Introduction to the Visual System
Receptor Profiles and Gabor Frames

Matilde Marcolli

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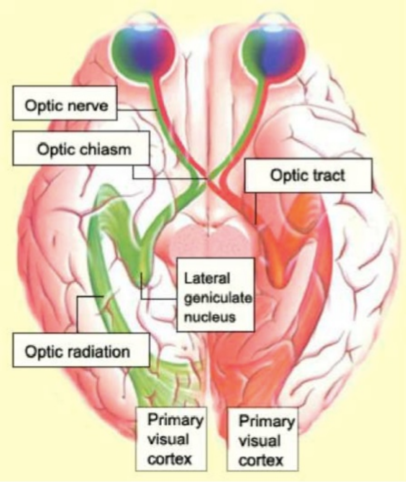
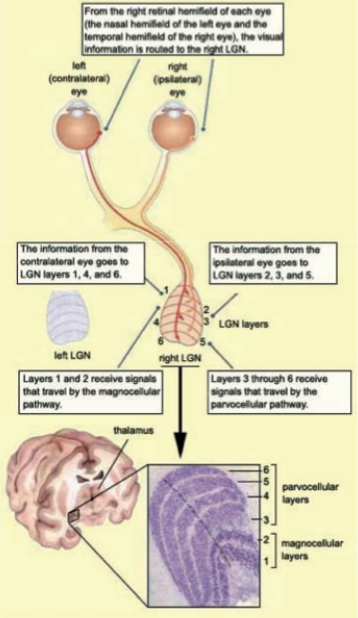
Geometry of Neuroscience

References for this lecture:

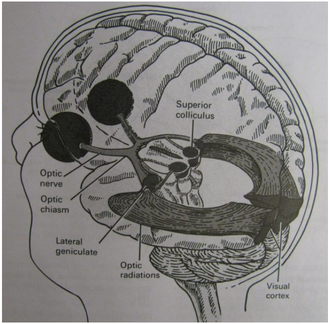
- Jean Petitot, *Neurogéométrie de la Vision*, Les Éditions de l'École Polytechnique, 2008
- D. Marr, *Vision: A Computational Investigation into the Human Representation and Processing of Visual Information* (1982), MIT Press, 2010
- S.Marcelja, *Mathematical description of the responses of simple cortical cells*, J Opt Soc Am A 70 (1980) 1297–1300
- Karlheinz Gröchenig, *Multivariate Gabor frames and sampling of entire functions of several variables*, Appl. Comput. Harmon. Anal. 31 (2011) 218–227

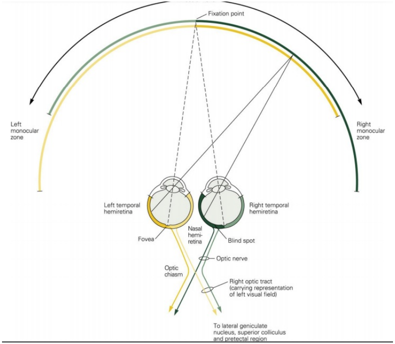
Magnocellular and Parvocellular Pathways



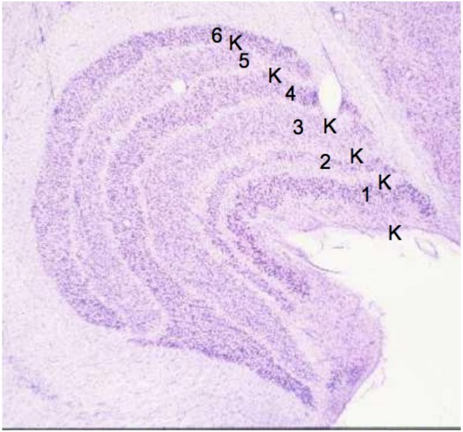


LGN: lateral geniculate nucleus



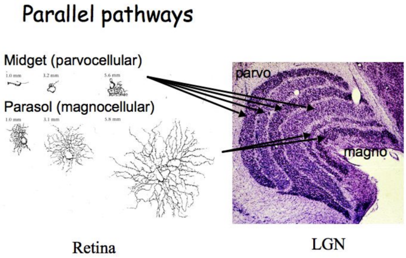


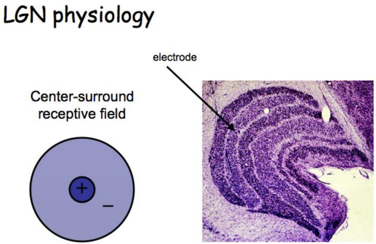
Magnocellular and Parvocellular (LGN structure)



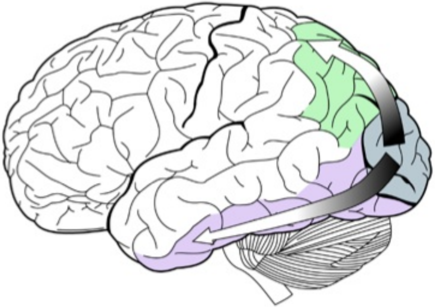
1-2: "Magno" fast and transient, large RF
2-6: "Parvo" slow and sustained, small RF
K: innervate extrastriate cortex

LGN: lateral geniculate nucleus



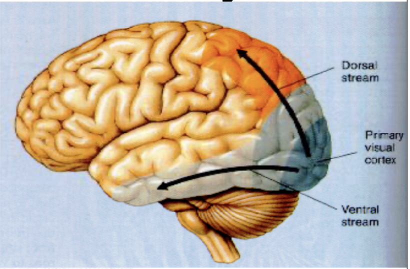


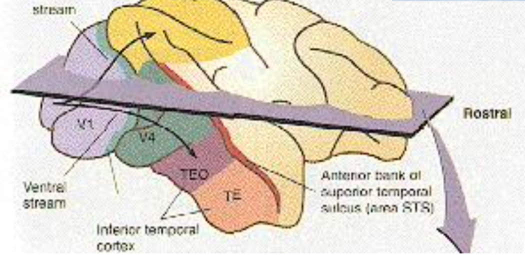
starting with the V1 cortex the magnocellular and parvocellular pathways part ways: dorsal and ventral (movement versus form recognition, where/what)



What & Where Pathways

“Vision is knowing what is where by looking” –David Marr





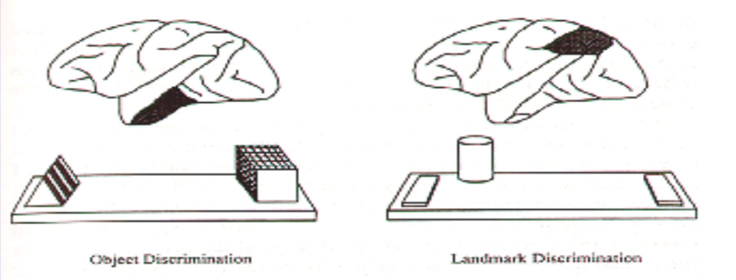
The Ungerleider & Mishkin (1982) Experiment

Task 1:
Object discrimination

- study an object
- select the familiar object (reward)

Task 2:
Landmark discrimination

- select foodwell closest to the TOWER

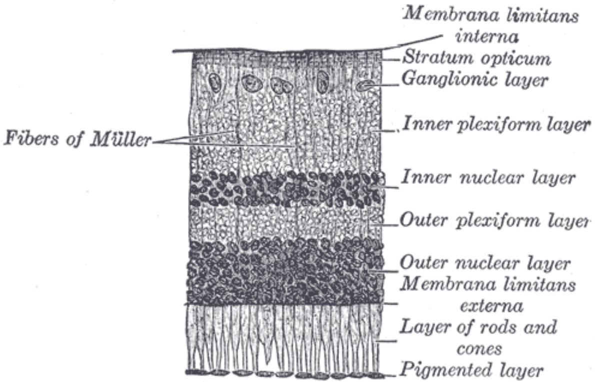


temporal lesions impair OBJECT TASK

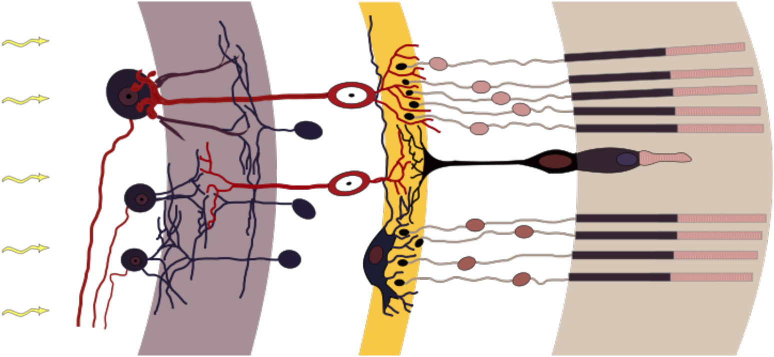
parietal lesions impair LANDMARK TASK

Layered structure of the retina

- ten layers of structure in the retina



- four main stages: photoreception, transmission to bipolar cells, transmission to ganglion cells containing photoreceptors (photosensitive ganglion cells), transmission along optic nerve



- incoming light (left) passes through nerve layers, reaches rods and cones (right) chemical change transmits signal back to bipolar and horizontal cells (yellow) and ganglion cells (purple) and to optic nerve

Visual cortex V1: Primary Visual Cortex

- D.H. Hubel, T.N. Wiesel, *Receptive fields, binocular interaction and functional architecture in the cat's visual cortex*, The Journal of Physiology. 160 (1962) 45, 106–154.

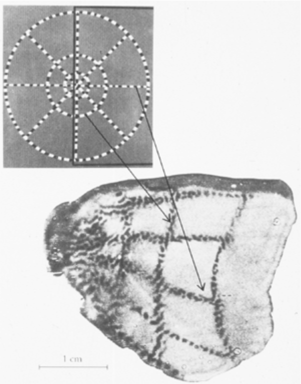
Main geometric features:

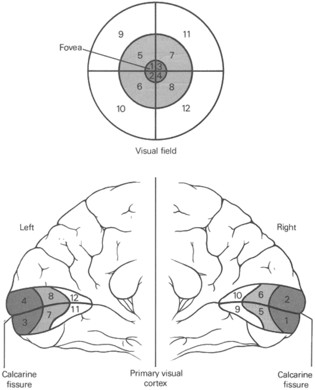
- Conformal mapping
- Receptor fields, filters
- Column structure
- Orientations

Features computed by V1

- Orientation
- Color
- Motion
- Binocular Disparity

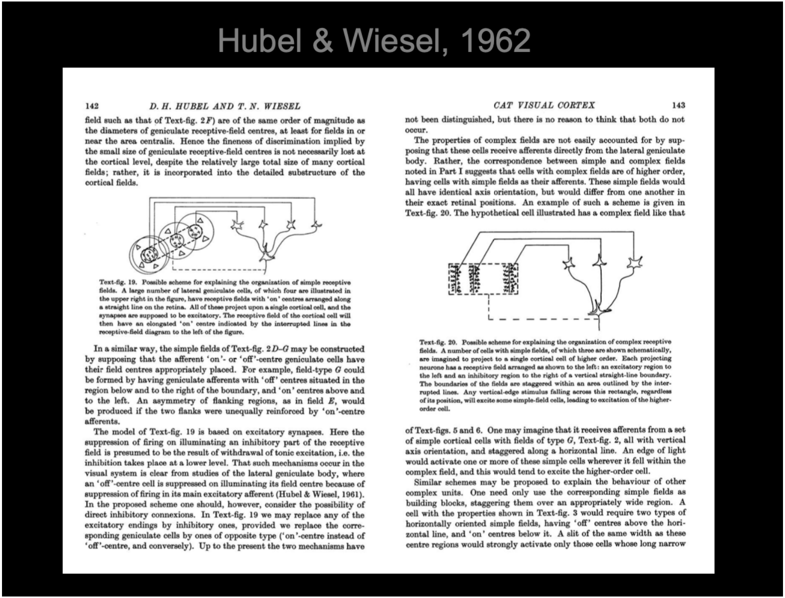
Retinotopic map





based on experiments of Tootell et al, 1982

Structure of the V1 cortex (Hubel & Wiesel 1962)



Hubel & Wiesel, 1962

field such as that of Text-fig. 2*F*) are of the same order of magnitude as the diameters of geniculate receptive-field centres, at least for fields in or near the area centralis. Hence the fineness of discrimination implied by the small size of geniculate receptive-field centres is not necessarily lost at the cortical level, despite the relatively large total size of many cortical fields; rather, it is incorporated into the detailed substructure of the cortical fields.

Text-fig. 19. Possible scheme for explaining the organization of simple receptive fields. A large number of lateral geniculate cells, of which four are illustrated in the upper right in the figure, have receptive fields with 'on' centres arranged along a straight line on the retina. All of these project upon a single cortical cell, and the synapses are supposed to be excitatory. The receptive field of the cortical cell will then have an elongated 'on' centre indicated by the interrupted lines in the receptive-field diagram to the left of the figure.

In a similar way, the simple fields of Text-fig. 2*D*–*G* may be constructed by supposing that the afferent 'on'- or 'off'-centre geniculate cells have their field centres appropriately placed. For example, field-type *G* could be formed by having geniculate afferents with 'off' centres situated in the region below and to the right of the boundary, and 'on' centres above and to the left. An asymmetry of flanking regions, as in field *E*, would be produced if the two flanks were unequally reinforced by 'on'-centre afferents.

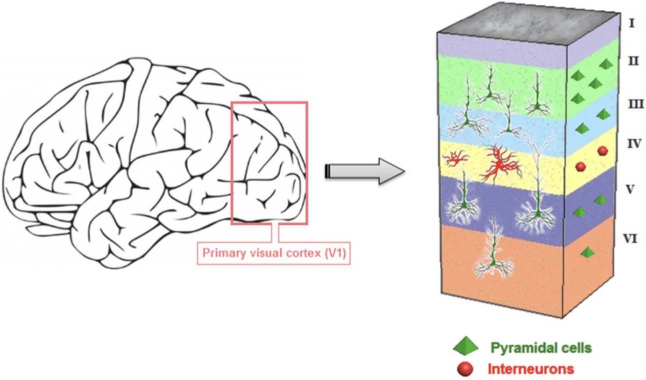
The model of Text-fig. 19 is based on excitatory synapses. Here the suppression of firing on illuminating an inhibitory part of the receptive field is presumed to be the result of withdrawal of tonic excitation, i.e. the inhibition takes place at a lower level. That such mechanisms occur in the visual system is clear from studies of the lateral geniculate body, where an 'off'-centre cell is suppressed on illuminating its field centre because of suppression of firing in its main excitatory afferent (Hubel & Wiesel, 1961). In the proposed scheme one should, however, consider the possibility of direct inhibitory connexions. In Text-fig. 19 we may replace any of the excitatory endings by inhibitory ones, provided we replace the corresponding geniculate cells by ones of opposite type ('on'-centre instead of 'off'-centre, and conversely). Up to the present the two mechanisms have not been distinguished, but there is no reason to think that both do not occur.

The properties of complex fields are not easily accounted for by supposing that these cells receive afferents directly from the lateral geniculate body. Rather, the correspondence between simple and complex fields noted in Part I suggests that cells with complex fields are of higher order, having cells with simple fields as their afferents. These simple fields would all have identical axis orientation, but would differ from one another in their exact retinal positions. An example of such a scheme is given in Text-fig. 20. The hypothetical cell illustrated has a complex field like that of Text-figs. 5 and 6. One may imagine that it receives afferents from a set of simple cortical cells with fields of type *G*, Text-fig. 2, all with vertical axis orientation, and staggered along a horizontal line. An edge of light would activate one or more of these simple cells wherever it fell within the complex field, and this would tend to excite the higher-order cell.

Text-fig. 20. Possible scheme for explaining the organization of complex receptive fields. A number of cells with simple fields, of which three are shown schematically, are imagined to project to a single cortical cell of higher order. Each projecting neurone has a receptive field arranged as shown to the left: an excitatory region to the left and an inhibitory region to the right of a vertical straight-line boundary. The boundaries of the fields are staggered within an area outlined by the interrupted lines. Any vertical-edge stimulus falling across this rectangle, regardless of its position, will excite some simple-field cells, leading to excitation of the higher-order cell.

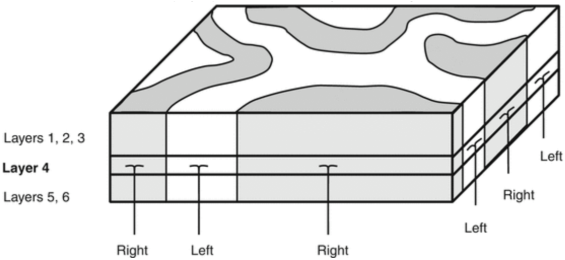
Similar schemes may be proposed to explain the behaviour of other complex units. One need only use the corresponding simple fields as building blocks, staggering them over an appropriately wide region. A cell with the properties shown in Text-fig. 3 would require two types of horizontally oriented simple fields, having 'off' centres above the horizontal line, and 'on' centres below it. A slit of the same width as these centre regions would strongly activate only those cells whose long narrow

Structure of the V1 cortex

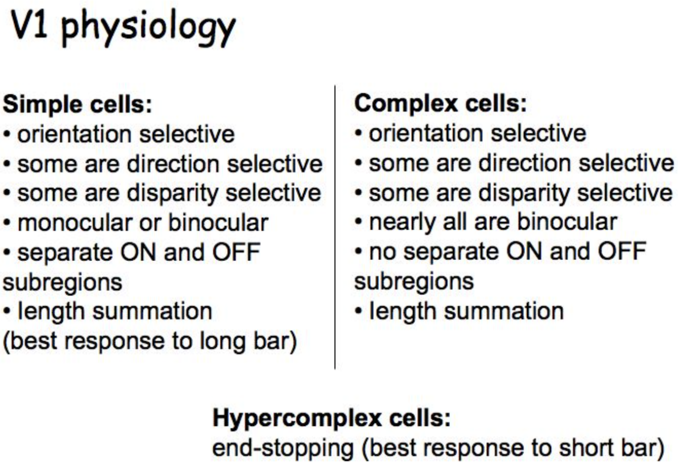


6 layers

ocular dominance columns



David Heeger's lecture notes NYU



V1 physiology

Simple cells:

- orientation selective
- some are direction selective
- some are disparity selective
- monocular or binocular
- separate ON and OFF subregions
- length summation (best response to long bar)

Complex cells:

- orientation selective
- some are direction selective
- some are disparity selective
- nearly all are binocular
- no separate ON and OFF subregions
- length summation

Hypercomplex cells:
end-stopping (best response to short bar)

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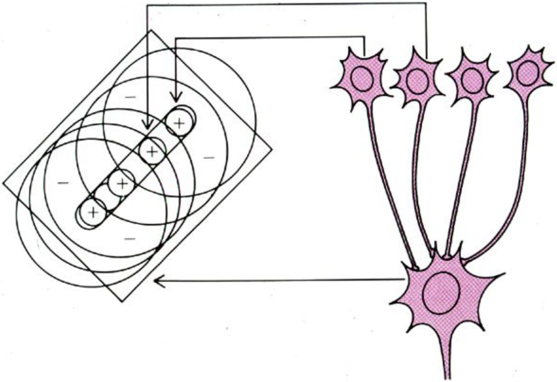
Simple cell sums LGN inputs



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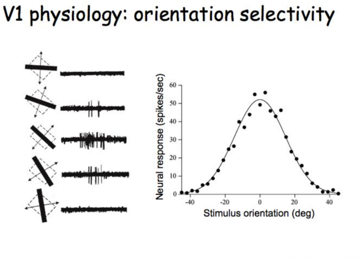
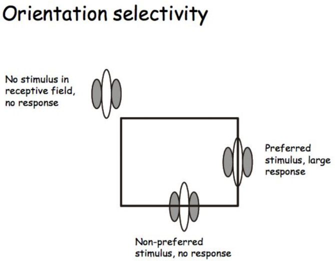
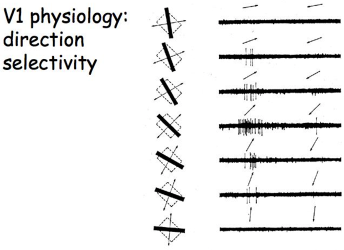
Orientation sensitivity

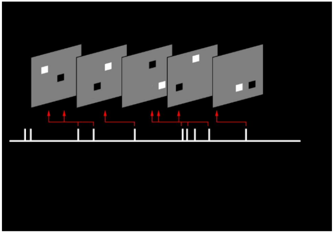


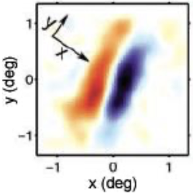
Hubel & Wiesel, 1968





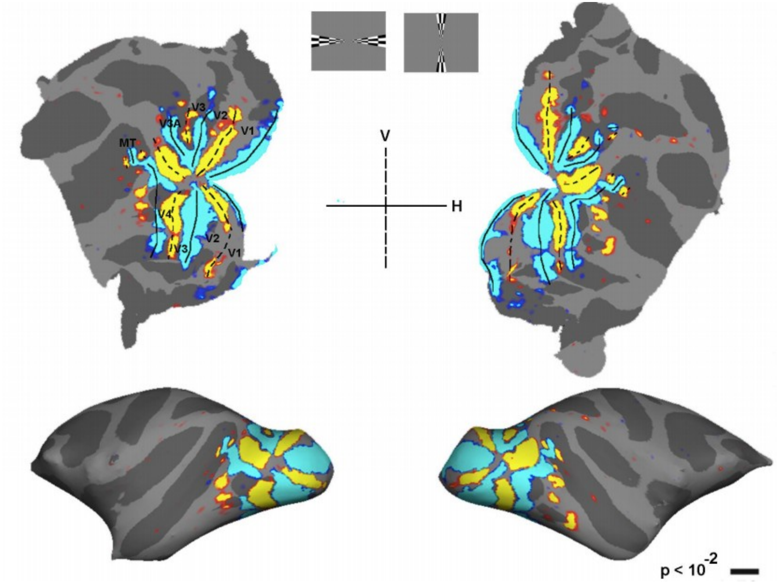
most V1 neurons are sensitive to a specific orientation, some also to a direction (oriented lines moving in a specific direction)



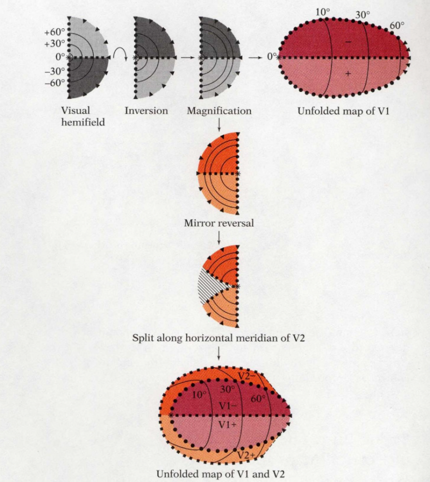


Receptor profiles sensitive to orientation: what type of signal analysis is performed by the V1 cortex? Gabor frames appears to be the mathematical formulation that captures the signal analysis properties of the V1 neurons

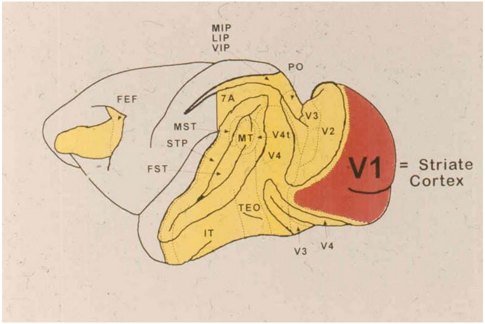
Visual system after V1

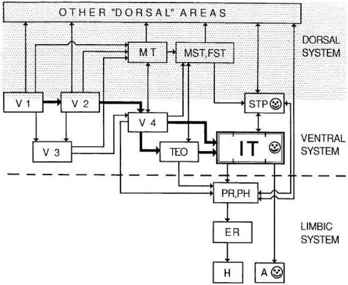


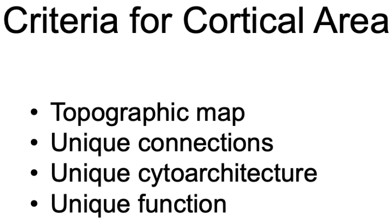
(Doris Tsao)


+60°
+30°
0°
−30°
−60°
Visual hemifield
Inversion
Magnification
10°
30°
60°
0°
−
+
Unfolded map of V1
Mirror reversal
Split along horizontal meridian of V2
V2−
10°
30°
60°
V1−
V1+
V2+
Unfolded map of V1 and V2

Inferior Temporal (IT) Cortex and Pathways







Criteria for Cortical Area

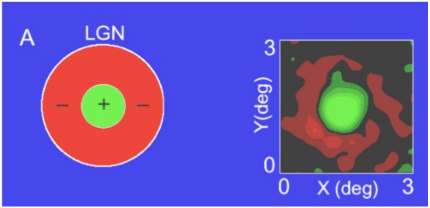
- Topographic map
- Unique connections
- Unique cytoarchitecture
- Unique function

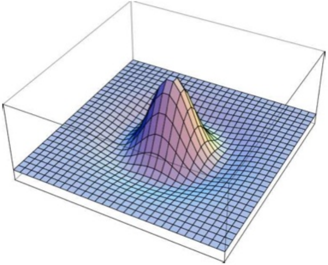
We'll mostly focus on the V1 part of the visual system looking at the interaction of geometry and signal analysis

Receptor Field

- the receptor field (RF) of a neuron in the visual cortex is an area D of the retina to which it responds, emitting a spike train
- receptor field has zones (ON) that respond positively to pointwise light stimuli and other zones (OFF) that respond negatively
- receptive profile (RP): a function $\varphi : D \to \mathbb{R}$ that measures the response of the neuron, positive on the ON zones, negative on the OFF zones
- the neuron acts as a filter
- by electrophysiology techniques it is possible to measure level sets of the receptor profiles φ of different visual neurons

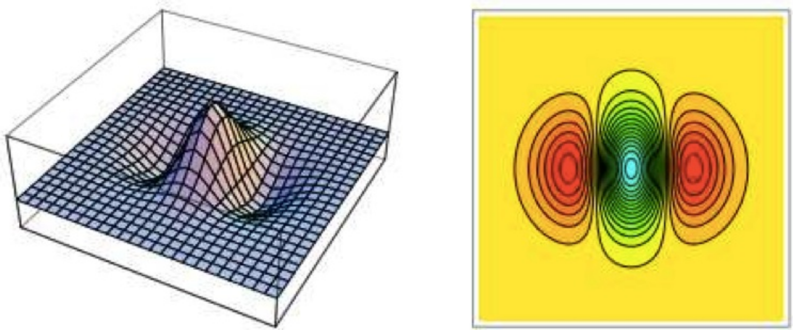
Example: receptor profile of a cell in the lateral geniculate nucleus





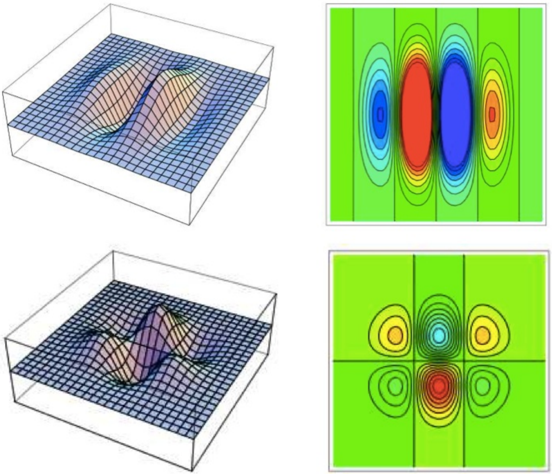
Petitot

Example: typical receptor profile of a neuron in the V1 cortex



- presence of a preferred direction; modeled by directional derivatives of two-variable Gaussians

Other Examples: more receptor profiles of neurons in the V1 cortex

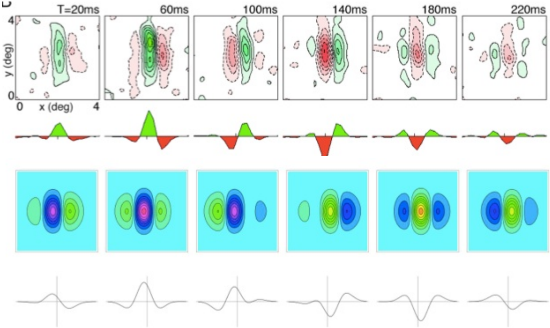


Spatio-temporal separability

- certain receptor profiles do not change their spatial pattern with time, only the intensity:

$$\varphi(x,y,t) = \varphi_{\text{spatial}}(x,y)\cdot\varphi_{\text{temporal}}(t)$$

- other receptor profiles do not have this kind of space/time dependence separation



Gaussian derivatives and Hermite polynomials

$$G_\sigma = G(x,\sigma) = \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left(-\frac{x^2}{2\sigma^2}\right)$$

$$\frac{d^n}{dx^n} \exp\left(-\frac{x^2}{2\sigma^2}\right) = (-1)^n\, H_{n,\sigma}(x)\, \exp\left(-\frac{x^2}{2\sigma^2}\right)$$

$$H_{n,\sigma}(x) = (\sqrt{2}\,\sigma)^{-n}\, H_n(\frac{x}{\sigma\sqrt{2}})$$

- Hermite polynomials $H_n(x)$

$$\frac{d^n}{dx^n} e^{-x^2} = (-1)^n H_n(x)\, e^{-x^2}$$

$$H_0(x) = 1,\ H_1(x) = 2x,\ H_2(x) = 4x^2 - 2,\ H_3(x) = 8x^3 - 12x, \cdots$$

- recursion relation

$$H_{n+1}(x) = 2x\, H_n(x) - H_n'(x)$$

- explicit form

$$H_n(x) = n! \sum_{m=0}^{\lfloor \frac{n}{2} \rfloor} \frac{(-1)^m}{m!(n-2m)!}(2x)^{n-2m}$$

- orthogonality: $L^2(\mathbb{R}, w(x)dx)$ with $w(x) = e^{-x^2}$

$$\int_{\mathbb{R}} H_n(x) H_m(x)\, w(x) dx = n! 2^n \sqrt{2}\, \delta_{n,m}$$

complete orthogonal basis

- generating function

$$\exp(2xs - s^2) = \sum_{n=0}^{\infty} H_n(x) \frac{s^n}{n!}$$

Fourier transform and power spectra

- Fourier transform

$$\mathcal{F}(f)(\omega) = \hat{f}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} f(x)\, e^{-ix\omega} dx$$

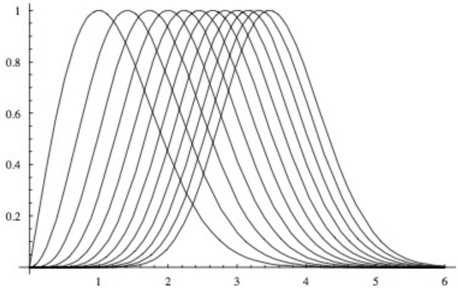
$$\mathcal{F}(G_\sigma)(\omega) = \frac{1}{\sqrt{2\pi}} \exp(-\frac{\sigma^2\omega^2}{2})$$

$$\mathcal{F}(D^n G_\sigma)(\omega) = (-i\omega)^n \mathcal{F}(G_\sigma), \quad \text{with } D^n = \frac{d^n}{dx^n}$$

Gaussian derivatives, when used as kernel operators (acting by convolution) act as bandpass filters

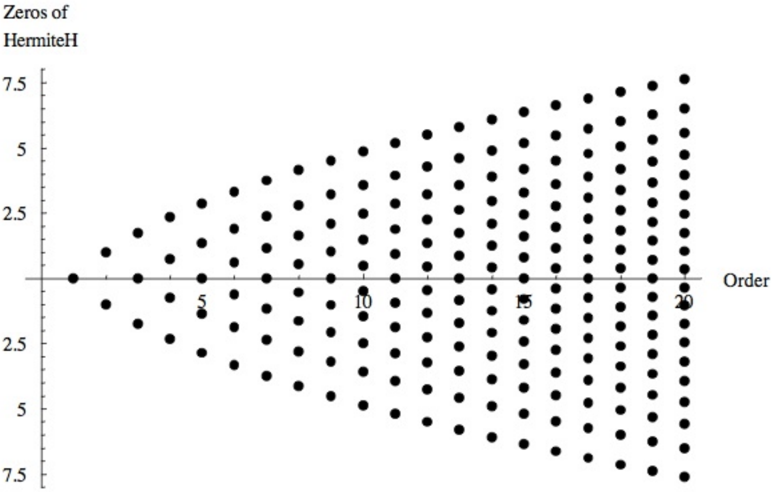
- maximum amplitude for $\mathcal{F}(D^n G_\sigma)$ at $\omega = \sqrt{n}$

shape of normalized power spectra $|\mathcal{F}(D^n G_\sigma)(\omega)/\mathcal{F}(D^n G_\sigma)(\sqrt{n})|$

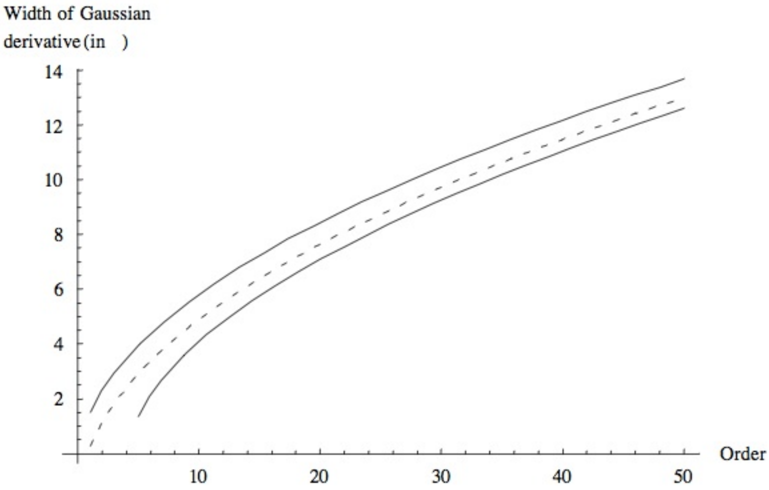


for $n = 1, \ldots, 12$ (left to right) with $\sigma = 1$

- width of a Gaussian derivative $D^n G_\sigma(x)$ is largest distance between zeros (zeros of Hermite polynomials): zeros of second derivatives $x = \pm\sigma$ one standard deviation from $x = 0$


Zeros of
HermiteH
7.5
5
2.5
2.5
5
7.5
5
10
15
20
Order

- no exact close formula for largest zero, but Zernicke and Szego estimates (above and below in plot)



Correlation between Gaussian Derivatives

$$r_{n,m} = \frac{\langle D^n G_\sigma, D^m G_\sigma \rangle}{\|D^n G_\sigma\| \cdot \|D^m G_\sigma\|}$$

Note: inner product and norm in $L^2(\mathbb{R}, dx)$ (non-weighted)

- $r_{n,n} = 1$ and $r_{n,m} = 0$ if $n - m$ odd
- for $n - m$ even, suppose $n > m$, after integrations by parts

$$\langle D^n G_\sigma, D^m G_\sigma \rangle = (-1)^{\frac{n-m}{2}} \|D^{\frac{n-m}{2}} G_\sigma\|^2$$

- Parseval identity (Plancherel theorem): Fourier transform preserves L^2-norm

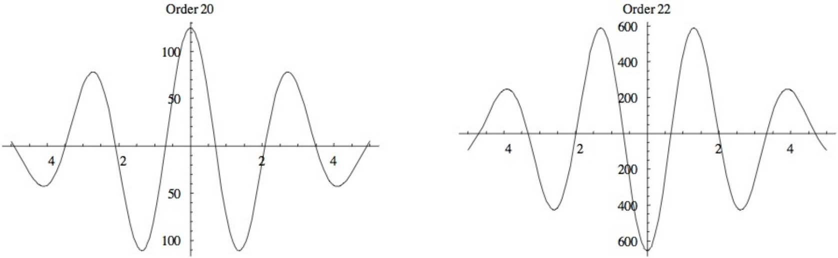
$$= (-1)^{\frac{n-m}{2}} \|(i\omega)^{\frac{n-m}{2}} \mathcal{F}(G_\sigma)(\omega)\|^2 = \frac{(-1)^{\frac{n-m}{2}}}{2\pi\sigma^{n+m+1}} \int_{\mathbb{R}} \omega^{n+m} e^{-\omega^2} d\omega$$

- last integral explicitly in terms of Γ-function

$$\int_{\mathbb{R}} x^{n+m} e^{-x^2} dx = \frac{1 + (-1)^{m+n}}{2} \Gamma(\frac{m+n+1}{2})$$

- resulting correlations:

$$r_{n,m} = \frac{(-1)^{\frac{n-m}{2}}(2\pi\sigma^{n+m+1})^{-1}\Gamma(\frac{m+n+1}{2})}{\sqrt{(2\pi\sigma^{2n+1})\Gamma(\frac{2n+1}{2})}\sqrt{(2\pi\sigma^{2m+1})\Gamma(\frac{2m+1}{2})}} = \frac{(-1)^{\frac{n-m}{2}}\Gamma(\frac{m+n+1}{2})}{(\Gamma(\frac{2n+1}{2})\Gamma(\frac{2m+1}{2}))^{1/2}}$$

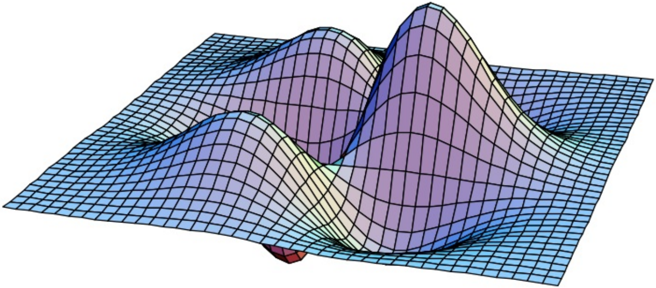


with $|n-m| = 2$ anticorrelation while very similar with large overlap when $|n-m| = 4$

- higher dimensional Gaussian Derivatives filters

$$G_\sigma(x,y) = G_{\sigma_x}(x) \cdot G_{\sigma_y}(y)$$

$$D_x^n D_y^m G_\sigma(x,y)$$



Gaussian Derivative filter $D_x D_y^2 G_\sigma(x,y)$ with $\sigma_x = \sigma_y = 2$

anisotropic if $\sigma_x \neq \sigma_y$ with anisotropy ratio σ_x/σ_y

Gabor Frames for Modulation

• if want filters tuned to waves at certain specific frequencies better use the Gabor functions

$$\mathcal{G}_{n,m}(a,b,\sigma;x) = e^{2\pi imbx} G_\sigma(x - na)$$

parameters $a, b, \sigma > 0$, and $n, m \in \mathbb{Z}$

• trigonometric functions $\sin(2\pi mbx)$ and $\cos(2\pi mbx)$ instead of Hermite polynomials:

- infinite number of zeros
- amplitude bounded by the Gaussian shape (Gaussian window)

• can approximate Gaussian Derivatives well by Gabor functions

• Gabor function model of cortical receptive fields (Marcejia, 1980) S.Marcelja, *Mathematical description of the responses of simple cortical cells*, J Opt Soc Am A 70 (1980) 1297–1300

Frames and non-orthonormal overcomplete expansions

- usual setting: Hilbert space $\mathcal{H}$, complete orthonormal system $\{\phi_n\}$, expansion $f = \sum_n c_n \phi_n$ with unique $c_n = \langle \phi_n, f \rangle$
- more generally Banach space $\mathcal{B}$, complete system $\{\phi_\alpha\}$ if every element $f \in \mathcal{B}$ can be approximated arbitrarily well in norm by linear combinations of ϕ_α
- system $\{\phi_\alpha\}_{\alpha \in \mathcal{J}}$ is overcomplete if it is a redundant complete set, namely it is complete and for some $j \in \mathcal{J}$ the set $\{\phi_\alpha\}_{\alpha \in \mathcal{J} \smallsetminus \{j\}}$ is also complete
- a set $\{\phi_\alpha\}$ in a Hilbert space $\mathcal{H}$ is a frame if there are constants $A, B > 0$ such that for all $f \in \mathcal{H}$

$$A \|f\|^2 \leq \sum_{\alpha \in \mathcal{J}} |\langle \phi_\alpha, f \rangle|^2 \leq B \|f\|^2$$

- non-orthogonal overcomplete expansion in a frame: frame $\{\phi_\alpha\}$, every $f \in \mathcal{H}$ has expansion $f = \sum_\alpha c_\alpha \phi_\alpha$ with $c = (c_\alpha) \in \ell^2(\mathcal{J})$ with $\|c\|_{\ell^2} \leq C \|f\|_{\mathcal{H}}$

Gabor frames (Lyubarskii, Seip)

- Gabor frame in dimension $d = 1$: set $\{\mathcal{G}_\lambda\}_{\lambda\in\Lambda}$ of Gabor functions

$$\mathcal{G}_\lambda(x) = e^{2\pi i \lambda_2 x} G(x-\lambda_1)$$

with $\lambda = (\lambda_1, \lambda_2) \in \Lambda \subset \mathbb{R}^2$ a lattice and with $G(x) = e^{-\pi x^2}$ is a frame if and only if lower Beurling density

$$\delta^-(\Lambda) > 1$$

- lower and upper Beurling density (or uniform densities) of a lattice $\Lambda \subset \mathbb{R}^2$

$$\delta^-(\Lambda) = \liminf_{r\to\infty} \frac{n_\Lambda^-(r)}{r^2}, \quad \delta^-(\Lambda) = \limsup_{r\to\infty} \frac{n_\Lambda^+(r)}{r^2}$$

where $n_\Lambda^\pm(r)$ are the largest and smallest number of points of Λ contained in the set $r\mathcal{I}$ where $\mathcal{I} \subset \mathbb{R}^2$ is a fixed compact set of measure 1 (and measure zero boundary: e.g. unit square)

Higher dimensional Gabor frames

- Gabor functions in d dimensions: lattice $\Lambda \subset \mathbb{R}^{2d}$

$$\mathcal{G}_\lambda(x) = \exp(2\pi i \lambda_2 \cdot x)\, G(x - \lambda_1)$$

$$G(x) = \exp(-\pi x \cdot x), \quad x \in \mathbb{R}^d$$

for $\lambda = (\lambda_1, \lambda_2) \in \Lambda$, with $\lambda_1 \in \mathbb{R}^d$

- Karlheinz Gröchenig, *Multivariate Gabor frames and sampling of entire functions of several variables*, Appl. Comput. Harmon. Anal. 31 (2011) 218–227
- lattice $\Lambda = A\mathbb{Z}^{2d}$ some $A \in \mathrm{GL}(2d, \mathbb{R})$

$$s(\Lambda) = |\det(A)|$$

$s(\Lambda)^{-1}$ measures lattice points per unit cube, gives density

- adjoint lattice $\Lambda^o = J(A^T)^{-1}\mathbb{Z}^{2d}$, with A^T transpose and

$$J = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}$$

- Wexler-Raz biorthogonality: if know $\{\mathcal{G}_\lambda\}_{\lambda\in\Lambda}$ satisfies

$$\sum_\lambda |\langle \mathcal{G}_\lambda, f\rangle|^2 \leq B\|f\|^2$$

for all $f \in L^2(\mathbb{R}^d)$ (Bessel sequence) then also frame iff there is another Bessel sequence

$$\mathcal{L}_\lambda(x) = \exp(2\pi i \lambda_2 \cdot x)\, L(x-\lambda_1)$$

where $L \in L^2(\mathbb{R}^d)$ satisfying biorthogonality

$$\langle L, \mathcal{G}_\mu\rangle = s(\Lambda)\cdot \delta_{\mu,0}, \quad \mu \in \Lambda^o$$

- density result: if $\{\mathcal{G}_\lambda\}_{\lambda\in\Lambda}$ is a frame then $s(\Lambda) < 1$ (based on Poisson summation formula, with $G \in \mathcal{S}(\mathbb{R}^d)$)
- open condition: if for $\Lambda = A\mathbb{Z}^{2d}$ the set $\{\mathcal{G}_\lambda\}_{\lambda\in\Lambda}$ is a frame, then there is an open neighborhood $\mathcal{V}$ of A in $\mathrm{GL}(2d,\mathbb{R})$ where also a frame

Gabor frames via complex analysis (Gröchenig)

- Gabor frames question (with Gaussian window) related to sampling and interpolation in Bargmann-Fock spaces

$$\|F\|_{\mathcal{F}}^2 = \int_{\mathbb{C}^d} |F(z)|^2 e^{-\pi|z|^2} dz < \infty$$

F entire function $z = (z_1, \ldots, z_d) \in \mathbb{C}^d$, with $|z|^2 = \sum_i z_i \bar{z}_i$

$$\langle F_1, F_2 \rangle_{\mathcal{F}} = \int_{\mathbb{C}^d} \overline{F_1(z)} F_2(z) e^{-\pi|z|^2} dz$$

- Bargmann transform unitary $\mathcal{B} : L^2(\mathbb{R}^d) \to \mathcal{F}$

$$(\mathcal{B}f)(z) = 2^{d/4} e^{-\pi z^2/2} \int_{\mathbb{R}^d} f(x) e^{-\pi x \cdot x} e^{2\pi x \cdot z} dx$$

- frames and sampling: $\{\mathcal{G}_{\bar{\lambda}}\}_{\bar{\lambda} \in \bar{\Lambda}}$ frame iff

$$\sum_{\lambda \in \Lambda} |F(\lambda)|^2 e^{-\pi|\lambda|^2} \asymp \|F\|_{\mathcal{F}}^2$$

for all $F \in \mathcal{F}$: Λ is a sampling of $\mathcal{F}$
because $|\langle f, \mathcal{G}_{\bar{z}} \rangle| = |\mathcal{B}f(z)| e^{-\pi|z|^2/2}$ and $F = \mathcal{B}f$

- sampling and interpolation: frame condition equivalent to sampling condition as above; by Wexler-Raz biorthogonality frame condition also equivalent to interpolation condition:

$$\exists F_o \in \mathcal{F}: \quad F_o(\mu) = \delta_{\mu,0}, \quad \forall \mu \in \Lambda^o$$

taking $F_o = \mathcal{B}L$ with L in biorthogonality

- addressing frame question by constructing solutions of the interpolation problem: entire functions with controlled growth and with zeros at all points of a given lattice
- in one dimension $d = 1$ lattices $\Lambda \subset \mathbb{C}$ a version of the Weierstrass sigma-function

$$\sigma_\Lambda(z) = \left(z \prod_{\lambda \in \Lambda \smallsetminus \{0\}} \left(1 - \frac{z}{\lambda}\right) e^{\frac{z}{\lambda} + \frac{z^2}{2\lambda^2}} \right) e^{az^2}$$

parameter $a \in \mathbb{C}$ can be fixed so that $|\sigma_\Lambda(z)| e^{-\frac{\pi}{2s(\Lambda)}|z|^2}$ is Λ-periodic, so get vanishing and growth: $\sigma_\Lambda(\mu) = 0 \ \forall \mu \in \Lambda$ and

$$|\sigma_\Lambda(z)| \leq C \, e^{\frac{\pi}{2s(\Lambda)}|z|^2}$$

Higher dimensional interpolation solutions

- restrict to complex lattices $\Lambda = A(\mathbb{Z}^d + i\mathbb{Z}^d) \subset \mathbb{C}^d$ with $A \in \mathrm{GL}(d,\mathbb{C})$: can also write as $\Lambda = A(\oplus_{j=1}^d L_j)$ with $L_j \subset \mathbb{C}$ normalized lattice $s(L_j) = 1$; size $s(\Lambda) = |\det(A)|^2$
- interpolation solution for complex Λ: entire function F_Λ

$$F_\Lambda(\lambda) = \delta_{\lambda,0}, \ \forall \lambda \in \Lambda, \quad \text{and} \ |F_\Lambda(z)| \leq Ce^{\pi \|A^{-1}\|^2_{op} \frac{|z|^2}{2}}$$

- construction for $\Lambda = \oplus_j L_j$

$$\sigma_0(z_1, \ldots, z_d) = \prod_{j=1}^d \frac{\sigma_{L_j}(z_j)}{z_j}, \quad \text{and} \ F_\Lambda(z) = \sigma_0(A^{-1}z)$$

previous properties of the Weierstrass sigma-functions σ_{L_j} determine vanishing and growth conditions for F_Λ

Frame Conditions: for a complex lattice $\Lambda = A(\oplus_j L_j)$

- adjoint lattice: for complex lattices $\Lambda^o = (A^*)^{-1}(\oplus_j L_j)$
- decompose $A = US$, unitary U (unitaries do not change frame/sampling/interpolation property) and S upper diagonal with characteristic numbers γ_j on diagonal
- then enough to check $\Lambda = S(\oplus_j L_j)$ with $\Lambda^o = (S^*)^{-1}(\oplus_j L_j)$
- assume $\gamma_j < 1$ for $j = 1, \ldots, d$
- entire function

$$F_\Lambda(z) = \prod_{j=1}^{d} \frac{\sigma_{L_j}(\gamma_j z_j)}{z_j}$$

$$|F_\Lambda(z)| \leq C \prod_j e^{\pi \gamma_j^2 |z_j|^2/2}$$

because $\gamma_j < 1$ entire function $F_\Lambda \in \mathcal{F}$

- also vanishing on $\mu = (S^*)^{-1}\lambda \in \Lambda^o$

Conclusions on Gabor Frames

- for Λ complex lattice with characteristic indices $0 < \gamma_j < 1$ the set $\{\mathcal{G}_\lambda\}_{\lambda\in\Lambda}$ is a frame for $L^2(\mathbb{R}^d)$

- Note: condition $\gamma_j < 1$ is sufficient but not necessary: there are lattices without this condition that give rise to frames

- for modeling of visual receptor fields especially interested in the case $d = 2$

$$\Lambda = A(L_1 \oplus L_2), \quad A = \begin{pmatrix} \gamma_1 & b \\ 0 & \gamma_2 \end{pmatrix}$$

1. $\gamma_1 < 1$ and $\gamma_2 < 1$: Gabor frame
2. $\gamma_2 \geq 1$: not Gabor frame
3. $\gamma_1 \geq 1$, $\gamma_2 < 1$ and $\gamma_1\gamma_2 < (\gamma_2^2 + |b|^2)^{1/2} < 1$: Gabor frame
4. $\gamma_1 \geq 1$, $\gamma_2 < 1$ and $\gamma_1\gamma_2 \geq (\gamma_2^2 + |b|^2)^{1/2}$: not Gabor frame
5. $\gamma_1 \geq 1$, $\gamma_2 < 1$ and $\gamma_1\gamma_2 < 1 \leq (\gamma_2^2 + |b|^2)^{1/2}$: not known

Conclusions about Reception Fields

- Receptor Profiles described accurately by Gabor functions in dimension $d = 2$
- Gabor functions act as a filters at specific frequencies, shaped with preferred directions
- if Gabor frame: good non-orthogonal overcomplete expansions for signal processing
- Question: when are the $d = 2$ Gabor frame conditions satisfied in the neuron receptor fields case?