Question 1)
Rod AB of length “l” and negligible weight lies in a vertical plane and is pinned to blocks A and B. The weight of each block is W, and the blocks are connected by an elastic cord which passes over a pulley at C. Neglecting friction between the blocks and the guides, determine the value of $\theta$ as a function of the tension in the cord, $T$ (known).

SOLUTION

Free-Body Diagram:

\[ + \sum M_D = 0: \quad \left[ l \cos(\theta + 30^\circ) \cos 30^\circ \right] W - \left[ l \cos(\theta + 30^\circ) \right] T + \left[ l \sin(\theta + 30^\circ) \cos 60^\circ \right] W + \left[ l \sin(\theta + 30^\circ) \right] T = 0 \]

or $W \cos(\theta + 60^\circ) + T \left[ \sin(\theta + 30^\circ) - \cos(\theta + 30^\circ) \right] = 0$

Using $\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$

$\sin(a+b) = \sin(a)\cos(b) + \cos(b)\sin(a)$

$W*(\cos(\theta)\cos(60) - \sin(\theta)\sin(60)) + T*(\sin(\theta)\cos(30) + \cos(\theta)\sin(30) - \cos(\theta)\cos(30) + \sin(\theta)\sin(30)) = 0$

$\Rightarrow -W\tan(\theta)\sin(60) + T\tan(\theta)(\cos(30) + \sin(30)) + W\cos(60) + T*(\sin(30) - \cos(30)) = 0$

$\Rightarrow \tan(\theta)(-W\sin(60) + T*(\cos(30) + \sin(30))) = T*(\cos(30) - \sin(30)) - W\cos(60)$

$\Rightarrow \theta = \tan^{-1}\left(\frac{T*(\cos(30) - \sin(30)) - W\cos(60)}{-W\sin(60) + T*(\cos(30) + \sin(30))}\right) = \tan^{-1}\left(\frac{T*(1 - \sqrt{3}) - W}{-W\sqrt{3} - T*(\sqrt{3} + 1)}\right)$
**Question 2)**
Knowing that \( w = 300 \text{ N/m} \), determine

a) The resultant concentrated load on the beam and its location.

b) The smallest distance \( a \) for which the vertical reaction at support B is 2.5 times that at A.

**SOLUTION**

The distributed load can be represented in terms of resultants:

\[
R_1 = (8 \text{ m})(300 \text{ N/m}) = 2400 \text{ N}
\]

\[
R_2 = \frac{1}{2}(8 - a) \times 2400 \text{ N/m} = 1200(8 - a) \text{ N}
\]

For equilibrium:

\[
\Sigma M_B = 0: \quad -8A_y + 4(2400) + \left[ \frac{1}{3}(8 - a) \right] [1200(8 - a)] = 0
\]

\[
A_y = 1200 + 50(8 - a)^2 \quad \ldots(1)
\]

\[
\Sigma M_A = 0: \quad 8B_y - 4(2400) - \left[ a + \frac{2}{3}(8 - a) \right] [1200(8 - a)] = 0
\]

\[
B_y = 1200 + 50(16 + a)(8 - a) \quad \ldots(2)
\]

(a) \( \Sigma F_y = 0: \quad A_y + B_y - 2400 - 1200(8 - a) = 0 \quad \ldots(3)
\]

The resultant concentrated load has magnitude \( 2400 + 1200(8-a) \)

\[
\Rightarrow \quad R_{\text{tot}} = 1200(10-a)
\]

The location of the resultant can be found by letting \( z \) be the horizontal distance from \( R_1 \):

\[
R_1 \times z = R_2 \times \left( (4 - (8-a)/3) - z \right)
\]

So

\[
z = \frac{R_2 \times (4 - (8-a)/3)}{R_1 + R_2} = \frac{1200(8-a) \times (12 - (8-a))}{1200 \times 3 \times (10-a)} = \frac{32 + 4a - a^2}{3 \times (10 - a)}
\]
Note that $4 + z$ is the horizontal distance from $A$ => total distance $= \frac{a^2 + 8a - 152}{3(a - 10)}$

b) To find $a$ where $B_y = 2.5A_y$, just set them equal:

$$(2.5) \times (1200 + 50 \times (8 - a)^2) = 1200 + 50(16 + a) \times (8 - a)$$

$\Rightarrow a = 3.36m, 5.78m$ (3.36 is smaller)