ME19a. HOMEWORK.

PROBLEM 17

A manufacturer advertises a line of centrifugal pumps of various sizes given by the radius of the impeller (the rotating part) denoted by R. These can be run by a motor at any rotating speed, N (in radians per sec). The manufacturer also states that the "operating condition" for this line of pumps is given by specific values of the "flow coefficient" and "head coefficient". The flow coefficient, ϕ , is defined by $\phi = Q/\pi N R^3$ where Q is the volume flow rate and the head coefficient, ψ , is defined by $\psi = \Delta p / \rho N^2 R^2$ where Δp is the total pressure rise across the pump and ρ is the fluid density. We denote the manufacturer's specific design values for the operating condition by ϕ_D and ψ_D and regard them as given constants.

We now wish to choose one of these pumps for a system in which we want a particular flow rate, Q, with a particular total pressure rise, Δp . How do we decide on the necessary size of pump (R) and the necessary speed, N?

PROBLEM 18

A wind tunnel is constructed primarily of 6 m diameter piping arranged with four 90° elbows as shown in the sketch below.



The working section is 3 m in diameter and is preceded by a nozzle and followed by a diffuser. A fan is installed to create the flow and is 80% efficient. If the tunnel is to achieve an air velocity of 80 m/s in the working section, find the power which must be provided to the fan (in HP where 1 $HP = 746 \ kg \ m^2/s^3$). Air at these speeds can be assumed essentially incompressible with a density of 1.2 kg/m^3 . Assume the following losses occur in the tunnel:

- 1. A loss in each of the four corner bends equivalent to a length of 20 diameters of the large piping.
- 2. A friction factor, f, of 0.02 in the 138 m of 6 m diameter pipe.

3. A total loss in the nozzle, working section and diffuser equivalent to one fifth of the velocity head (the $\frac{1}{2}\rho u^2$) in the working section.

PROBLEM 19

The tailrace (discharge pipe) of a hydro-electric turbine installation is at an elevation, h, below the water level in the reservoir:



The frictional losses in the penstock (the pipe leading to the turbine) and the tailrace are represented by the loss coefficient, k, based on the mean velocity, U, in those pipes (which have the same cross-sectional area, A). The flow discharges to atmospheric pressure at the exit from the tailrace. The water density is denoted by ρ and the acceleration due to gravity by g.

- (a) What is the drop in total head across the turbine in terms of U, h, k and g?
- (b) What is the power developed by the turbine assuming that it is 90% efficient? (Answer in terms of U, h, k, ρ, A and g.)
- (c) What is the optimum velocity, U, which will produce the maximum power output from the turbine assuming that h, k, A, ρ and g are constant? (Answer in terms of h, k, and g.)

PROBLEM 20

Find the speed of propagation, c, of small amplitude, traveling water waves of wavelength, λ , on an ocean of infinite extent but finite depth, H. The answer is an expression involving H, λ , and the acceleration due to gravity, g.