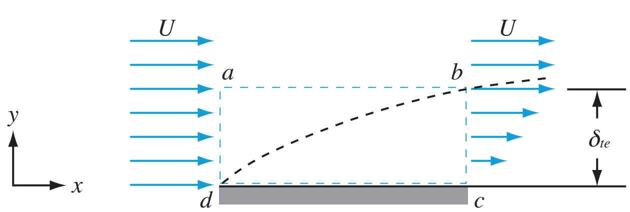
EXAMPLE PROBLEM 2

This isn't really a problem, but it is something you should *definitely* read and understand. The **Karman Momentum Integral Equation** (KMIE) has appeared on every problem set since the midterm. It is very important that you understand where this comes from and the significance of each term. It also offers a perfect opportunity to combine a control-volume approach using the mass and momentum integral equations from the first half of the term to the boundary-layer analysis from the second half.

That said, please carefully read the additional document posted with this review at,
http://www.its.caltech.edu/~mefm/me19b/handouts/scan_fox9-4_momentumintegraleqn.pdf.
The material was scanned from Fox, R.W., McDonald, A.T., and Pritchard, P.J., *Introduction to Fluid Mechanics*, 6th ed., John Wiley & Sons, 2004, §9-4, pp. 415-420. It is very well written, easy to understand, and presented in a manner that is easy to follow.

EXAMPLE PROBLEM 3

A fluid, with density ρ, flows with speed U over a flat plate with length L and width w (assume that $L > w$). At the trailing edge, the boundary-layer thickness is δ_{te}. Assume the velocity profile within the boundary layer is linear, as shown, and that the flow is two-dimensional (flow conditions are independent of z).



(a) Using control volume *abcd*, shown by dashed lines, compute the mass flow rate across surface *ab*. Determine the drag force on the upper surface of the plate. Explain how this (viscous) drag can be computed from the given data even though we do not know the fluid viscosity.

(b) The flat plate is turned so that w is now the length parallel to the flow, and L perpendicular to it (recall $L > w$). Is the boundary-layer thickness at the trailing edge greater or less than the previous case? Should we expect the drag to increase or decrease?

(c) Now, assuming we know the fluid viscosity, $\nu = \mu/\rho$, compute the drag using boundary layer equations.

EXAMPLE PROBLEM 4

Heated air at 1 atm and 35°C is to be transported in a 150 m long circular duct at a rate of 0.35 m^3/s. If the head loss in the pipe is not to exceed 20 m, determine the minimum diameter of the duct.