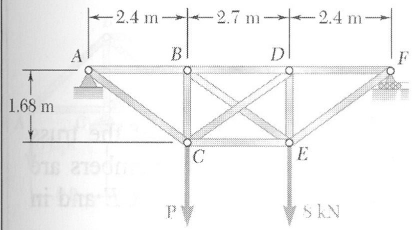
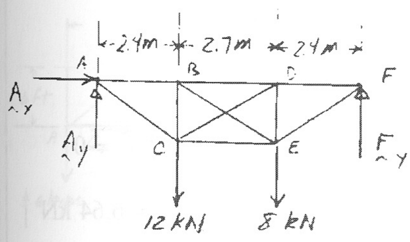


PROBLEM 6.64

The diagonal members in the center panel of the truss shown are very slender and can act only in tension; such members are known as *counters*. Determine the force in members *BD* and *CE* and in the counter which is acting when $P = 12$ kN.

SOLUTION

FBD Truss:



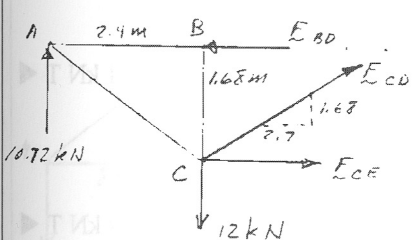
$$\rightarrow \Sigma F_x = 0: \quad \mathbf{A}_x = 0$$

$$\circlearrowleft \Sigma M_F = 0: \quad (2.4 \text{ m})(8 \text{ kN}) + (5.1 \text{ m})(12 \text{ kN})$$

$$-(7.5 \text{ m})A_y = 0, \qquad \mathbf{A}_y = 10.72 \text{ kN} \uparrow$$

Since only *CD* can provide an upward force necessary for equilibrium, it must be in tension, and $F_{BE} = 0$

FBD Section *ABC*:



$$\uparrow \Sigma F_y = 0: \quad 10.72 \text{ kN} - 12 \text{ kN} + \frac{1.68}{3.18} F_{CD} = 0$$

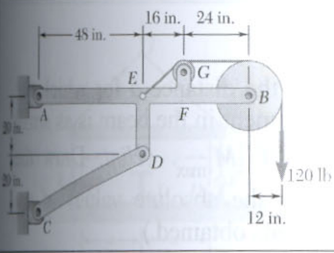
$$F_{CD} = 2.4229 \text{ kN}, \qquad F_{CD} = 2.42 \text{ kN T} ◀$$

$$\circlearrowleft \Sigma M_C = 0: \quad (1.68 \text{ m})F_{BD} - (2.4 \text{ m})(10.72 \text{ kN}) = 0$$

$$F_{BD} = 15.3143 \text{ kN}, \qquad F_{BD} = 15.31 \text{ kN C} ◀$$

$$\rightarrow \Sigma F_x = 0: \quad F_{CE} + \frac{2.7}{3.18}(2.4229 \text{ kN}) - 15.3143 \text{ kN} = 0$$

$$F_{CE} = 13.26 \text{ kN T} ◀$$

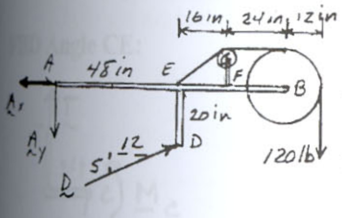


PROBLEM 7.50

Neglecting the size of the pulley at G, (*a*) draw the shear and bending-moment diagrams for the beam AB, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

FBD AB + Pulley & Cord:



$$\circlearrowleft \Sigma M_A = 0: \ (48 \text{ in.})\left(\frac{5}{13}D\right) + (20 \text{ in.})\left(\frac{12}{13}D\right) - (100 \text{ in.})(120 \text{ lb}) = 0$$

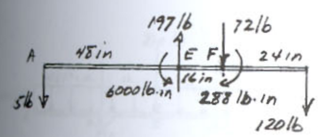
$\mathbf{D} = 325 \text{ lb} \nearrow$ so $\mathbf{D}_x = 300 \text{ lb} \longrightarrow$, $\mathbf{D}_y = 125 \text{ lb} \uparrow$

$\uparrow \Sigma F_y = 0: \ -A_y + 125 \text{ lb} - 120 \text{ lb} = 0 \qquad \mathbf{A}_y = 5 \text{ lb} \downarrow$

Neglecting the diameter of pulley G, the cord EG has slope 3/4,

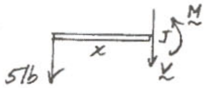
and tension 120 lb, $\mathbf{E}_x = 96 \text{ lb} \longrightarrow$, $\qquad \mathbf{E}_y = 72 \text{ lb} \uparrow$

(*a*)



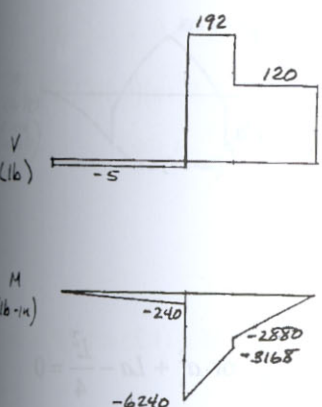
Beam AB with forces at D and G replaced by forces and couples at E and F. Horizontal forces are omitted to avoid clutter.

Along AE:

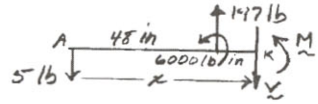


$\uparrow \Sigma F_y = 0: \quad -5 \text{ lb} - V = 0, \qquad V = -5 \text{ lb}$

$\circlearrowleft \Sigma M_J = 0: \ x(5 \text{ lb}) + M = 0, \qquad M = (-5 \text{ lb})x$



Along EF:

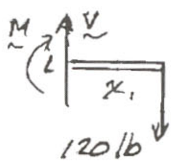


$\uparrow \Sigma F_y = 0: \quad -5 \text{ lb} + 197 \text{ lb} - V = 0, \qquad V = 192 \text{ lb}$

$\circlearrowleft \Sigma M_K = 0: \quad M + 6000 \text{ lb} \cdot \text{in.} + x(5 \text{ lb}) - (x - 48 \text{ in.})(197 \text{ lb}) = 0$

$$M = (192 \text{ lb})x - 15456 \text{ lb} \cdot \text{in.}$$

Along FB:



$\uparrow \Sigma F_y = 0: \quad V - 120 \text{ lb} = 0, \ V = 120 \text{ lb}$

$\circlearrowleft \Sigma M_L = 0: \ -M - x_1(120 \text{ lb}) = 0$

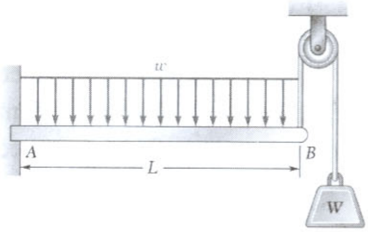
$$M = -(120 \text{ lb})x_1$$

(*b*)

From diagrams, $\qquad |V|_{max} = 192.0 \text{ lb along } EF \blacktriangleleft$

$|M|_{max} = 6240 \text{ lb} \cdot \text{in.} = 520 \text{ lb} \cdot \text{ft at } E \blacktriangleleft$

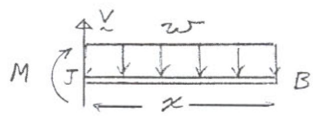
PROBLEM 7.57



In order to reduce the bending moment in the cantilever beam AB, a cable and counterweight are permanently attached at end B. Determine the magnitude of the counterweight for which the maximum absolute value of the bending moment in the beam is as small as possible and the corresponding value of $|M|_{max}$. Consider (*a*) the case when the distributed load is permanently applied to the beam, (*b*) the more general case when the distributed load may either be applied or removed.

SOLUTION

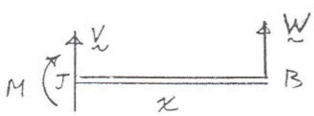
M due to distributed load:



$$\circlearrowleft \Sigma M_J = 0: \quad -M - \frac{x}{2}wx = 0$$

$$M = -\frac{1}{2}wx^2$$

M due to counter weight:

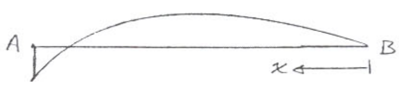


$$\circlearrowleft \Sigma M_J = 0: \quad -M + xw = 0$$

$$M = wx$$

(*a*) **Both applied:**

$$M = W_x - \frac{w}{2}x^2 \qquad \frac{dM}{dx} = W - wx = 0 \text{ at } x = \frac{W}{w}$$



And here $M = \dfrac{W^2}{2w} > 0$ so M_{max}; M_{min} must be at $x = L$

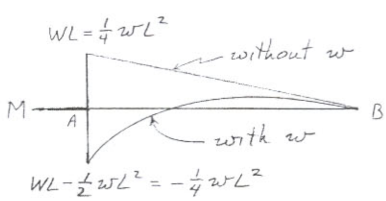
So $M_{min} = WL - \dfrac{1}{2}wL^2$. For minimum $|M|_{max}$ set $M_{max} = -M_{min}$, so

$$\frac{W^2}{2w} = -WL + \frac{1}{2}wL^2 \text{ or } W^2 + 2wLW - w^2L^2 = 0$$

$$W = -wL \pm \sqrt{2w^2L^2} \text{(need+)} \qquad W = \left(\sqrt{2} - 1\right)wL = 0.414wL \blacktriangleleft$$

$$M_{max} = \frac{W^2}{2w} = \frac{\left(\sqrt{2}-1\right)^2}{2}wL^2 \qquad M_{max} = 0.858wL^2 \blacktriangleleft$$

(*b*) ***w*** **may be removed**



Without w, $\qquad M = Wx,\ M_{max} = WL$ at A

With w (see part *a*) $\qquad M = Wx - \dfrac{w}{2}x^2, \qquad M_{max} = \dfrac{W^2}{2w}$ at $x = \dfrac{W}{w}$

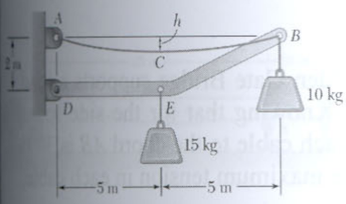
$$M_{min} = WL - \frac{1}{2}wL^2 \text{ at } x = L$$

PROBLEM 7.57 CONTINUED

For minimum M_{max}, set $M_{max}(\text{no } w) = -M_{min}(\text{with } w)$

$$WL = -WL + \frac{1}{2}wL^2 \rightarrow W = \frac{1}{4}wL \rightarrow \qquad M_{max} = \frac{1}{4}wL^2 \blacktriangleleft$$

With $\qquad W = \frac{1}{4}wL \blacktriangleleft$

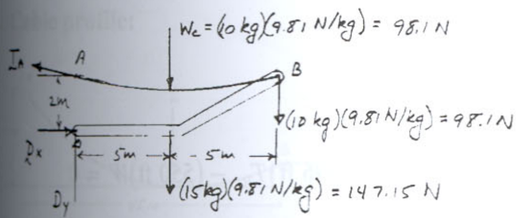


PROBLEM 7.109

The total mass of cable ACB is 10 kg. Assuming that the mass of the cable is distributed uniformly along the horizontal, determine (*a*) the sag h, (*b*) the slope of the cable at A.

SOLUTION

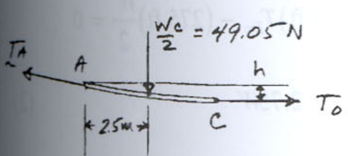
FBD whole:



$$\circlearrowleft \Sigma M_D = 0: \quad (2\text{m})T_{Ax} - (5\text{ m})(98.1 + 147.15)\text{ N} = 0$$

$$T_{Ax} = 1103.6\text{ N}$$

FBD half-cable:



$$\rightarrow \Sigma F_x = 0: \quad T_0 - T_{Ax} = 0, \qquad T_0 = 1103.6\text{ N}$$

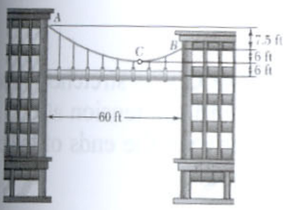
$$\uparrow \Sigma F_y = 0: \quad T_{Ay} = 49.05\text{ N} = 0, \qquad T_{Ay} = 49.05\text{ N}$$

$$\circlearrowleft \Sigma M_A = 0: \quad h(1103.6\text{ N}) - (2.5\text{ m})(49.05\text{ N}) = 0$$

$$h = 0.11111\text{ m}$$

(*a*) $h = 111.1\text{ mm}$ ◀

(*b*) $\theta = \tan^{-1}\frac{T_{Ay}}{T_{Ax}} = \tan^{-1}\frac{49.05}{1103.6} = 2.5449°, \qquad \theta = 2.54°$ ◀

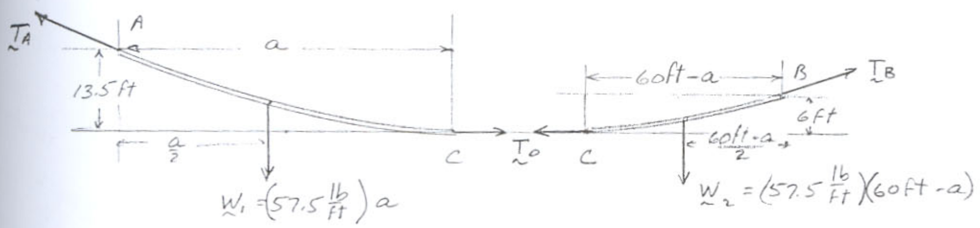


PROBLEM 7.157

A steam pipe weighting 50 lb/ft that passes between two buildings 60 ft apart and is supported by a system of cables as shown. Assuming that the weight of the cable is equivalent to a uniformly distributed loading of 7.5 lb/ft, determine (*a*) the location of the lowest point *C* of the cable, (*b*) the maximum tension in the cable.

UTION

AC: **FBD CB:**



$$\circlearrowleft \Sigma M_A = 0: (13.5 \text{ ft})T_0 - \frac{a}{2}(57.5 \text{ lb/ft})a = 0$$

$$T_0 = \left(2.12963 \text{ lb/ft}^2\right)a^2 \qquad (1)$$

$$\circlearrowleft \Sigma M_B = 0: \frac{60 \text{ ft} - a}{2}(57.5 \text{ lb/ft})(60 \text{ ft} - a) - (6 \text{ ft})T_0 = 0$$

$$6T_0 = \left(28.75 \text{ lb/ft}^2\right)\left[3600 \text{ ft}^2 - (120 \text{ ft})a + a^2\right] \qquad (2)$$

Using (1) in (2), $0.55a^2 - (120 \text{ ft})a + 3600 \text{ ft}^2 = 0$

Solving: $a = (108 \pm 72)$ ft, $a = 36$ ft (180 ft out of range)

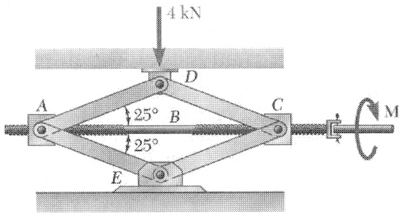
So *C* is 36 ft from *A*

(*a*) *C* is 6 ft below and 24 ft left of *B* ◀

$$T_0 = 2.1296 \text{ lb/ft}^2(36 \text{ ft})^2 = 2760 \text{ lb}$$

$$W_1 = (57.5 \text{ lb/ft})(36 \text{ ft}) = 2070 \text{ lb}$$

(*b*) $T_{\max} = T_A = \sqrt{T_0^2 + W_1^2} = \sqrt{(2760 \text{ lb})^2 + (2070 \text{ lb})^2} = 3450 \text{ lb}$ ◀

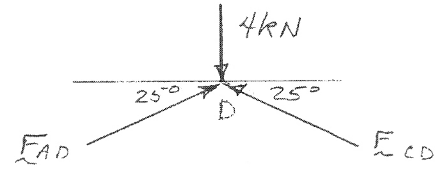


PROBLEM 8.70

The position of the automobile jack shown is controlled by a screw *ABC* that is single-threaded at each end (right-handed thread at *A*, left-handed thread at *C*). Each thread has a pitch of 2 mm and a mean diameter of 7.5 mm. If the coefficient of static friction is 0.15, determine the magnitude of the couple **M** that must be applied to raise the automobile.

SOLUTION

FBD joint *D*:



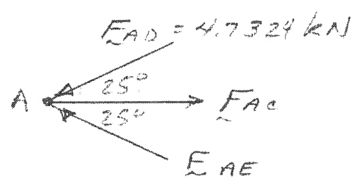
By symmetry:

$$F_{AD} = F_{CD}$$

$$\uparrow \Sigma F_y = 0: \quad 2F_{AD}\sin 25° - 4\text{ kN} = 0$$

$$F_{AD} = F_{CD} = 4.7324\text{ kN}$$

FBD joint *A*:



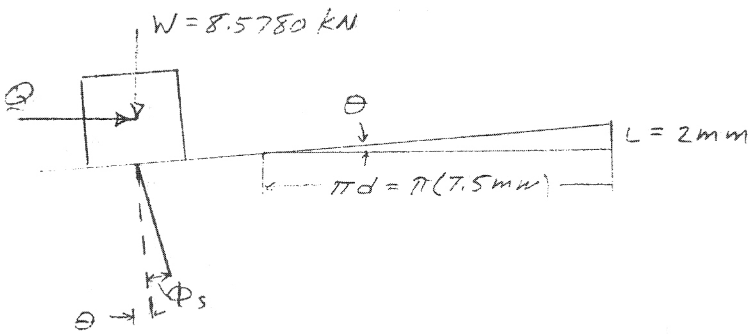
By symmetry:

$$F_{AE} = F_{AD}$$

$$\rightarrow \Sigma F_x = 0: \quad F_{AC} - 2(4.7324\text{ kN})\cos 25° = 0$$

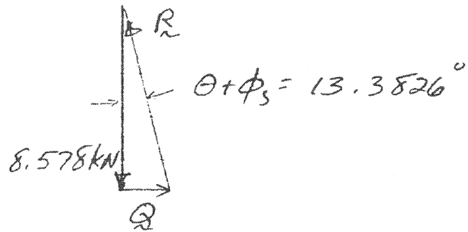
$$F_{AC} = 8.5780\text{ kN}$$

Block and incline *A*:



$$\theta = \tan^{-1}\frac{2\text{ mm}}{\pi(7.5\text{ mm})} = 4.8518°$$

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.15 = 8.5308°$$



$$Q = (8.578 \text{ kN})\tan(13.3826°)$$

$$= 2.0408 \text{ kN}$$

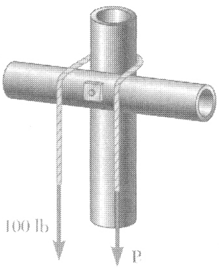
Couple at A: $$M_A = rQ$$

$$= \left(\frac{7.5}{2} \text{ mm}\right)(2.0408 \text{ kN})$$

$$= 7.653 \text{ N}\cdot\text{m}$$

By symmetry: Couple at C: $$M_C = 7.653 \text{ N}\cdot\text{m}$$

Total couple $M = 2(7.653 \text{ N}\cdot\text{m})$ $$M = 15.31 \text{ N}\cdot\text{m}$$ ◀

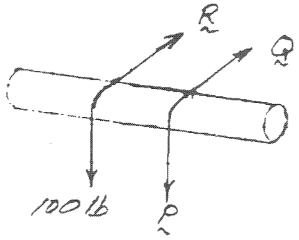


PROBLEM 8.106

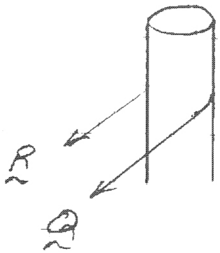
Knowing that the coefficient of static friction is 0.30 between the rope and the horizontal pipe and that the smallest value of P for which equilibrium is maintained is 20 lb, determine (*a*) the largest value of P for which equilibrium is maintained, (*b*) the coefficient of static friction between the rope and the vertical pipe.

SOLUTION

Horizontal pipe



Vertical pipe



Contact angles $\beta_H = \dfrac{\pi}{2}$

$\mu_{sH} = 0.30$

Contact angle $\beta_V = \pi$

$\mu_{sV} = ?$

For P_{min}, the 100 lb force impends downward, and

$$100 \text{ lb} = \left(e^{\mu_{sH}\frac{\pi}{2}}\right)R = \left(e^{\mu_{sH}\frac{\pi}{2}}\right)\left(e^{\mu_{sV}\pi}\right)Q = \left(e^{\mu_{s\pi}\frac{\pi}{2}}\right)\left(e^{\mu_{sV}\pi}\right)\left(e^{\mu_{sH}\frac{\pi}{2}}\right)P$$

$$100 \text{ lb} = \left[e^{\pi(0.30+\mu_{sV})}\right](20 \text{ lb}), \text{ so } e^{\pi(0.30+\mu_{sV})} = 5$$

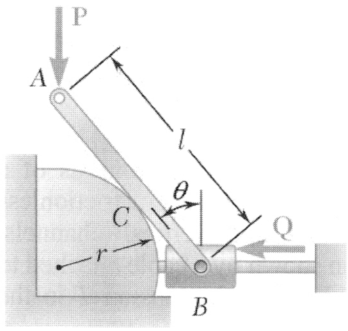
(*a*) For P_{max} the force **P** impends downward, and the ratios are reversed, so $P_{max} = 5(100 \text{ lb}) = 500 \text{ lb}$ ◀

(*b*) $\pi(0.30 + \mu_{sV}) = \ln 5$

$$\mu_{sV} = \frac{1}{\pi}\ln 5 - 0.30 = 0.21230$$

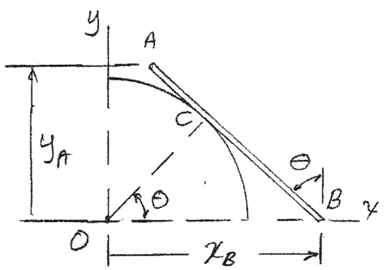
$\mu_{sV} = 0.212$ ◀

PROBLEM 10.26



A slender rod of length l is attached to a collar at B and rests on a portion of a circular cylinder of radius r. Neglecting the effect of friction, determine the value of θ corresponding to the equilibrium position of the mechanism when $l = 14$ in., $r = 5$ in., $P = 75$ lb, and $Q = 150$ lb.

SOLUTION



Geometry

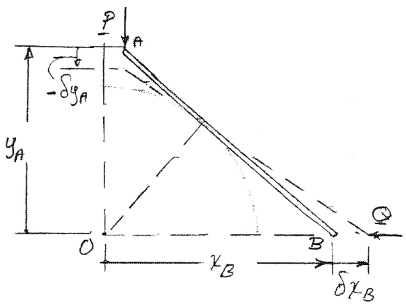
$$OC = r$$

$$\cos\theta = \frac{OC}{OB} = \frac{r}{x_B}$$

$$x_B = \frac{r}{\cos\theta}$$

$$\delta x_B = \frac{r\sin\theta}{\cos^2\theta}\delta\theta$$

$$y_A = l\cos\theta; \qquad \delta y_A = -l\sin\theta\,\delta\theta$$



Virtual Work:

$$\delta U = 0: \quad P(-\delta y_A) - Q\delta x_B = 0$$

$$Pl\sin\theta\,\delta\theta - Q\frac{r\sin\theta}{\cos^2\theta}\delta\theta = 0$$

$$\cos^2\theta = \frac{Qr}{Pl} \tag{1}$$

Then, with $l = 14$ in., $r = 5$ in., $P = 75$ lb, and $Q = 150$ lb

$$\cos^2\theta = \frac{(150\text{ lb})(5\text{ in.})}{(75\text{ lb})(14\text{ in.})} = 0.7143$$

or $\theta = 32.3115°$ $\qquad$ $\theta = 32.3°$ ◀