ME19a. **HOMEWORK.** **Oct. 13, 2009. Due Oct. 20**

PROBLEM 9

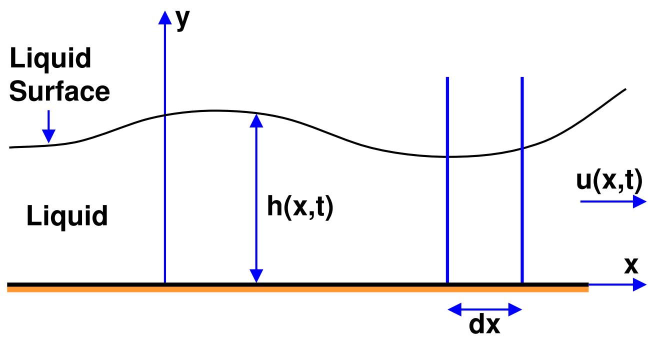
Consider the flow of a fluid in which the fluid elements are traveling with velocity, u, in the x direction (this is the only non-zero velocity of the fluid which it is necessary to consider in this problem). A succession of fluid elements travel through the Eulerian point, $x = x_o$ with a velocity $u = u_o$ and subsequently accelerate according to

$$u = (u_o/x_o^2)x^2$$

However the flow is **steady**. Chemical constituents within the fluid are reacting in such a way that the concentration, c, of one of the constituents is increasing with time at a rate denoted by α (a constant). If the concentration at the point $x = x_o$ has a known and constant value denoted by c_o find an expression for the concentration elsewhere as a function of x, x_o, u_o, c_o and α.

PROBLEM 10

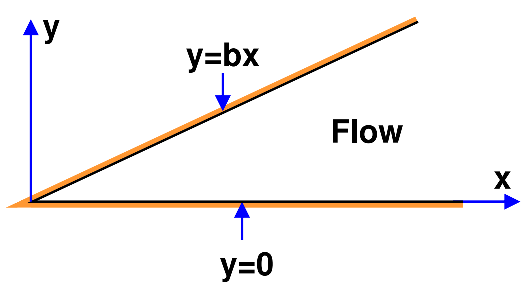
Construct from first principles an equation for the conservation of mass which governs the planar flow (in the xy plane) of an incompressible liquid lying on a flat horizontal plane:



The depth, $h(x, t)$, is a function of x and time, t. Examine an Eulerian element of width, dx, as shown above (it extends from $y = 0$ to $y = \infty$) and assume that the velocity, $u(x, t)$, of the water in the positive x direction is **independent of** y. Then utilize conservation of mass to obtain a partial differential equation connecting the depth, $h(x, t)$, and the velocity, $u(x, t)$. Neglect surface tension. [This is one of the equations of what is known as "shallow water wave theory".]

PROBLEM 11

A planar, incompressible flow within a wedge-shaped region bounded by solid walls at $y = 0$ and $y = bx$ has a velocity, u, in the x direction given by $u = A(y - ax)$ where A and a are constants:



Find expressions for the streamfunction, ψ, and the velocity, v, in the y direction. Determine the relation between a and b. Sketch some of the streamlines of the flow.

[Do **not** use the no-slip condition which is violated in the above problem. Later, in class, we shall discuss the issues associated with the no-slip condition.]

PROBLEM 12

Consider the following streamfunction, ψ, for a planar incompressible flow:

$$\psi = Ur\left(1 - \frac{r_0^2}{r^2}\right)\sin\theta$$

where U and r_0 are constants and r, θ are polar coordinates.

(a) Find and sketch the streamline corresponding to $r = r_0$.

(b) Find and add to your sketch the streamlines for $\theta = 0, r > r_0$ and for $\theta = \pi, r > r_0$. Note on your sketch the value of ψ along these lines and along the streamline for $r = r_0$.

(c) Make a rough estimate of some other streamlines with $\psi > 0$ and show the form of these streamlines in your sketch.

(d) What is the magnitude and direction of the flow for $r \gg r_0$?

(e) Guided by your sketch, estimate what real flow might have the above streamfunction.

Note: In polar coordinates, the velocities in the r and θ directions, denoted respectively by u_r and u_θ, are given by

$$u_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} \quad ; \quad u_\theta = -\frac{\partial\psi}{\partial r}$$