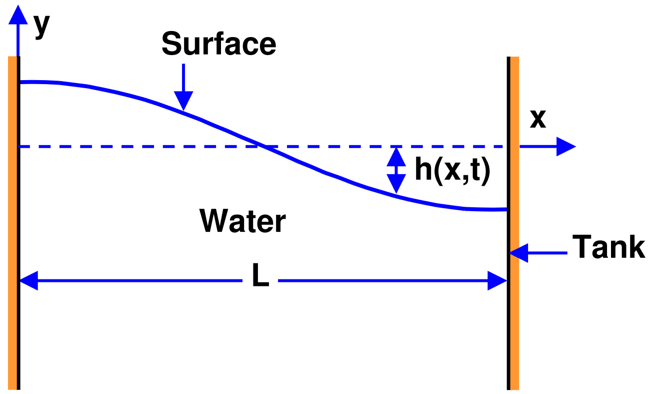
PROBLEM 21

Water is sloshing back and forth between two infinite vertical walls separated by a distance L:



The flow is assumed to be planar, incompressible, inviscid potential flow. The free surface is devoid of surface tension and is at constant atmospheric pressure. Its position is described by $h(x,t)$ as indicated in the sketch. The wave height, $h(x,t)$, is small so that the assumptions of linear water wave theory may be used. An appropriate velocity potential for this flow is

$$\phi = Ae^{ky}\cos kx \sin \omega t$$

where A, k and ω are undetermined constants.

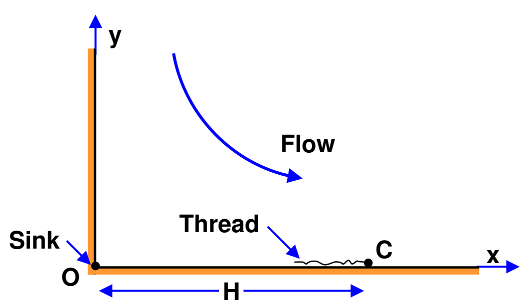
(a) What are the four boundary conditions which a solution to this flow must satisfy ?

(b) Find the series of values which are possible for the wavelength, λ ($\lambda = 2\pi/k$), of the free surface waves. Each of these wavelengths corresponds to a particular mode of sloshing.

(c) Use the kinematic condition on the free surface to determine the shape of the free surface, $h(x,t)$, as a function of A, k, ω, x and t.

(d) Use the dynamic condition on the free surface to determine the frequency, f ($f = \omega/2\pi$), for each of the modes of sloshing. Denote the acceleration due to gravity by g.

PROBLEM 22



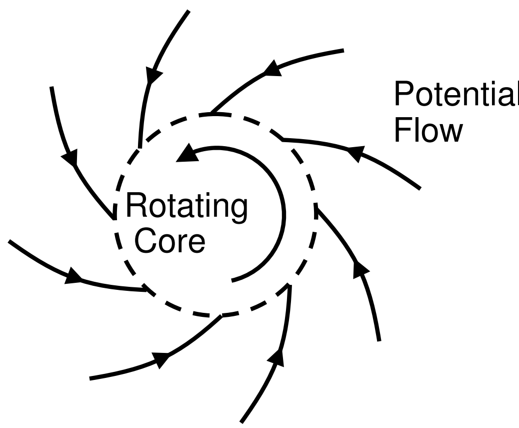
The flow in the neighborhood of a corner in a rectangular ventilation duct is to be modelled as a planar potential flow of an incompressible, inviscid fluid and is therefore given by the streamfunction, $\psi = Axy$, where A is assumed known:

This flow is then changed by withdrawing fluid through pipes connected to the walls at the origin, O; fluid is thereby withdrawn at a volumetric rate of q per unit depth normal to the sketch. Construct the velocity potential for the modified flow and find expressions for the velocity components in terms of x, y, A and q.

A piece of thread is attached by one end to a point, C, which is at a distance, H, from the origin. The flow will extend the free end of this thread either toward the origin or toward $x = \infty$. Find the condition under which it will extend toward the origin.

PROBLEM 23

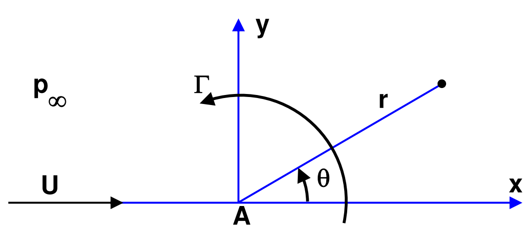
A hurricane can be visualized as a planar incompressible flow consisting of a rotating circular core surrounded by a potential flow:



A particular hurricane has a core of radius 40 m and air is sucked into this core at a volume flow rate per unit depth perpendicular to the diagram of 5000 m^2/s. Furthermore the pressure difference between the air far away from the hurricane and the air at the edge of the core is 1500 $kg/m^2\ s$. The velocity of the air far from the core is assumed to be negligible. The density of the air is assumed uniform and constant at 1.2 kg/m^3. Find the angular rate of rotation of the hurricane and the velocity of the wind at the edge of the core.

PROBLEM 24

A planar incompressible potential flow is generated by superposition of:



1. A uniform stream with velocity potential Ux.
2. A doublet with velocity potential $UR^2 \cos\theta / r$ at the point A in the sketch above.

3. A potential vortex at the point A with circulation, Γ, and velocity potential, $\Gamma\theta/2\pi$.

This generates the flow around a cylinder of radius, R, whose center is at A; the cylinder is also spinning in the counterclockwise direction. Find the velocity and pressure on the surface of the cylinder as a function of angular position, θ. Neglecting shear stresses and considering only the pressures on the surface of the cylinder, find the total force on the cylinder per unit depth normal to the sketch. This is probably most readily done by separately evaluating the drag (the component of the force in the direction of the uniform stream, in other words the direction x) and the lift (the component of the force in the direction, y, perpendicular to the uniform stream). Denote the fluid density by ρ and the pressure far from the cylinder by p_∞.