Please hand in homework to Cheryl Geer in Room 119 Thomas by 5pm on Nov. 24

## PROBLEM 25

A two-dimensional potential (or free) vortex is located near an infinite wall at a distance $h$ from the wall:


The velocity and pressure of the flow far from the vortex are $U$ and $p_{\infty}$ respectively. The flow is irrotational and planar and the fluid is incompressible. The strength or circulation of the vortex is $\Gamma$. Find the velocity potential for this flow and make a sketch of the streamlines and the equipotentials. Find the force on the wall per unit depth normal to the sketch if the pressure on the underside of the wall is $p_{\infty}$.

## PROBLEM 26

A cylindrical artic hut is subjected to a crosswind as shown in the figure:


The interior of the hut is ventilated to the outside through a small vent at a position, $\theta$, as indicated. Hence the pressure inside the hut (assumed uniform and constant) is the same as the pressure just outside the vent. Assuming potential flow over the hut find the angle, $\theta$, at which the net vertical lift on the hut is zero. Neglect the thickness of the wall of the hut and assume that the vent has no effect on the exterior flow. Also assume that the air is incompressible.

## PROBLEM 27

The incompressible, axisymmetric potential flow around a sphere can be generated by superposition of a uniform stream $(\phi=U x)$ and a three-dimensional doublet whose potential is given by $A \cos \theta / r^{2}$ where $A$ is a constant representing the doublet strength. The coordinates $r, \theta$ are centered on the doublet and the direction $x(x=$ $r \cos \theta$ ) is in the direction of the uniform stream:


On the basis of this information construct the velocity potential for potential flow around a sphere of radius $R$ in terms of $U, R$ and the coordinates $r, \theta$. What is the maximum velocity on the surface of the sphere?

## PROBLEM 28

For the purposes of estimating the drag force on a spherical body (radius, $R$ ) in a uniform stream (velocity, $U$, and density, $\rho$ ) it is assumed that the pressure distribution over the upstream side (facing the oncoming stream) is the same as in potential flow whereas the pressure on the downstream side is constant simulating the conditions in a wake. Moreover the pressures match at the "equator", $\theta=\pi / 2$ (where $\theta$ is the angle measured from the front stagnation point). Find the drag, $F_{D}$, on the sphere as a function of $\rho, R$ and $U$. Evaluate the "drag coefficient" defined as $C_{D}=F_{D} /\left(0.5 \rho U^{2} \pi R^{2}\right)$.

## PROBLEM 29

A finite difference method is to be used with a mesh having a uniform node spacing, $h$, in the $x$ and $y$ directions to solve for the quantity, $f$, which is governed by the following partial differential equation:

$$
\begin{equation*}
f \frac{\partial^{2} f}{\partial x^{2}}=-4\left(\frac{\partial f}{\partial y}\right)^{2} \tag{1}
\end{equation*}
$$

Determine the finite difference form of this equation at the node 0 that utilizes values of $f$ at the nodes $0,1,2$, 3 and 4 as shown below:


