Noncommutative Geometry models for Particle Physics and Cosmology, Lecture III

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Villa de Leyva school, July 2011

This lecture based on

- Matilde Marcolli, Elena Pierpaoli, *Early universe models from Noncommutative Geometry*, arXiv:0908.3683
- Daniel Kolodrubetz, Matilde Marcolli, *Boundary conditions of the RGE flow in the noncommutative geometry approach to particle physics and cosmology*, Phys. Lett. B, Vol.693 (2010) 166–174, arXiv:1006.4000

Re-examine RGE flow; gravitational terms in the asymptotic form of the spectral action; cosmological timeline; running in the very early universe; inflation and other gravitational phenomena

Asymptotic expansion of spectral action for large energies

$$
\begin{aligned}
S = & \frac{1}{2\kappa_0^2}\int R\sqrt{g}\,d^4x + \gamma_0\int\sqrt{g}\,d^4x \\
+ & \alpha_0\int C_{\mu\nu\rho\sigma}\,C^{\mu\nu\rho\sigma}\sqrt{g}\,d^4x + \tau_0\int R^*R^*\sqrt{g}\,d^4x \\
+ & \frac{1}{2}\int |DH|^2\sqrt{g}\,d^4x - \mu_0^2\int |H|^2\sqrt{g}\,d^4x \\
- & \xi_0\int R\,|H|^2\sqrt{g}\,d^4x + \lambda_0\int |H|^4\sqrt{g}\,d^4x \\
+ & \frac{1}{4}\int (G^i_{\mu\nu}\,G^{\mu\nu i} + F^\alpha_{\mu\nu}\,F^{\mu\nu\alpha} + B_{\mu\nu}\,B^{\mu\nu})\sqrt{g}\,d^4x
\end{aligned}
$$

A modified gravity model with non minimal coupling to Higgs and minimal coupling to gauge theories

Coefficients and parameters:

$$\frac{1}{2\kappa_0^2} = \frac{96 f_2 \Lambda^2 - f_0 \mathfrak{c}}{24\pi^2} \qquad \gamma_0 = \frac{1}{\pi^2}(48 f_4 \Lambda^4 - f_2 \Lambda^2 \mathfrak{c} + \frac{f_0}{4}\mathfrak{d})$$

$$\alpha_0 = -\frac{3 f_0}{10\pi^2} \qquad \tau_0 = \frac{11 f_0}{60\pi^2}$$

$$\mu_0^2 = 2\frac{f_2 \Lambda^2}{f_0} - \frac{\mathfrak{e}}{\mathfrak{a}} \qquad \xi_0 = \frac{1}{12} \qquad \lambda_0 = \frac{\pi^2 \mathfrak{b}}{2 f_0 \mathfrak{a}^2}$$

- f_0, f_2, f_4 free parameters, $f_0 = f(0)$ and, for $k > 0$,

$$f_k = \int_0^\infty f(v) v^{k-1} dv.$$

- $\mathfrak{a}, \mathfrak{b}, \mathfrak{c}, \mathfrak{d}, \mathfrak{e}$ functions of Yukawa parameters of νMSM

$$\begin{aligned}
\mathfrak{a} &= \mathrm{Tr}(Y_\nu^\dagger Y_\nu + Y_e^\dagger Y_e + 3(Y_u^\dagger Y_u + Y_d^\dagger Y_d)) \\
\mathfrak{b} &= \mathrm{Tr}((Y_\nu^\dagger Y_\nu)^2 + (Y_e^\dagger Y_e)^2 + 3(Y_u^\dagger Y_u)^2 + 3(Y_d^\dagger Y_d)^2) \\
\mathfrak{c} &= \mathrm{Tr}(MM^\dagger) \qquad \mathfrak{d} = \mathrm{Tr}((MM^\dagger)^2) \\
\mathfrak{e} &= \mathrm{Tr}(MM^\dagger Y_\nu^\dagger Y_\nu).
\end{aligned}$$

Two different perspectives on running: parameters $\mathfrak{a}, \mathfrak{b}, \mathfrak{c}, \mathfrak{d}, \mathfrak{e}$ run with RGE

- The relation between coefficients κ_0, γ_0, α_0, τ_0, μ_0, ξ_0, λ_0 and parameters $\mathfrak{a}, \mathfrak{b}, \mathfrak{c}, \mathfrak{d}, \mathfrak{e}$ hold only at Λ_{unif}: constraint on initial conditions; independent running
- There is a range of energies $\Lambda_{min} \leq \Lambda \leq \Lambda_{unif}$ where the running of coefficients κ_0, γ_0, α_0, τ_0, μ_0, ξ_0, λ_0 determined by running of $\mathfrak{a}, \mathfrak{b}, \mathfrak{c}, \mathfrak{d}, \mathfrak{e}$ and relation continues to hold (very early universe only)

The first perspective requires independent running of gravitational parameters with given condition at unification; the second perspective allows for interesting gravitational phenomena in the very early universe specific to NCG model only

The first approach followed in

- Ali Chamseddine, Alain Connes, Matilde Marcolli, *Gravity and the Standard Model with neutrino mixing*, ATMP 11 (2007) 991–1090, arXiv:hep-th/0610241

For modified gravity models

$$\int \left(\frac{1}{2\eta} C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} - \frac{\omega}{3\eta} R^2 + \frac{\theta}{\eta} R^* R^* \right) \sqrt{g}\, d^4x$$

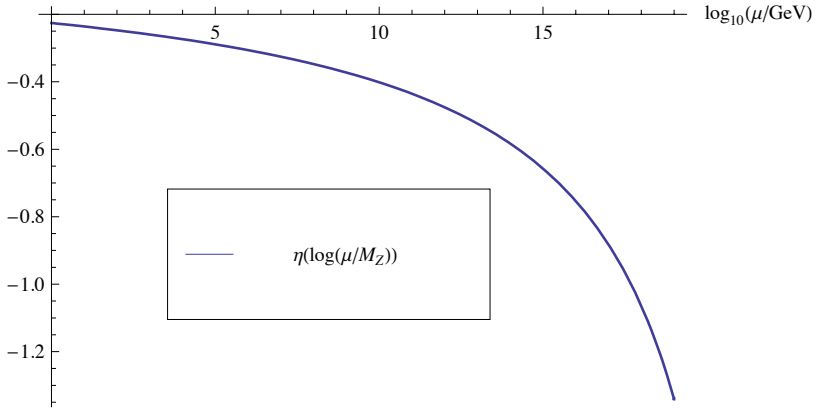
Running of gravitational parameters (Avramidi, Codello–Percacci, Donoghue) by

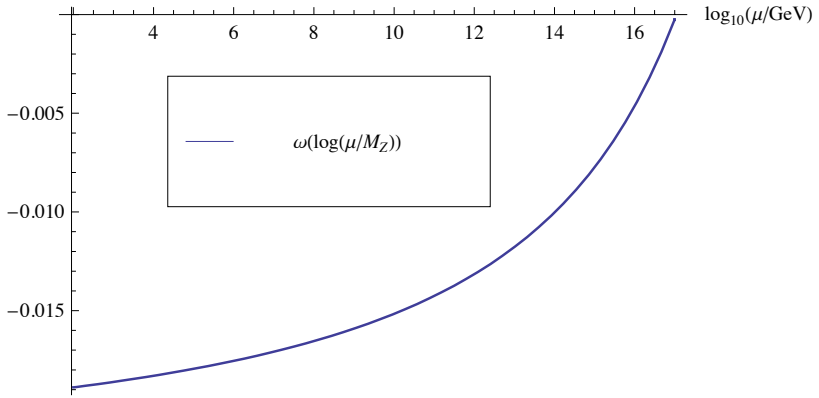
$$\beta_\eta = -\frac{1}{(4\pi)^2} \frac{133}{10} \eta^2$$

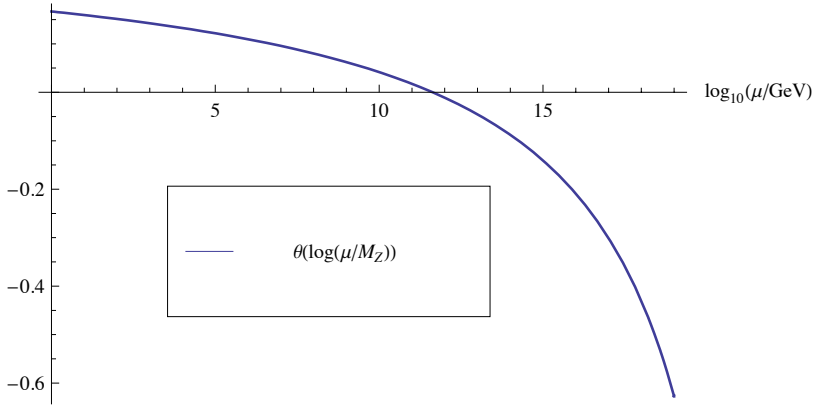
$$\beta_\omega = -\frac{1}{(4\pi)^2} \frac{25 + 1098\,\omega + 200\,\omega^2}{60} \eta$$

$$\beta_\theta = \frac{1}{(4\pi)^2} \frac{7(56 - 171\,\theta)}{90} \eta.$$

With relations at unification get running like







Plausible values in low energy limit (near fixed points)

Look then at second point of view: M.M.-E.Pierpaoli (2009)

Renormalization group equations for SM with right handed neutrinos and Majorana mass terms, from unification energy (2×10^{16} GeV) down to the electroweak scale (10^2 GeV)

AKLRS S. Antusch, J. Kersten, M. Lindner, M. Ratz, M.A. Schmidt *Running neutrino mass parameters in see-saw scenarios*, JHEP 03 (2005) 024.

Remark: RGE analysis in [CCM] only done using minimal SM

1-loop RGE equations: $\Lambda \frac{df}{d\Lambda} = \beta_f(\Lambda)$

$$16\pi^2\ \beta_{g_i} = b_i\, g_i^3 \quad \text{with } (b_{SU(3)}, b_{SU(2)}, b_{U(1)}) = (-7, -\frac{19}{6}, \frac{41}{10})$$

$$16\pi^2\ \beta_{Y_u} = Y_u(\frac{3}{2}Y_u^\dagger Y_u - \frac{3}{2}Y_d^\dagger Y_d + \mathfrak{a} - \frac{17}{20}g_1^2 - \frac{9}{4}g_2^2 - 8g_3^2)$$

$$16\pi^2\ \beta_{Y_d} = Y_d(\frac{3}{2}Y_d^\dagger Y_d - \frac{3}{2}Y_u^\dagger Y_u + \mathfrak{a} - \frac{1}{4}g_1^2 - \frac{9}{4}g_2^2 - 8g_3^2)$$

$$16\pi^2\ \beta_{Y_\nu} = Y_\nu(\frac{3}{2}Y_\nu^\dagger Y_\nu - \frac{3}{2}Y_e^\dagger Y_e + \mathfrak{a} - \frac{9}{20}g_1^2 - \frac{9}{4}g_2^2)$$

$$16\pi^2\ \beta_{Y_e} = Y_e(\frac{3}{2}Y_e^\dagger Y_e - \frac{3}{2}Y_\nu^\dagger Y_\nu + \mathfrak{a} - \frac{9}{4}g_1^2 - \frac{9}{4}g_2^2)$$

$$16\pi^2\ \beta_M = Y_\nu Y_\nu^\dagger M + M(Y_\nu Y_\nu^\dagger)^T$$

$$16\pi^2\ \beta_\lambda = 6\lambda^2 - 3\lambda(3g_2^2 + \frac{3}{5}g_1^2) + 3g_2^4 + \frac{3}{2}(\frac{3}{5}g_1^2 + g_2^2)^2 + 4\lambda\mathfrak{a} - 8\mathfrak{b}$$

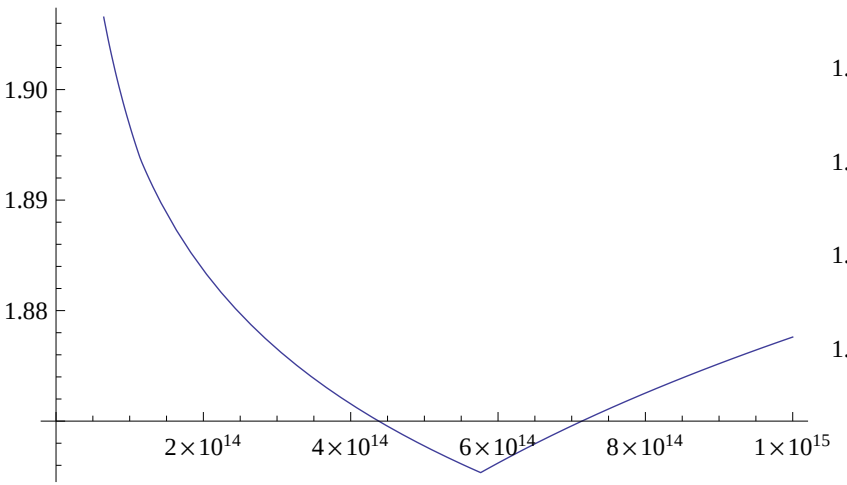
Note: different normalization from [CCM] and 5/3 factor included in g_1^2

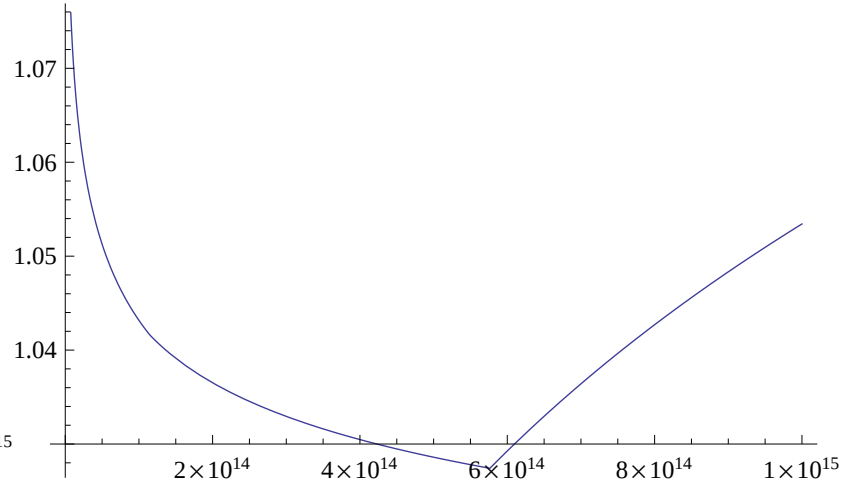
Method of AKLRS: non-degenerate spectrum of Majorana masses, different effective field theories in between the three see-saw scales:

- RGE from unification Λ_{unif} down to first see-saw scale (largest eigenvalue of M)
- Introduce $Y_\nu^{(3)}$ removing last row of Y_ν in basis where M diagonal and $M^{(3)}$ removing last row and column.
- Induced RGE down to second see-saw scale
- Introduce $Y_\nu^{(2)}$ and $M^{(2)}$, matching boundary conditions
- Induced RGE down to first see-saw scale
- Introduce $Y_\nu^{(1)}$ and $M^{(1)}$, matching boundary conditions
- Induced RGE down to electoweak energy Λ_{ew}

Use effective field theories $Y_\nu^{(N)}$ and $M^{(N)}$ between see-saw scales

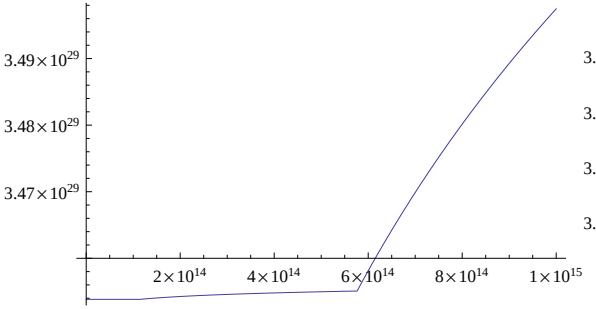
Running of coefficients $\mathfrak{a}, \mathfrak{b}$ with RGE

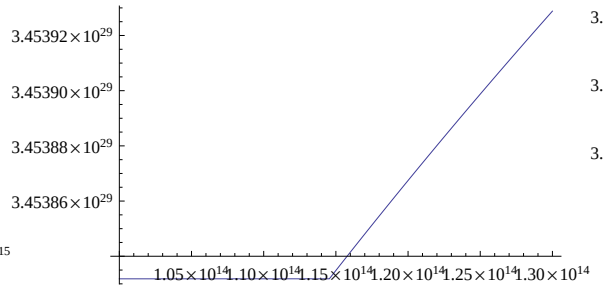


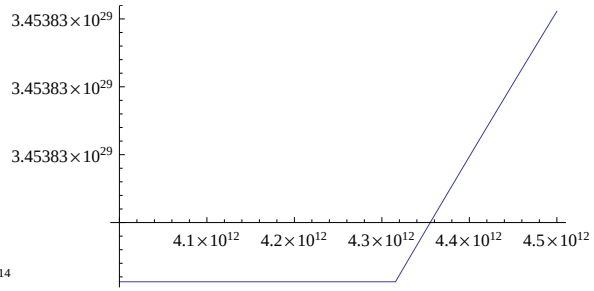


Coefficients $\mathfrak{a}$ and $\mathfrak{b}$ near the top see-saw scale

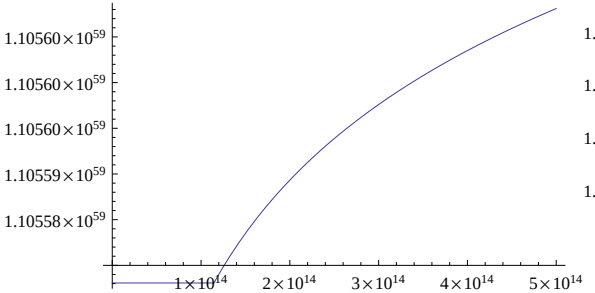
Running of coefficient $\mathfrak{c}$ with RGE

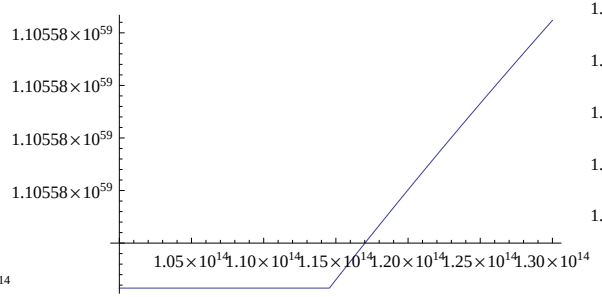


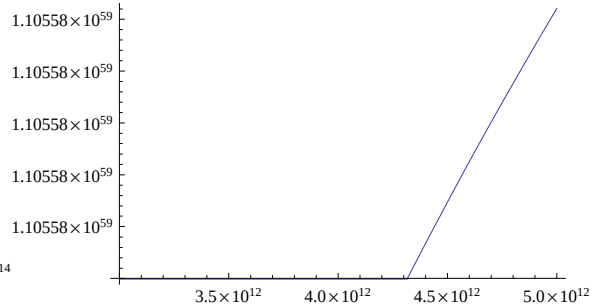




Running of coefficient $\mathfrak{d}$ with RGE

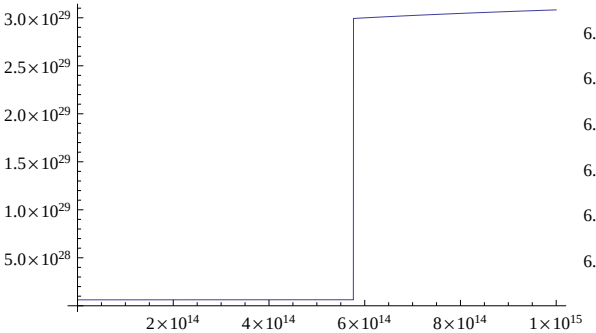


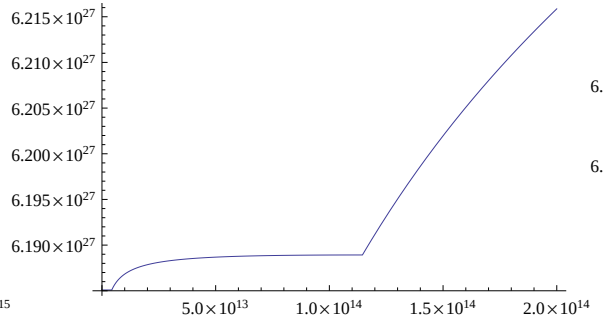


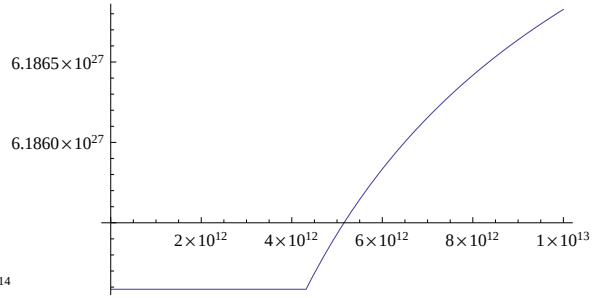


Effect of the three see-saw scales

Running of coefficient $\mathfrak{e}$ with RGE







Effect of the three see-saw scales

With default boundary conditions at unification of AKLRS

...but sensitive dependence on the initial conditions $\Rightarrow$ fine tuning

Changing initial conditions: maximal mixing conditions at unification

$$\zeta = exp(2\pi i/3)$$

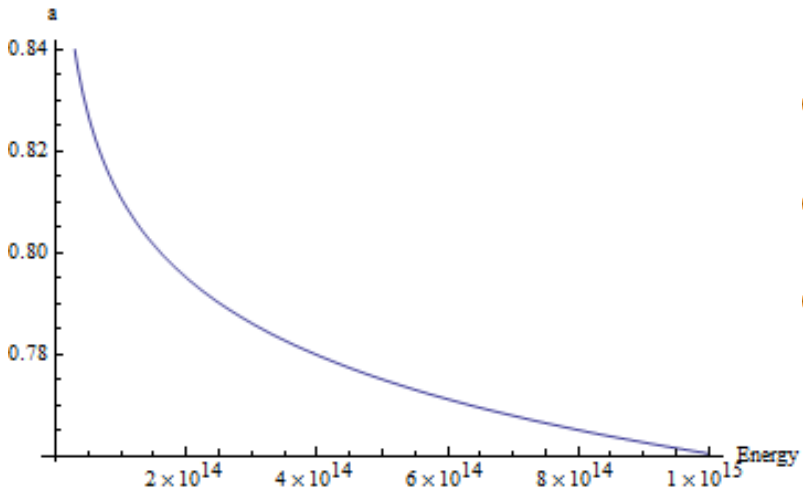
$$U_{PMNS}(\Lambda_{unif}) = \frac{1}{3}\begin{pmatrix} 1 & \zeta & \zeta^2 \\ \zeta & 1 & \zeta \\ \zeta^2 & \zeta & 1 \end{pmatrix}$$

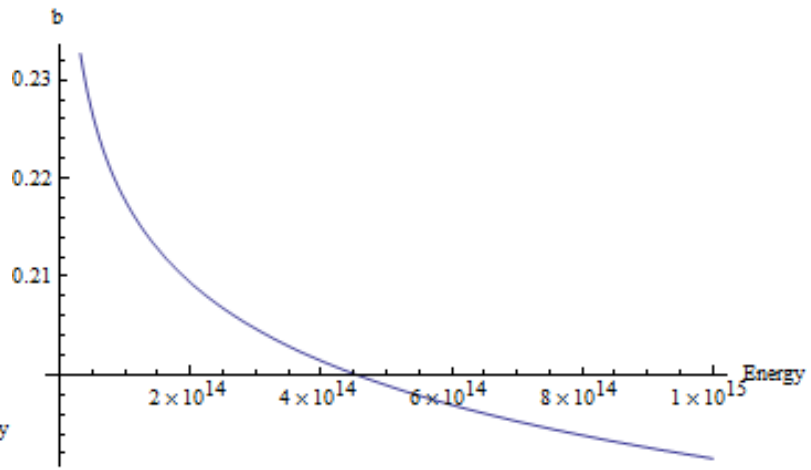
$$\delta_{(\uparrow 1)} = \frac{1}{246}\begin{pmatrix} 12.2\times 10^{-9} & 0 & 0 \\ 0 & 170\times 10^{-6} & 0 \\ 0 & 0 & 15.5\times 10^{-3} \end{pmatrix}$$

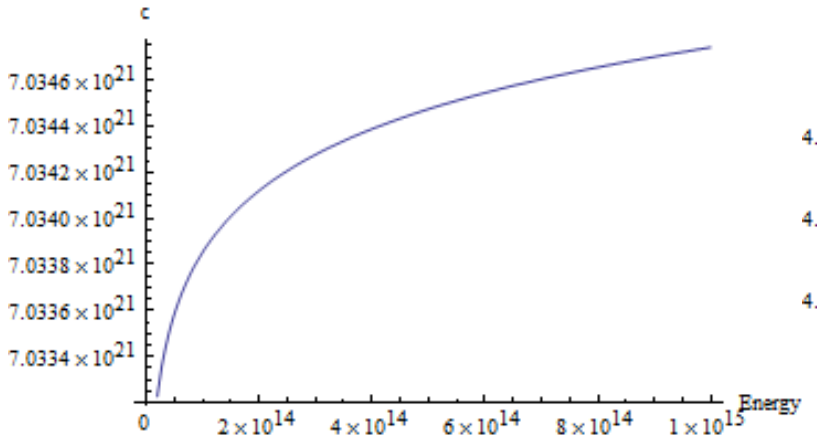
$$Y_\nu = U^\dagger_{PMNS}\delta_{(\uparrow 1)}U_{PMNS}$$

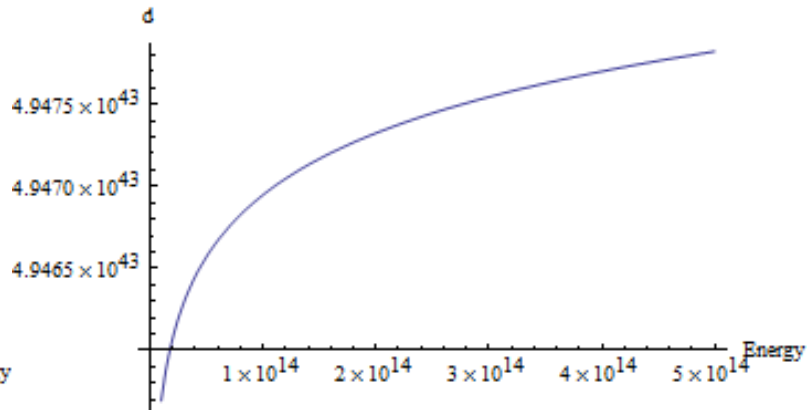
- Daniel Kolodrubetz, Matilde Marcolli, *Boundary conditions of the RGE flow in the noncommutative geometry approach to particle physics and cosmology*, arXiv:1006.4000

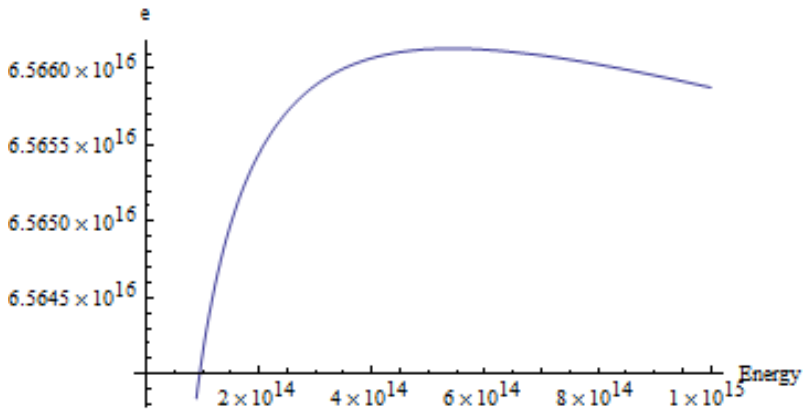
Effect on coefficients running



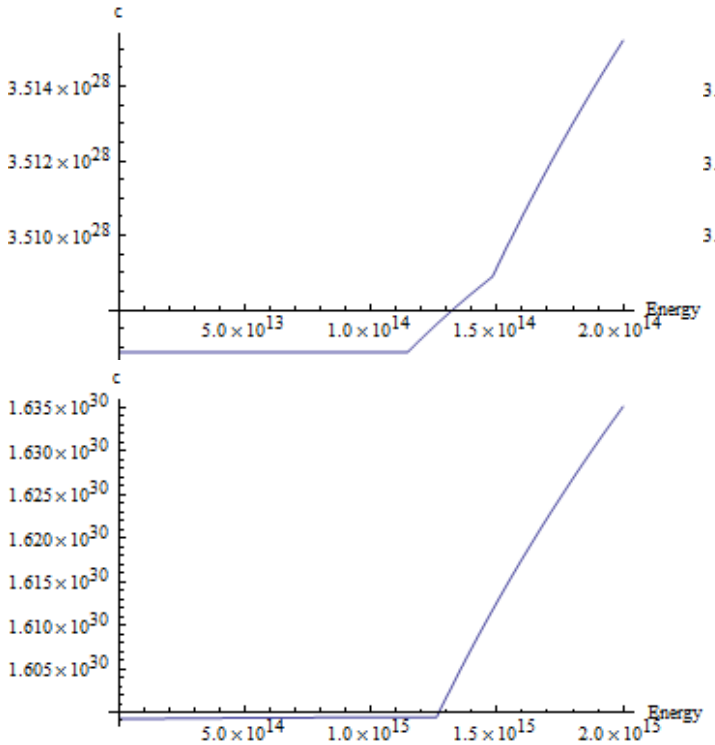


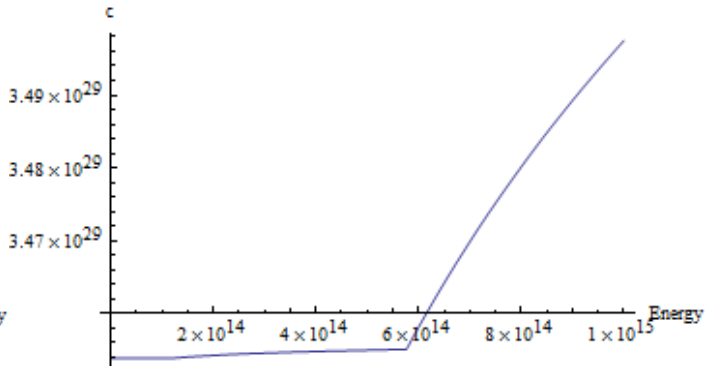

c
7.0346 × 10^21
7.0344 × 10^21
7.0342 × 10^21
7.0340 × 10^21
7.0338 × 10^21
7.0336 × 10^21
7.0334 × 10^21
0
2 × 10^14
4 × 10^14
6 × 10^14
8 × 10^14
1 × 10^15
Energy


d
4.9475 × 10^43
4.9470 × 10^43
4.9465 × 10^43
1 × 10^14
2 × 10^14
3 × 10^14
4 × 10^14
5 × 10^14
Energy


e
6.5660 × 10^16
6.5655 × 10^16
6.5650 × 10^16
6.5645 × 10^16
2 × 10^14
4 × 10^14
6 × 10^14
8 × 10^14
1 × 10^15
Energy

Further evidence of sensitive dependence: changing only one parameter in diagonal matrix Y_ν get running of top term:





Geometric constraints at unification energy

- λ parameter constraint

$$\lambda(\Lambda_{unif}) = \frac{\pi^2}{2f_0}\frac{\mathfrak{b}(\Lambda_{unif})}{\mathfrak{a}(\Lambda_{unif})^2}$$

- Higgs vacuum constraint

$$\frac{\sqrt{\mathfrak{a}f_0}}{\pi} = \frac{2M_W}{g}$$

- See-saw mechanism and $\mathfrak{c}$ constraint

$$\frac{2f_2\Lambda^2_{unif}}{f_0} \leq \mathfrak{c}(\Lambda_{unif}) \leq \frac{6f_2\Lambda^2_{unif}}{f_0}$$

- Mass relation at unification

$$\sum_{generations} (m_\nu^2 + m_e^2 + 3m_u^2 + 3m_d^2)|_{\Lambda=\Lambda_{unif}} = 8M_W^2|_{\Lambda=\Lambda_{unif}}$$

Choice of initial conditions at unification:

- Compatibility with low energy values: experimental constraints
- Compatibility with geometric constraints at unification

It is possible to modify boundary conditions to achieve both compatibilities

Example: using maximal mixing conditions but modify parameters in the Majorana mass matrix and initial condition of Higgs parameter to satisfy geometric constraints

Cosmology timeline

- Planck epoch: $t \leq 10^{-43}\, s$ after the Big Bang (unification of forces with gravity, quantum gravity)
- Grand Unification epoch: $10^{-43}\, s \leq t \leq 10^{-36}\, s$ (electroweak and strong forces unified; Higgs)
- Electroweak epoch: $10^{-36}\, s \leq t \leq 10^{-12}\, s$ (strong and electroweak forces separated)
- Inflationary epoch: possibly $10^{-36}\, s \leq t \leq 10^{-32}\, s$

- NCG SM preferred scale at unification; RGE running between unification and electroweak $\Rightarrow$ info on inflationary epoch.
- Remark: Cannot extrapolate to modern universe, nonperturbative effects in the spectral action and phase transitions in the RGE flow

Cosmological implications of the NCG SM

- Linde's hypothesis (antigravity in the early universe)
- Primordial black holes and gravitational memory
- Gravitational waves in modified gravity
- Gravity balls
- Varying effective cosmological constant
- Higgs based slow-roll inflation
- Spontaneously arising Hoyle-Narlikar in EH backgrounds

Effective gravitational constant

$$G_{\mathrm{eff}} = \frac{\kappa_0^2}{8\pi} = \frac{3\pi}{192 f_2 \Lambda^2 - 2 f_0 \mathfrak{c}(\Lambda)}$$

Effective cosmological constant

$$\gamma_0 = \frac{1}{4\pi^2}(192 f_4 \Lambda^4 - 4 f_2 \Lambda^2 \mathfrak{c}(\Lambda) + f_0 \mathfrak{d}(\Lambda))$$

Conformal non-minimal coupling of Higgs and gravity

$$\frac{1}{16\pi G_{\rm eff}}\int R\sqrt{g}d^4x-\frac{1}{12}\int R\,|H|^2\sqrt{g}d^4x$$

Conformal gravity

$$\frac{-3f_0}{10\pi^2}\int C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}\sqrt{g}d^4x$$

$C^{\mu\nu\rho\sigma}$ = Weyl curvature tensor (trace free part of Riemann tensor)

$$C_{\lambda\mu\nu\kappa}=R_{\lambda\mu\nu\kappa}-\frac{1}{2}(g_{\lambda\nu}R_{\mu\kappa}-g_{\mu\nu}R_{\lambda\kappa}+g_{\mu\kappa}R_{\lambda\nu})+\frac{1}{6}(g_{\lambda\nu}g_{\mu\kappa}-g_{\lambda\kappa}g_{\mu\nu})$$

An example: $G_{\rm eff}(\Lambda_{ew}) = G$ (at electroweak phase transition $G_{\rm eff}$ is already modern universe Newton constant)
$1/\sqrt{G} = 1.22086 \times 10^{19}$ GeV $\Rightarrow f_2 = 7.31647 \times 10^{32}$

$$G_{\rm eff}^{-1}(\Lambda) \sim \frac{96 f_2 \Lambda^2}{24\pi^2}$$

Term $\mathfrak{e}/\mathfrak{a}$ lower order
Dominant terms in the spectral action:

$$\Lambda^2 \left(\frac{1}{2\tilde{\kappa}_0^2} \int R\sqrt{g} d^4x - \tilde{\mu}_0^2 \int |H|^2 \sqrt{g} d^4x \right)$$

$\tilde{\kappa}_0 = \Lambda \kappa_0$ and $\tilde{\mu}_0 = \mu_0/\Lambda$, where $\mu_0^2 \sim \frac{2 f_2 \Lambda^2}{f_0}$

Detectable by gravitational waves:
Einstein equations $R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = \kappa_0^2 T^{\mu\nu}$

$$g_{\mu\nu} = a(t)^2 \begin{pmatrix} -1 & 0 \\ 0 & \delta_{ij} + h_{ij}(x) \end{pmatrix}$$

trace and traceless part of $h_{ij} \Rightarrow$ Friedmann equation

$$-3\left(\frac{\dot{a}}{a}\right)^2 + \frac{1}{2}\left(4\left(\frac{\dot{a}}{a}\right)\dot{h} + 2\ddot{h}\right) = \frac{\tilde{\kappa}_0^2}{\Lambda^2}\, T_{00}$$

$\Lambda(t) = 1/a(t)$ (f_2 large) Inflationary epoch: $a(t) \sim e^{\alpha t}$
NCG model solutions:

$$h(t) = \frac{3\pi^2 T_{00}}{192 f_2 \alpha^2} e^{2\alpha t} + \frac{3\alpha}{2} t + \frac{A}{2\alpha} e^{-2\alpha t} + B$$

Ordinary cosmology:

$$(\frac{4\pi G T_{00}}{\alpha} + \frac{3\alpha}{2}) t + \frac{A}{2\alpha} e^{-2\alpha t} + B$$

Radiation dominated epoch: $a(t) \sim t^{1/2}$
NCG model solutions:

$$h(t) = \frac{4\pi^2 T_{00}}{288 f_2} t^3 + B + A \log(t) + \frac{3}{8} \log(t)^2$$

Ordinary cosmology:

$$h(t) = 2\pi G T_{00} t^2 + B + A \log(t) + \frac{3}{8} \log(t)^2$$

Same example, special case:

$$R \sim \frac{2\tilde{\kappa}_0^2 \tilde{\mu}_0^2 \mathfrak{a} f_0}{\pi^2} \sim 1 \quad \text{and} \quad H \sim \sqrt{\mathfrak{a} f_0}/\pi$$

Leaves conformally coupled matter and gravity

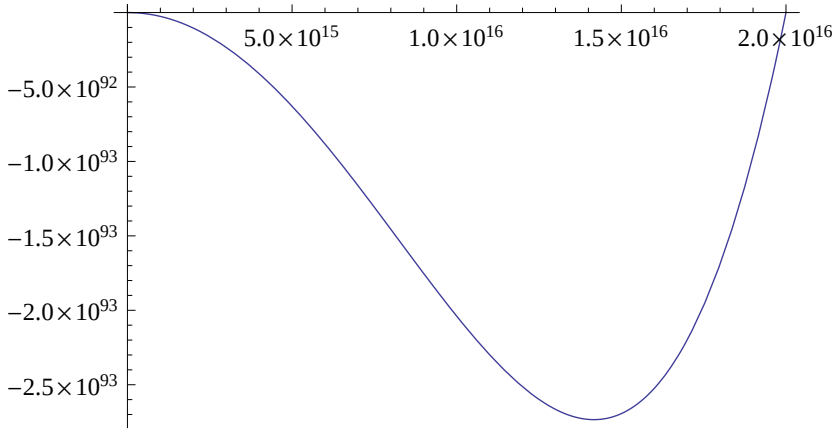
$$\begin{aligned} S_c = \quad & \alpha_0 \int C_{\mu\nu\rho\sigma}\, C^{\mu\nu\rho\sigma} \sqrt{g}\, d^4x + \frac{1}{2} \int |DH|^2 \sqrt{g}\, d^4x \\ & -\xi_0 \int R\, |H|^2 \sqrt{g}\, d^4x + \lambda_0 \int |H|^4 \sqrt{g}\, d^4x \\ & + \frac{1}{4} \int (G^i_{\mu\nu}\, G^{\mu\nu i} + F^\alpha_{\mu\nu}\, F^{\mu\nu\alpha} + B_{\mu\nu}\, B^{\mu\nu}) \sqrt{g}\, d^4x. \end{aligned}$$

A Hoyle-Narlikar type cosmology, normally suppressed by dominant Einstein–Hilbert term, arises when $R \sim 1$ and $H \sim v$, near see-saw scale.

Cosmological term controlled by additional parameter f_4, vanishing condition:

$$f_4 = \frac{(4 f_2 \Lambda^2 \mathfrak{c} - f_0 \mathfrak{d})}{192 \Lambda^4}$$

Example: vanishing at unification $\gamma_0(\Lambda_{unif}) = 0$



Running of $\gamma_0(\Lambda)$: possible inflationary mechanism

The λ_0-ansatz

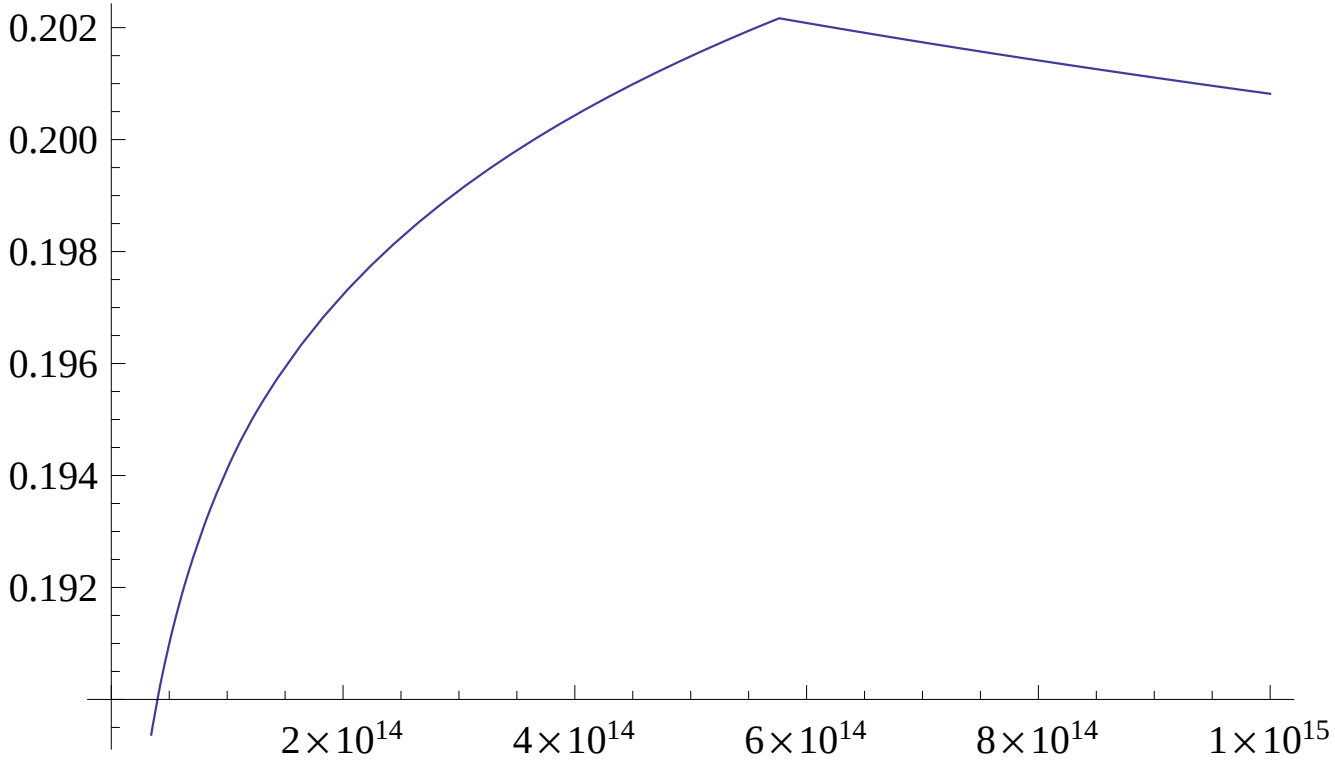
$$\lambda_0|_{\Lambda=\Lambda_{unif}} = \lambda(\Lambda_{unif})\frac{\pi^2\mathfrak{b}(\Lambda_{unif})}{f_0\mathfrak{a}^2(\Lambda_{unif})},$$

- Run like $\lambda(\Lambda)$ but change boundary condition to $\lambda_0|_{\Lambda=\Lambda_{unif}}$
- Run like

$$\lambda_0(\Lambda) = \lambda(\Lambda)\frac{\pi^2\mathfrak{b}(\Lambda)}{f_0\mathfrak{a}^2(\Lambda)}$$

For most of our cosmological estimates no serious difference

The running of $\lambda_0(\Lambda)$ near the top see-saw scale



Tongue-in-cheek remark:

- Higgs mass estimate in [CCM] from low energy limit of λ (running with RGE of minimal SM)

$$\sqrt{2\lambda}\frac{2M}{g} \sim 170\,\mathrm{GeV}$$

  Higgs vacuum $2M/g \sim 246$ GeV

- Estimate using the ansatz for $\lambda_0(\Lambda)$:

$$\sqrt{2\lambda_0}\frac{2M}{g} < 158\,\mathrm{GeV}$$

Tevatron collaboration: projected window of exclusion for the Higgs starts at 158 GeV

Linde's hypothesis antigravity in the early universe

- A.D. Linde, *Gauge theories, time-dependence of the gravitational constant and antigravity in the early universe*, Phys. Letters B, Vol.93 (1980) N.4, 394–396

Based on a conformal coupling

$$\frac{1}{16\pi G}\int R\sqrt{g}d^4x - \frac{1}{12}\int R\,\phi^2\sqrt{g}d^4x$$

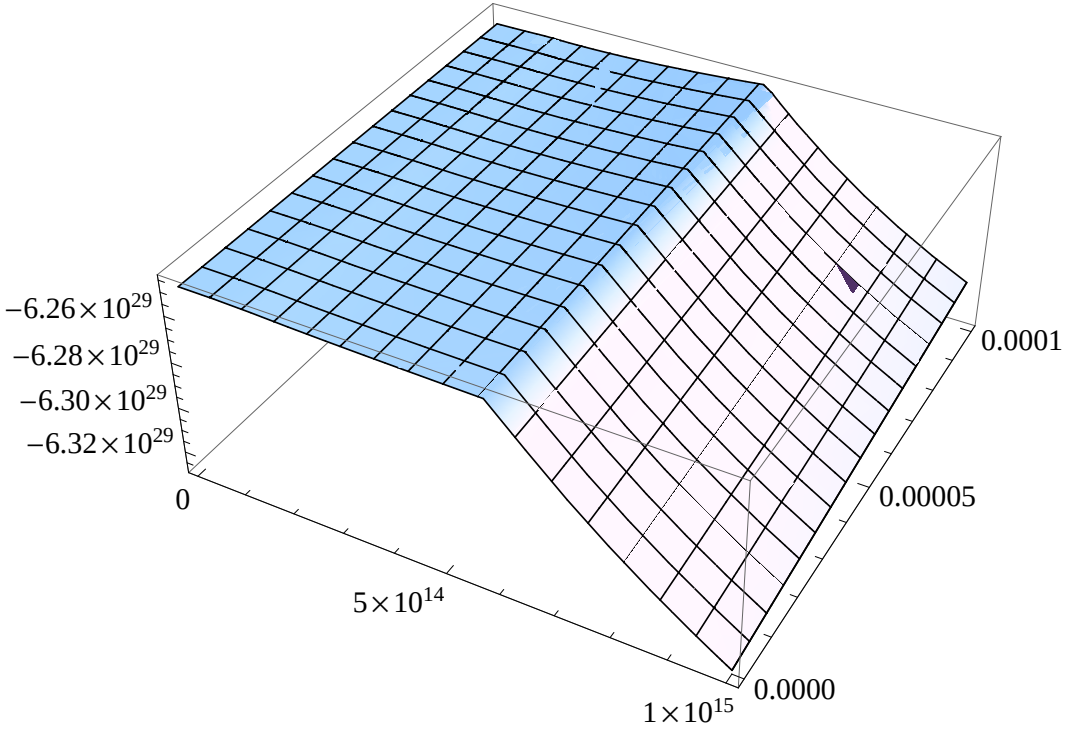
giving an effective

$$G_{\rm eff}^{-1} = G^{-1} - \frac{4}{3}\pi\phi^2$$

In the NCG SM model two sources of negative gravity

- Running of $G_{\rm eff}(\Lambda)$
- Conformal coupling to the Higgs field

Example of effective $G_{\rm eff}^{-1}(\Lambda, f_2)$ near the top see-saw scale



Example: fixing $G_{\rm eff}(\Lambda_{unif}) = G$ gives a phase of negative gravity with conformal gravity becoming dominant near sign change of $G_{\rm eff}(\Lambda)^{-1}$ at $\sim 10^{12}$ GeV

Gravity balls

$$G_{\mathrm{eff},H} = \frac{G_{\mathrm{eff}}}{1 - \frac{4\pi}{3} G_{\mathrm{eff}} |H|^2}$$

combines running of G_{eff} with Linde mechanism

Suppose f_2 such that $G_{\mathrm{eff}}(\Lambda) > 0$

$$\begin{cases} G_{\mathrm{eff},H} < 0 \quad \text{for } |H|^2 > \dfrac{3}{4\pi G_{\mathrm{eff}}(\Lambda)}, \\ G_{\mathrm{eff},H} > 0 \quad \text{for } |H|^2 < \dfrac{3}{4\pi G_{\mathrm{eff}}(\Lambda)}. \end{cases}$$

Unstable and stable equilibrium for H:

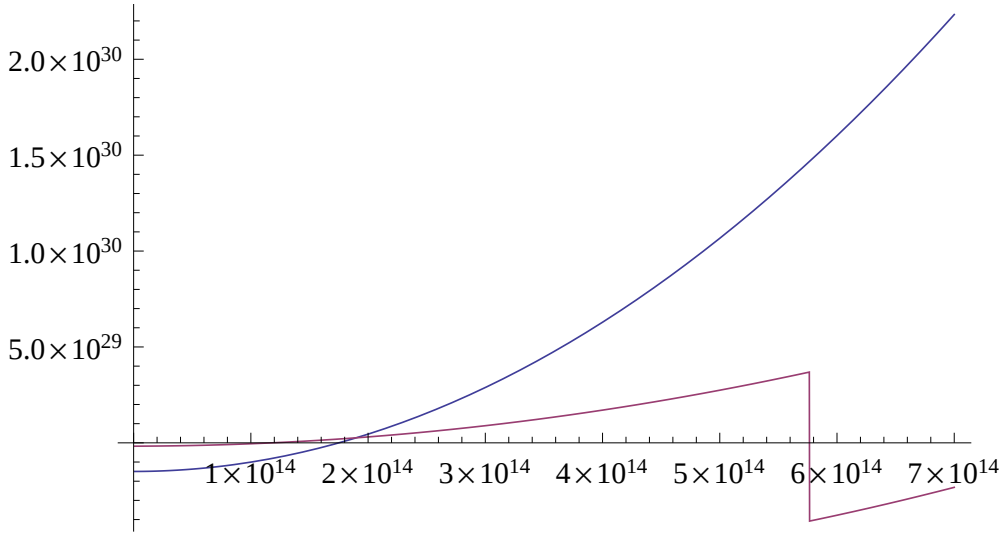
$$\ell_H(\Lambda, f_2) := \frac{\mu_0^2}{2\lambda_0}(\Lambda) = \frac{2\frac{f_2\Lambda^2}{f_0} - \frac{\mathfrak{e}(\Lambda)}{\mathfrak{a}(\Lambda)}}{\lambda(\Lambda)\frac{\pi^2\mathfrak{b}(\Lambda)}{f_0\mathfrak{a}^2(\Lambda)}} = \frac{(2f_2\Lambda^2\mathfrak{a}(\Lambda) - f_0\mathfrak{e}(\Lambda))\mathfrak{a}(\Lambda)}{\pi^2\lambda(\Lambda)\mathfrak{b}(\Lambda)}$$

(with λ_0-ansatz)

Negative gravity regime where

$$\ell_H(\Lambda, f_2) > \frac{3}{4\pi G_{\mathrm{eff}}(\Lambda, f_2)}$$

An example of transition to negative gravity



Gravity balls: regions where $|H|^2 \sim 0$ unstable equilibrium (positive gravity) surrounded by region with $|H|^2 \sim \ell_H(\Lambda, f_2)$ stable (negative gravity): possible model of dark energy

Primordial black holes (Zeldovich–Novikov, 1967)

- I.D. Novikov, A.G. Polnarev, A.A. Starobinsky, Ya.B. Zeldovich, *Primordial black holes*, Astron. Astrophys. 80 (1979) 104–109
- J.D. Barrow, *Gravitational memory?* Phys. Rev. D Vol.46 (1992) N.8 R3227, 4pp.

Caused by: collapse of overdense regions, phase transitions in the early universe, cosmic loops and strings, inflationary reheating, etc

Gravitational memory: if gravity balls with different $G_{\mathrm{eff},H}$ primordial black holes can evolve with different $G_{\mathrm{eff},H}$ from surrounding space

Evaporation of PBHs by Hawking radiation

$$\frac{d\mathcal{M}(t)}{dt} \sim -(G_{\rm eff}(t)\mathcal{M}(t))^{-2}$$

with Hawking temperature $T = (8\pi G_{\rm eff}(t)\mathcal{M}(t))^{-1}$.
In terms of energy:

$$\mathcal{M}^2\, d\mathcal{M} = \frac{1}{\Lambda^2 G_{\rm eff}^2(\Lambda, f_2)} d\Lambda$$

With or without gravitational memory depending on $G_{\rm eff}$ behavior

Evaporation of PBHs linked to γ-ray bursts

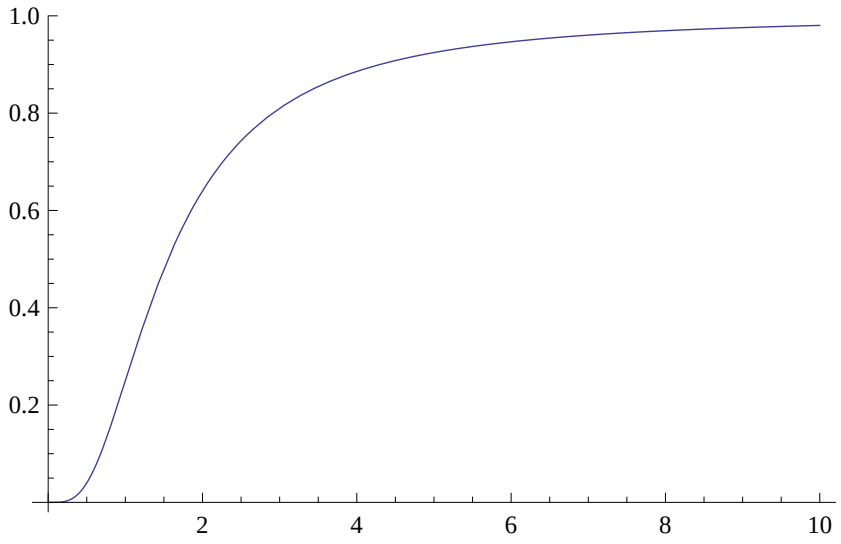
Higgs based slow-roll inflation

dSHW A. De Simone, M.P. Hertzberg, F. Wilczek, *Running inflation in the Standard Model*, hep-ph/0812.4946v2

Minimal SM and non-minimal coupling of Higgs and gravity.

$$\xi_0 \int R\,|H|^2 \sqrt{g}\, d^4x$$

Non-conformal coupling $\xi_0 \neq 1/12$, running of ξ_0
Effective Higgs potential: inflation parameter $\psi = \sqrt{\xi_0}\kappa_0|H|$



inflationary period $\psi >> 1$, end of inflation $\psi \sim 1$, low energy regime $\psi << 1$

In the NCG SM have $\xi_0 = 1/12$ but same Higgs based slow-roll inflation due to κ_0 running (say $\kappa_0 > 0$)

$$\psi(\Lambda) = \sqrt{\xi_0(\Lambda)}\kappa_0(\Lambda)|H| = \sqrt{\frac{\pi^2}{96 f_2 \Lambda^2 - f_0 \mathfrak{c}(\Lambda)}}\,|H|$$

Einstein metric $g^E_{\mu\nu} = f(H) g_{\mu\nu}$, for $f(H) = 1 + \xi_0 \kappa_0 |H|^2$
Higgs potential

$$V_E(H) = \frac{\lambda_0 |H|^4}{(1 + \xi_0 \kappa_0^2 |H|^2)^2}$$

For $\psi >> 1$ approaches constant; usual quartic potential for $\psi << 1$

Conclusion:
Various possible inflation scenarios in the very early universe from running of coefficients of the spectral action according to the relation to Yukawa parameters: phases and regions of negative gravity, variable gravitational and cosmological constants, inflation potential from nonmininal coupling of Higgs to gravity

Main problem: these effects depend on choice of initial conditions at unification (sensitive dependence) and several of these scenarios are ruled out when moving boundary conditions

Next episode

- The problem of cosmic topology
- The Poisson summation formula
- Nonperturbative computation of the spectral action on 3-dimensional space forms
- Slow-roll inflation: potential, slow-roll coefficients, power spectra
- Slow-roll inflation from the nonperturbative spectral action and cosmic topology