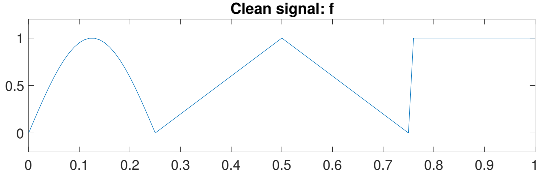
On the Frame Condition

$$A\|f\|^2 \leq \sum_{i=1}^{\infty} < f, \phi_i >^2 \leq B\|f\|^2$$

- $A > 0$: $\{\phi_i\}_{i=1}^{\infty}$ is a complete basis.
- $B < \infty$: $\{\phi_i\}_{i=1}^{\infty}$ is not "too redundant".
- Many theoretical results based on $0 < A \leq B < \infty$, such as existence of a dual frame, stability of the reconstruction, and etc...
- The smaller B/A is, the more stable the reconstruction is. In other words, the more robust the reconstruction is against the noise in the measurements $\{< f, \phi_i >\}_{i=1}^{\infty}$.

Reconstruction from noisy measurements

- Clean signal/image $f \in \mathbf{R}^{101}$.



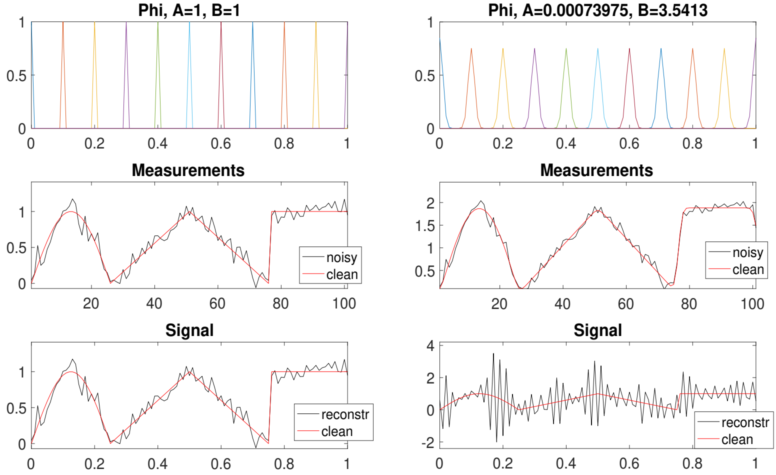
- Basis/frame $\{\phi_i\}_{i=1}^{101}$, with different (A, B)'s.
- Noisy measurements

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_{101} \end{bmatrix} = \begin{bmatrix} \phi_1^T \\ \phi_2^T \\ \vdots \\ \phi_{101}^T \end{bmatrix} f + \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_{101} \end{bmatrix}, \qquad \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_{101} \end{bmatrix} \sim \mathcal{N}(0, 0.1^2).$$

- Reconstruct the signal

$$\hat{f} = \Phi^{-1}\mathbf{y}.$$

Numerical Example



Numerical Example, continued