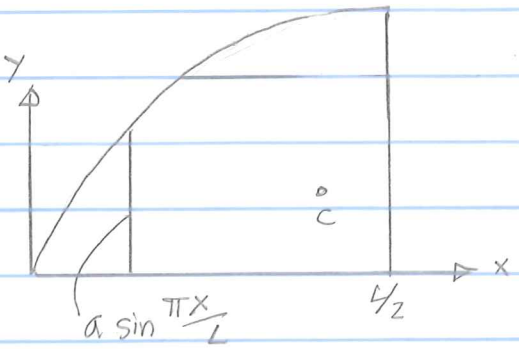
ME 35a HW#4 Solution

5.48



$$\int dA = \int_0^{L/2} a \sin\frac{\pi x}{L}\,dx = -a\frac{L}{\pi}\cos\frac{\pi x}{L}\Big]_0^{L/2} = \frac{aL}{\pi}$$

$$\int x\,dA = \int_0^{L/2} x a \sin\frac{\pi x}{L}\,dx = \left(-\frac{Lax}{\pi}\cos\frac{\pi x}{L} + \frac{L^2 a}{\pi^2}\sin\frac{\pi x}{L}\right)\Big]_0^{L/2} = \frac{aL^2}{\pi^2}$$

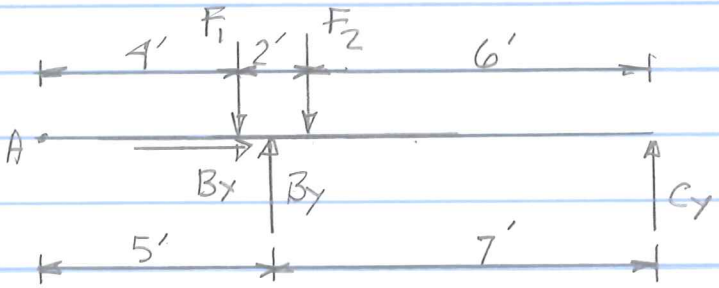
$$\int y\,dA = \int_0^{L/2} \underbrace{\frac{a^2}{2}\sin^2\frac{\pi x}{L}}\,dx = \frac{a^2}{2}\left(\frac{x}{2} - \frac{\sin 2\pi x/L}{4\pi/L}\right)_0^{L/2} = \frac{a^2 L}{8}$$

(underbrace: column height times half column height)

$x_c = \int x\,dA / \int dA = \boxed{L/\pi}$

$y_c = \int y\,dA / \int dA = = \boxed{\frac{\pi a}{8}}$

5.74 FBD:



$F_1 = \frac{1}{2}\cdot(400\,\text{lb/ft} - 300\,\text{lb/ft})\cdot 12\,\text{ft} = 600\,\text{lb}$

$F_2 = 300\,\text{lb/ft} \cdot 12\,\text{ft} = 3600\,\text{lb}$

$\Sigma M_B = 0 = F_1\cdot 1\,\text{ft} - F_2\cdot 1\,\text{ft} + C_y\cdot 7\,\text{ft} \Rightarrow \boxed{C_y = 429\,\text{lb}}$

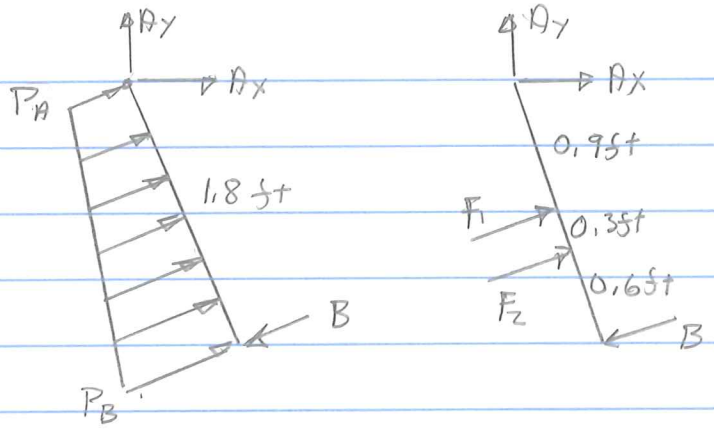
$\Sigma M_C = 0 = F_1\cdot 8\,\text{ft} + F_2\cdot 6\,\text{ft} - B_y\cdot 7\,\text{ft} \Rightarrow \boxed{B_y = 3771\,\text{lb}}$

check $\Sigma F_y = 0 = B_y + C_y - F_1 - F_2 = 0$ ✓

$\Sigma F_x = 0 \Rightarrow \boxed{B_x = 0}$

5.90

FBD:



$p_A = 62.4\ lb/ft^3 \cdot 1.7ft = 106.08\ psf$

$p_B = 62.4\ lb/ft^3 \cdot (1.7ft + 1.8ft \cdot \cos 30^\circ) = 203.35\ psf$

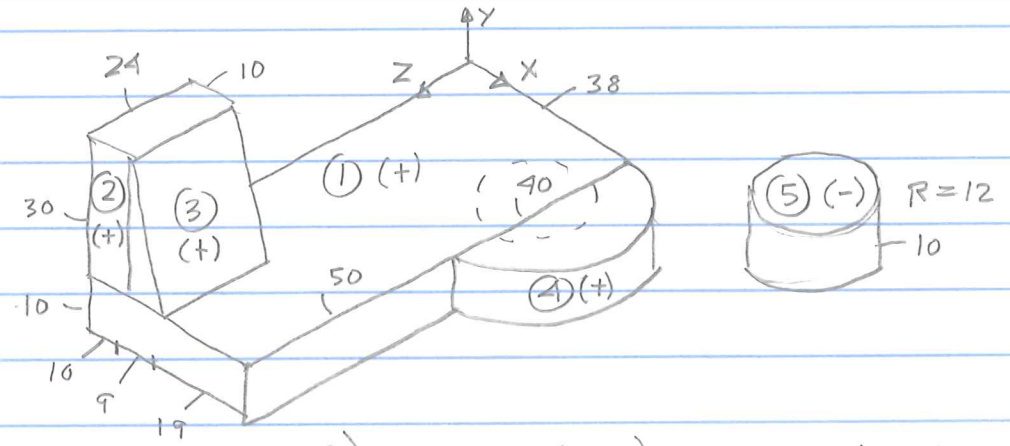
$F_1 = p_A \cdot (1.8ft)^2 = 343.70\ lb$

$F_2 = \frac{1}{2}(p_B - p_A) \cdot (1.8ft)^2 = 157.58\ lb$

$\sum M_A = 0 = F_1 \cdot 0.9ft + F_2 \cdot 1.2ft - B \cdot 1.8ft \Rightarrow \boxed{B = 277\ lb}$

direction of B is as shown (normal to gate)

5.104



i	$V_i\ (mm^3)$	$X_{ci}\ (mm)$	$V_i \cdot X_{ci}\ (mm^4)$
(1)	34200	19	649800
(2)	7200	5	36000
(3)	3240	10+3=13	42120
(4)	6283	$38 + \frac{4}{3}\frac{20}{\pi} = 46.488$	292084
(5)	-4524	38	-171,912
Totals	46399		848092

$X_c = \sum V_i \cdot X_{ci} / \sum V_i = \boxed{18.28mm}$