PROBLEM 8.14

The 20-lb block A and the 40-lb block B are at rest on an incline as shown. Knowing that the coefficient of static friction is 0.25 between all surfaces of contact, determine the value of $\theta$ for which motion is impending.

SOLUTION

FBD's:

A:

\[ \Sigma F_y = 0: \quad N_1 - 20 \text{ lb} = 0, \quad N_1 = 20 \text{ lb} \]

Impending slip: \[ F_1 = \mu_s N_1 = (0.25)(20 \text{ lb}) = 5 \text{ lb} \]

\[ \Sigma F_x = 0: \quad -T + 5 \text{ lb} = 0, \quad T = 5 \text{ lb} \]

\[ \Sigma F_y' = 0: \quad N_2 - (20 \text{ lb} + 40 \text{ lb}) \cos \theta - (5 \text{ lb}) \sin \theta = 0 \]

\[ N_2 = (60 \text{ lb}) \cos \theta - (5 \text{ lb}) \sin \theta \]

Impending slip: \[ F_2 = \mu_s N_2 = (0.25)(60 \cos \theta - 5 \sin \theta) \text{ lb} \]

\[ \Sigma F_x' = 0: \quad -F_2 - 5 \text{ lb} - (5 \text{ lb}) \cos \theta + (20 \text{ lb} + 40 \text{ lb}) \sin \theta = 0 \]

\[ -20 \cos \theta + 58.75 \sin \theta - 5 = 0 \]

Solving numerically, \[ \theta = 23.4^\circ \]
PROBLEM 8.67

The square-threaded worm gear shown has a mean radius of 1.5 in. and a lead of 0.375 in. The larger gear is subjected to a constant clockwise couple of 7.2 kip·in. Knowing that the coefficient of static friction between the two gears is 0.12, determine the couple that must be applied to shaft $AB$ to rotate the large gear counterclockwise. Neglect friction in the bearings at $A$, $B$, and $C$.

SOLUTION

FBD large gear:

\[ \Sigma M_C = 0: \quad (12 \text{ in.}) W - 7.2 \text{ kip·in.} = 0, \quad W = 0.600 \text{ kips} \]
\[ = 600 \text{ lb} \]

Block on incline:

\[ \theta = \tan^{-1} \frac{0.375 \text{ in.}}{2\pi (1.5 \text{ in.})} = 2.2785^\circ \]
\[ \phi_s = \tan^{-1} \mu_s = \tan^{-1} 0.12 = 6.8428^\circ \]
\[ Q = W \tan(\theta + \phi_s) \]
\[ = (600 \text{ lb}) \tan 9.1213^\circ = 96.333 \text{ lb} \]

FBD worm gear:

\[ \Sigma M_B = 0: \quad (1.5 \text{ in.})(96.333 \text{ lb}) - M = 0 \]
\[ M = 144.5 \text{ lb·in.} \]
PROBLEM 8.114

A differential band brake is used to control the speed of a drum. Determine the minimum value of the coefficient of static friction for which the brake is self-locking when the drum rotates counterclockwise.

SOLUTION

FBD Lever:

If brake is self-locking, no force $P$ is required

\[ \Sigma M_B = 0: \quad (2 \text{ in.})T_C - (7.5 \text{ in.})T_A = 0 \]

\[ T_C = 3.75T_A \]

For impending slip on drum: $T_C = T_A e^{\mu_s \beta}$

\[ : \quad e^{\mu_s \beta} = 3.75, \quad \text{or} \quad \mu_s = \frac{1}{\beta} \ln(3.75) \]

With $\beta = \frac{7\pi}{6}$,

\[ \mu_s = 0.361 \]

PROBLEM 10.37

Knowing that the constant of spring $CD$ is $k$ and that the spring is unstretched when $\theta = 0$, determine the value of $\theta$, where $0 \leq \theta \leq 90^\circ$, corresponding to equilibrium for the given data.

$P = 600 \text{ N}$, $l = 800 \text{ mm}$, $k = 4 \text{ kN/m}$

SOLUTION

From geometry:

$y_A = l \sin \theta$

$\delta y_A = l \cos \theta \delta \theta$

$x_C = l \cos \theta + l \sin \theta$

$= l (\cos \theta + \sin \theta)$

$y_C = l \sin \theta - l \cos \theta$

$= l (\sin \theta - \cos \theta)$

$l_{CD} = l \sqrt{(\cos \theta + \sin \theta)^2 + [(\sin \theta - \cos \theta) - (-1)]^2}$

$= l \sqrt{3 + 2 \sin \theta - 2 \cos \theta}$

$\delta l_{CD} = \frac{\cos \theta + \sin \theta}{\sqrt{3 + 2 \sin \theta - 2 \cos \theta}} \delta \theta$

and

$F_{SP} = k (l_{CD} - l)$

$= kl \left(\sqrt{3 + 2 \sin \theta - 2 \cos \theta} - 1\right)$

Virtual Work:

$\frac{\delta U}{\delta \theta} = 0: \quad P \delta y_A - F_{SP} \delta l_{CD} = 0$

or

$P (l \cos \theta \delta \theta) - kl \left(\sqrt{3 + 2 \sin \theta - 2 \cos \theta} - 1\right) \left[l \frac{\cos \theta + \sin \theta}{\sqrt{3 + 2 \sin \theta - 2 \cos \theta}} \delta \theta\right] = 0$

or

$\left(1 - \frac{1}{\sqrt{3 + 2 \sin \theta - 2 \cos \theta}}\right) (1 + \tan \theta) = \frac{P}{kl}$

$= \frac{600 \text{ N}}{(4000 \text{ N/m})(0.8 \text{ m})}$

$= 0.1875$

Solving numerically

$\theta = 10.77^\circ$
PROBLEM 10.50

Denoting by $\mu_s$ the coefficient of static friction between the block attached to rod ACE and the horizontal surface, derive expressions in terms of $P$, $\mu_s$, and $\theta$ for the largest and smallest magnitudes of the force $Q$ for which equilibrium is maintained.

SOLUTION

For the linkage:

$$+\Sigma M_B = 0: \quad -x_A + \frac{x_A}{2}P = 0 \quad \text{or} \quad A = \frac{P}{2}$$

Then:

$$F = \mu_s A = \mu_s \frac{P}{2} = \frac{1}{2} \mu_s P$$

Now

$$x_A = 2l \sin \theta$$

$$\delta x_A = 2l \cos \theta \delta \theta$$

and

$$y_F = 3l \cos \theta$$

$$\delta y_F = -3l \sin \theta \delta \theta$$

Virtual Work:

$$\delta U = 0: \quad (Q_{\text{max}} - F) \delta x_A + P \delta y_F = 0$$

$$\left( Q_{\text{max}} - \frac{1}{2} \mu_s P \right) (2l \cos \theta \delta \theta) + P (-3l \sin \theta \delta \theta) = 0$$

or

$$Q_{\text{max}} = \frac{3}{2} P \tan \theta + \frac{1}{2} \mu_s P$$

$$Q_{\text{max}} = \frac{P}{2} (3 \tan \theta + \mu_s)$$

For $Q_{\text{min}}$, motion of $A$ impedes to the right and $F$ acts to the left. We change $\mu_s$ to $-\mu_s$ and find

$$Q_{\text{min}} = \frac{P}{2} (3 \tan \theta - \mu_s)$$