Structures of Randomness

Matilde Marcolli

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Randomness and culture

In the post World War II years, the concept of *randomness* penetrated the collective consciousness of European culture like never before...

it profoundly altered its main modes of expression: the arts and the sciences

a major shift in consciousness and world view

Europe at the end of WWII

London

Warsaw

Berlin 

Leningrad 

Aerial bombing and randomness

• most of the destruction of European cities was caused by aerial bombing

• a new form of warfare (Guernica, 1937)

• ... but never before seen on such immense inhuman scale as during WW2

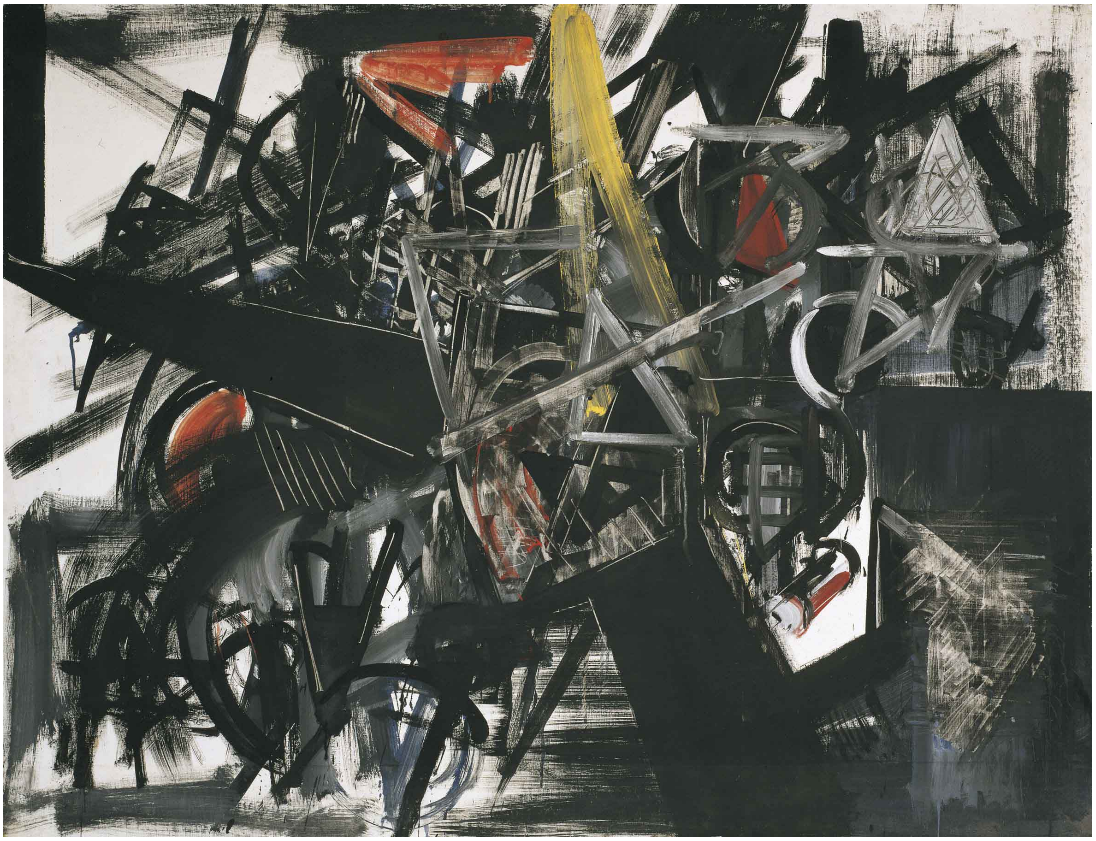
• life and death, survival and destruction were entirely left to ... the *pure randomness* of falling bombs

Traces

At the end of WW2, traces of humanity were left amidst the desolation

Georges Mathieu, Evanescence, 1945

European art began to come to terms with the need for a new language of expression, articulating remnants and traces in the surrounding chaos, in the arbitrariness of destruction



Emilio Vedova, Sbarramento, 1951

- Arte Informale/Art Informel: Alberto Burri, Emilio Vedova, Georges Mathieu, Antoni Tàpies ...
- Abstract Expressionism: Hans Hofmann, Janet Sobel, Jackson Pollock, Lee Krasner ...
- Conceptual Art: Sol LeWitt ...
- Generative Art: Ellsworth Kelly, Hiroshi Kawano, ...

Randomness also developed as a compositional form in music and literature

Randomness and the Cosmos

Randomness becomes a poetic way of reconnecting a devastated humanity to the cosmos, in a universe where the illusion of a supernatural god no longer makes sense

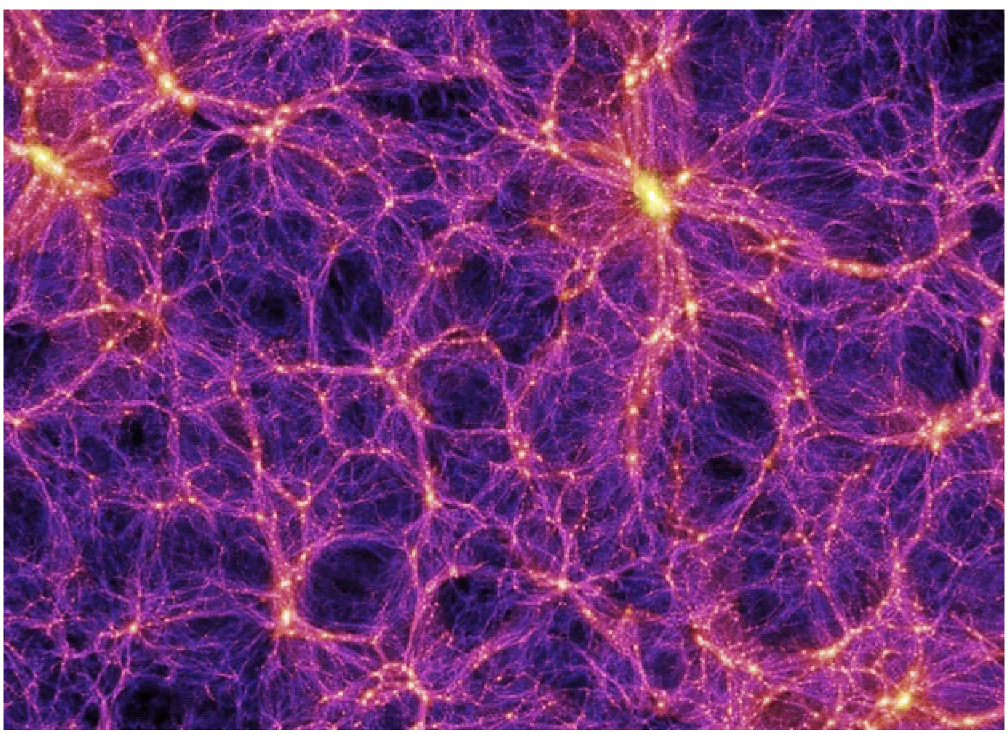
Janet Sobel, Milky Way, 1945

The Cosmic Web

• Until recently ($\sim$ 1989) astronomers believed galaxy clusters were uniformly distributed in the universe

• in 1989 Margaret Geller and John Huchra discovered a larger scale structure, the "great wall": galaxies accumulate on a 2-dimensional structure

• since then: walls and filaments, separated by voids, a foam structure (cosmic web)



... but what is randomness?

Probabilities and frequencies

• a set (alphabet) $A = \{\ell_1, \ldots, \ell_N\}$ (or faces of a dice, or graphical symbols,...)

• probabilities assigned to the ℓ_i:

$$\{p_1, \ldots, p_N\} \quad p_i \geq 0 \quad p_1 + \cdots + p_N = 1$$

(Fair coin: $p_1 = 1/2 = p_2$)

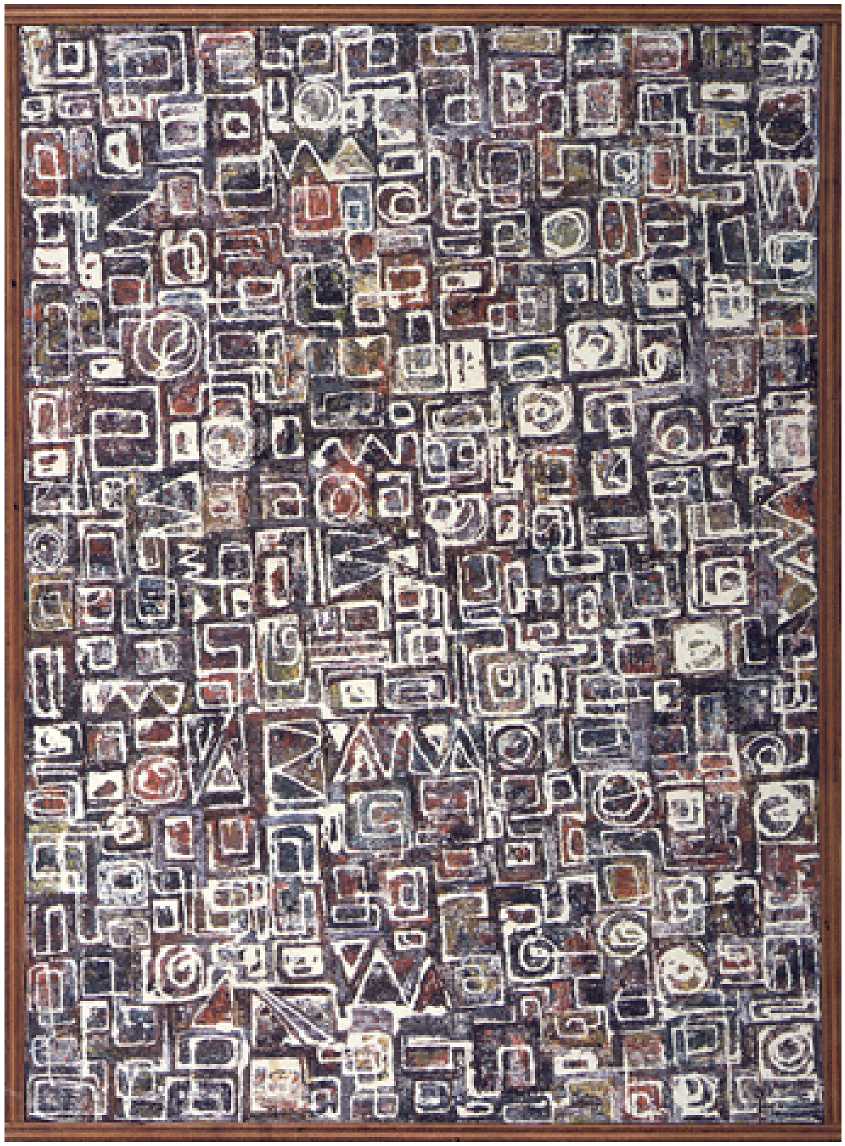
• Over longer and longer sequences in the alphabet the *frequencies* of occurrence of the symbols approaches the *probability* (law of large numbers)

• the Shannon entropy (information)

$$S = -\sum_{i=1}^{N} p_i \log(p_i)$$

average information (in bits) in an event drawn according to the probability distribution

• Shannon entropy maximal for uniform $p_i = 1/N$: very difficult to predict outcome, minimal for $p_1 = 1$ and $p_i = 0$: easy to predict... measure of "unpredictability"



Lee Krasner, Composition, 1949

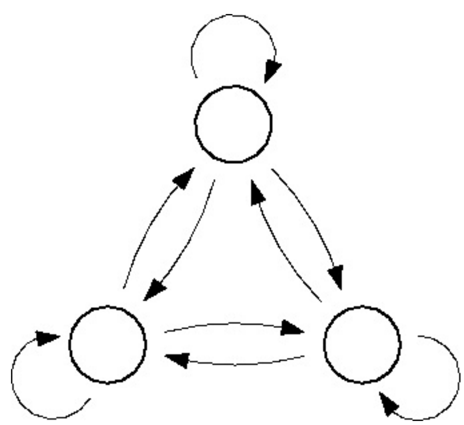
...but esthetically chosen frequencies/probabilities more "interesting" when neither uniform nor extremal: intermediate non-uniform where Shannon entropy neither maximal nor minimal

Markov chain: with one step memory

- p_{ij} = probability of going from state i to state j ($p_{ij} \geq 0$ and $\sum_j p_{ij} = 1$)

- π_i = probabilities of initial state ($\pi_i \geq 0$ and $\sum_i \pi_i = 1$)

$$\sum_i \pi_i p_{ij} = \pi_j$$

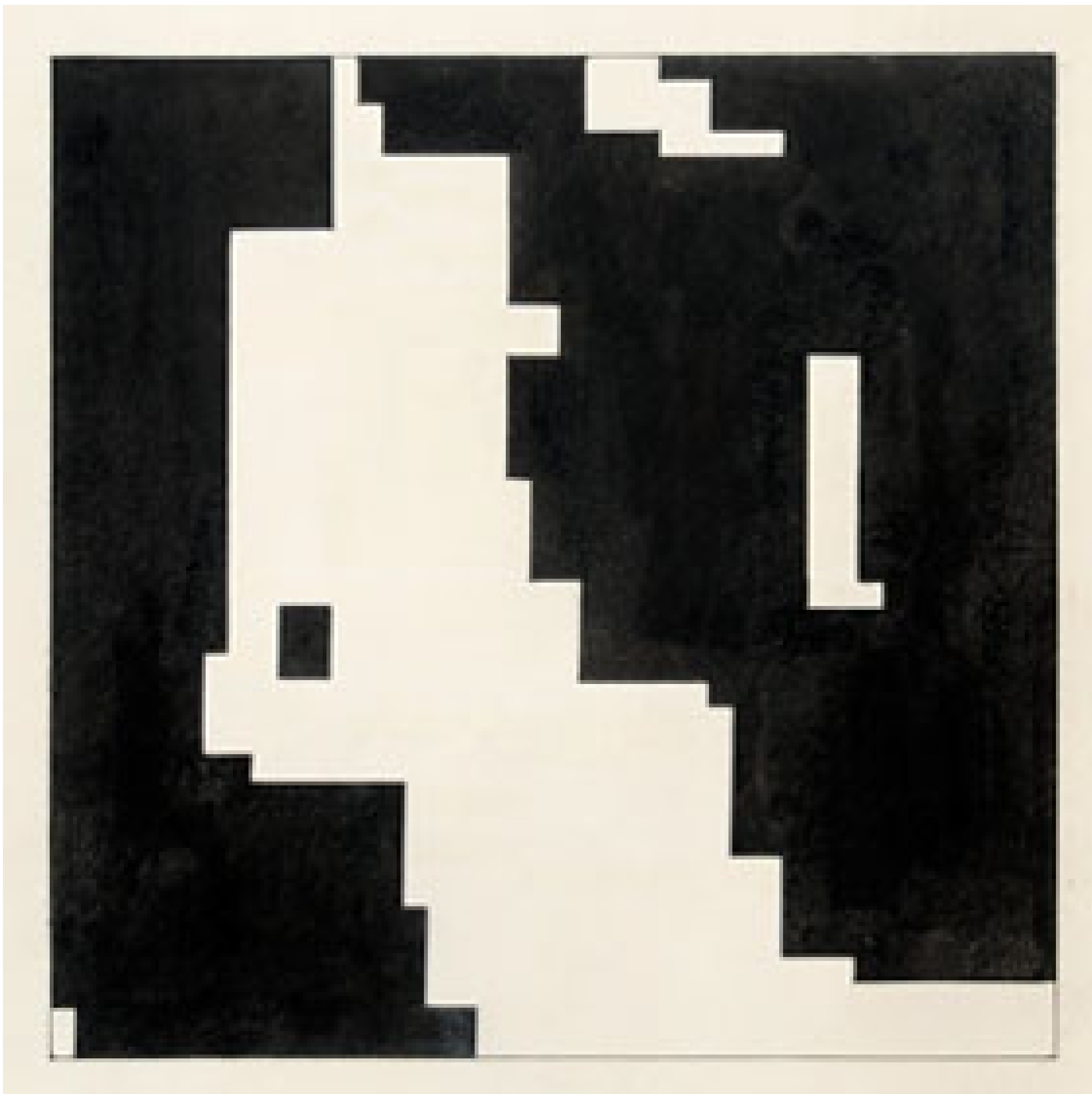


- Entropy:

$$S = -\sum_i \pi_i \sum_j p_{ij} \log(p_{ij})$$

Generative Art (produced with the intervention of autonomous agents – computer programs, cellular automata, neural networks – involving random processes)

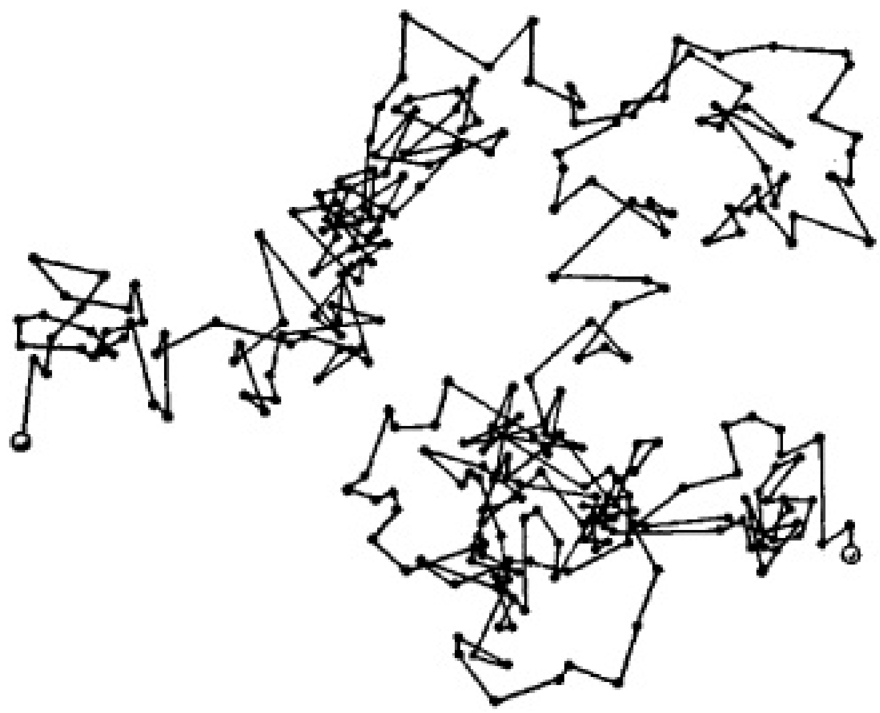
Hiroshi Kawano, Design 3-1: Color Markov Chain Pattern, 1964



Hiroshi Kawano, Design 2-3: Markov Chain Pattern, 1964

Brownian motion and stochastic processes

Serious interest in randomness in science began in the XIX century with the study of Brownian motion, rigorously explained by Einstein in 1905

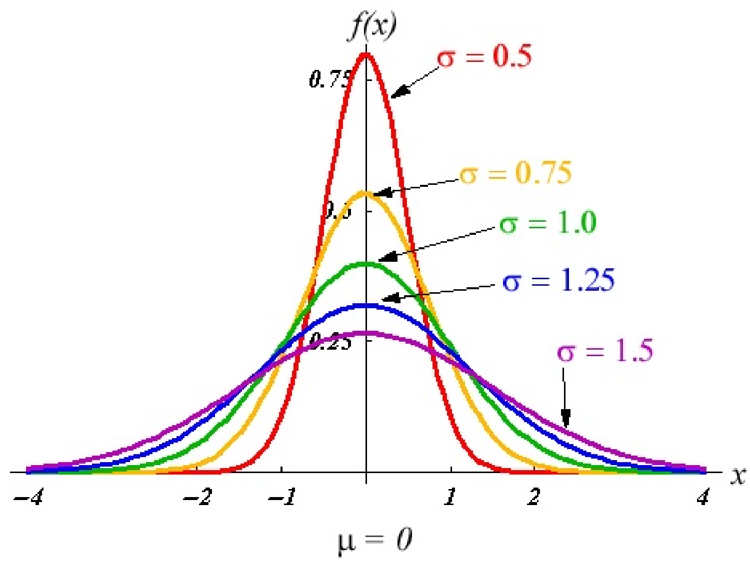


Brownian motion: random “dance” of particles in a fluid... first described by Lucretius in his “De Rerum Natura” for dust particles in the air

Camille McPhee, Mutated Brownian Motion II, 2011

Einstein's model of Brownian motion: diffusion equation for density of Brownian particles gives

$$\rho(x,t) = \frac{e^{-\frac{x^2}{4Dt}}}{\sqrt{4\pi Dt}}$$



• Norbert Wiener: continuous-time stochastic process $W_t - W_s \sim N(0, t-s)$ **normal distribution**

$$N(\mu,\sigma) = \frac{e^{-\frac{(x-\mu)^2}{2\sigma}}}{\sqrt{2\pi\sigma}}$$

$W_0 = 0$, W_t almost surely continuous, $W_{t_1} - W_{s_1}$ and $W_{t_2} - W_{s_2}$ independent random variables for non-overlapping time intervals $s_1 < t_1 \leq s_2 < t_2$

Camille McPhee, Mutated Brownian Motion, 2011

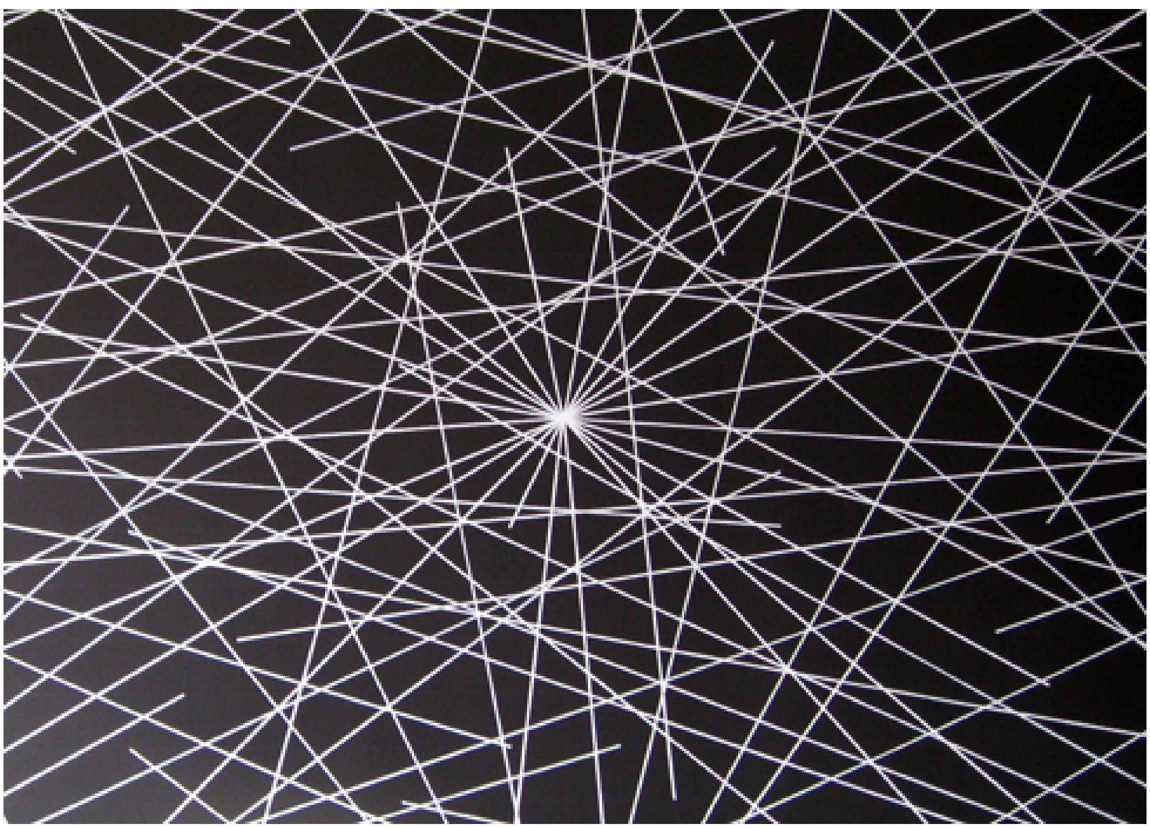
Elena Kozhevnikova, Brownian Motion II, 2013(?)

Uniform randomness

"Lines not short, not straight, crossing and touching, drawn at random, using four colors, uniformly dispersed with maximum density, covering the entire surface of the wall" (Sol Lewitt)

Sol LeWitt's wall drawing being made at Dia Beacon

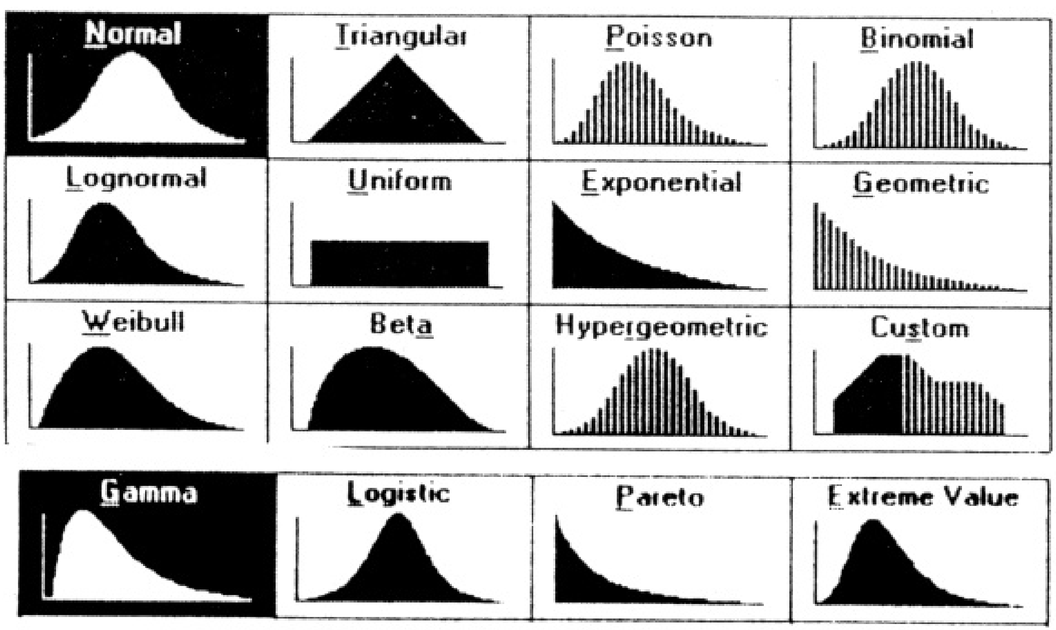
... but what does *"at random ... uniformly dispersed with maximum density"* mean? what *kind of randomness*?



Sol LeWitt, wall drawing 289, 1978

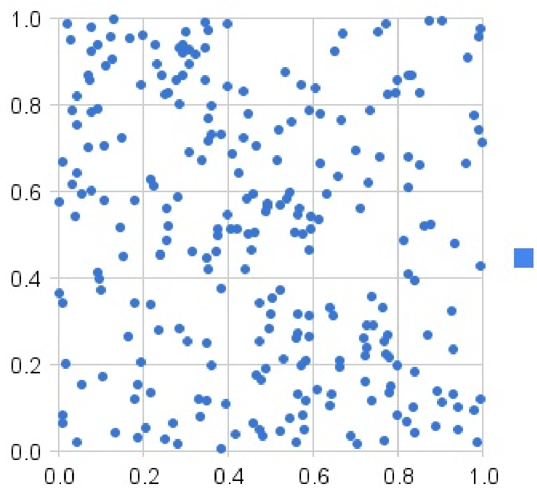
Different kinds of randomness

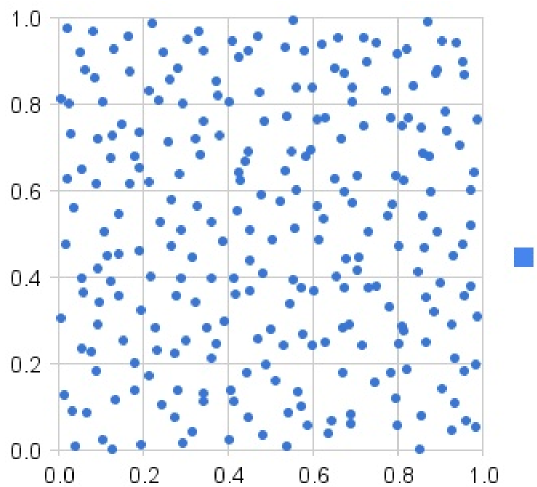
Continuous: Normal distribution, uniform distribution, triangular distribution,...



Discrete: Bernoulli distribution, Poisson distribution, hypergeometric distribution,...

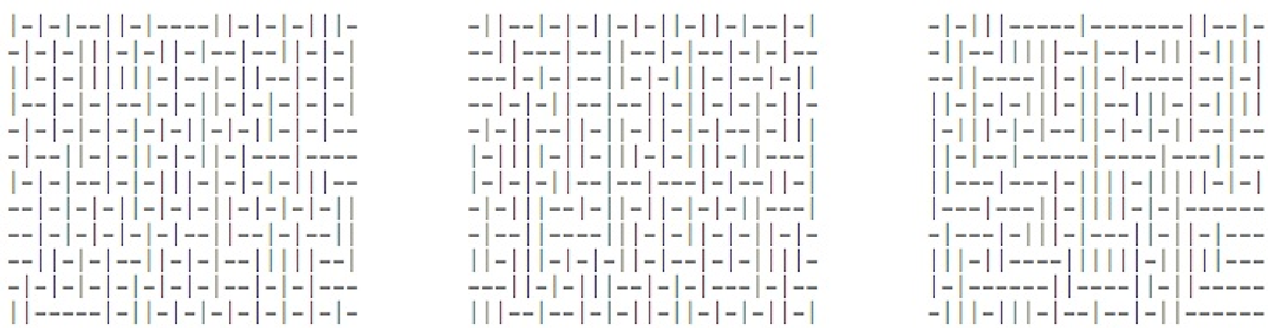
What is truly random?





which of these plots used the uniform distribution?

Randomness is not always what meets the eyes



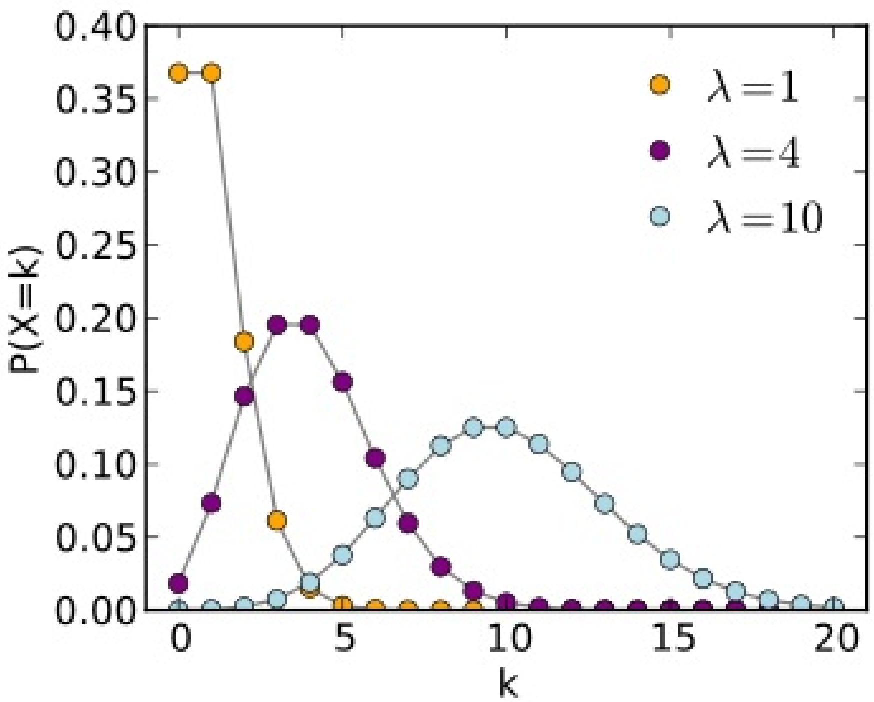
(A) (B) (C)

which of these plots is drawn using a uniform Bernoulli distribution? (vertical and horizontal dash both have probability $1/2$ *and next choice independent of previous one)*

The Poisson distribution

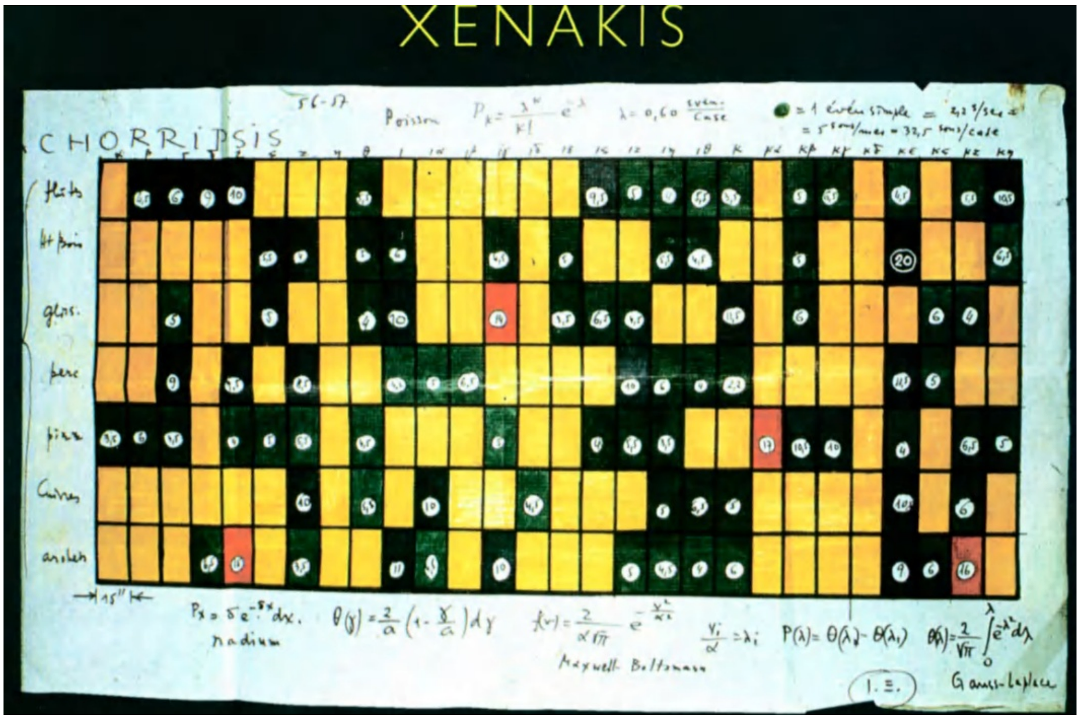
Discrete random variable X:
Probability of $X = k$ is

$$P_k = \frac{\lambda^k}{k!} e^{-\lambda}$$



Iannis Xenakis's *Achorripsis*
($A\chi o\varsigma$ = sound, $\rho\iota\psi\eta$ = cast)

The Poisson distribution in musical composition (Iannis Xenakis)



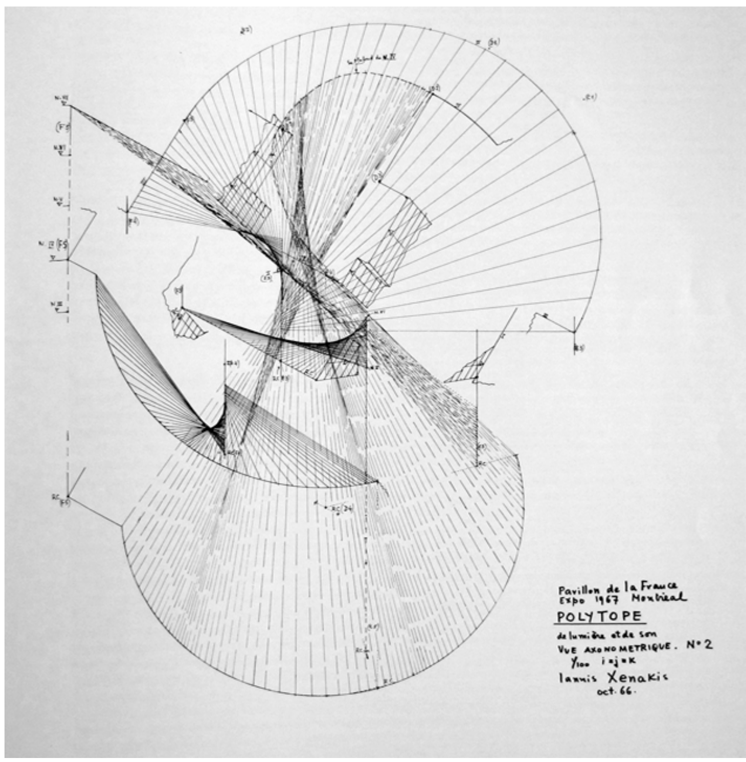
Xenakis distributed different “sound densities” over the 28×7 “cells” with probabilities generated by Poisson distribution, $\lambda = 0.6$

https://www.youtube.com/watch?v=WasFTDq0dJI

Linda M. Arsenault, *Iannis Xenakis’s Achorripsis: the matrix game*, Computer Music Journal, Vol.26 (2002) N.1, 58–72.

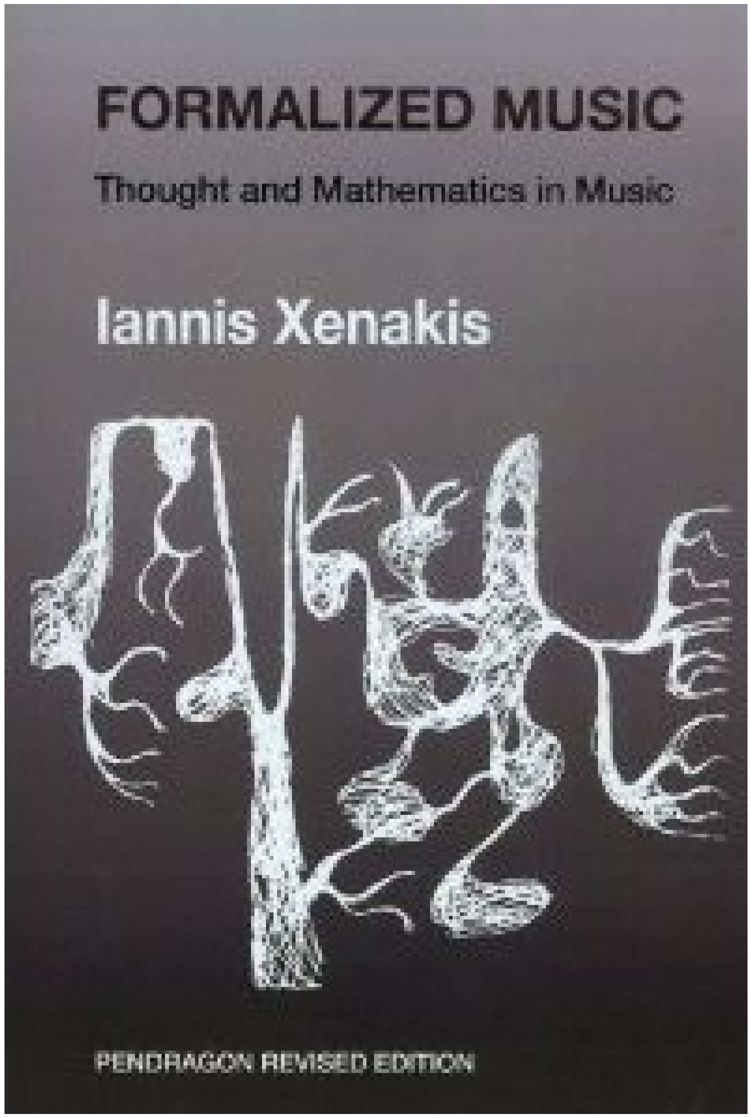
Iannis Xenakis and stochastic music

engineer, architect, worked with Le Corbusier, one of the most famous music composers of postwar Europe



"*Every probability function* is a particular stochastic variation, which *has its own personality*... Poisson, exponential, normal, uniform, Cauchy, logistic distributions"

S.Kanach, C.Lovelace, *Iannis Xenakis: composer, architect, visionary*, The Drawing Center, 2011.



Iannis Xenakis, *Formalized music: thought and mathematics in music*, Pendragon Press, 1992.

All the colors of noise

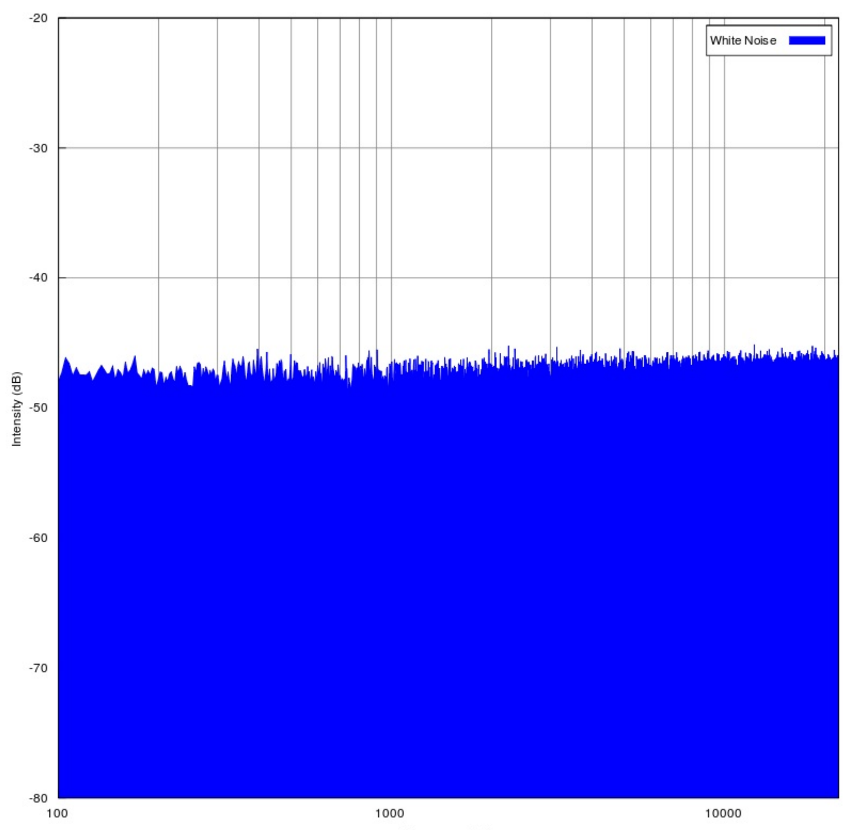
A random signal $x(t)$: decompose into frequencies (truncated Fourier transform)

$$\widehat{x}_T(\omega) = \frac{1}{\sqrt{T}} \int_0^T x(t) e^{-i\omega t} dt$$

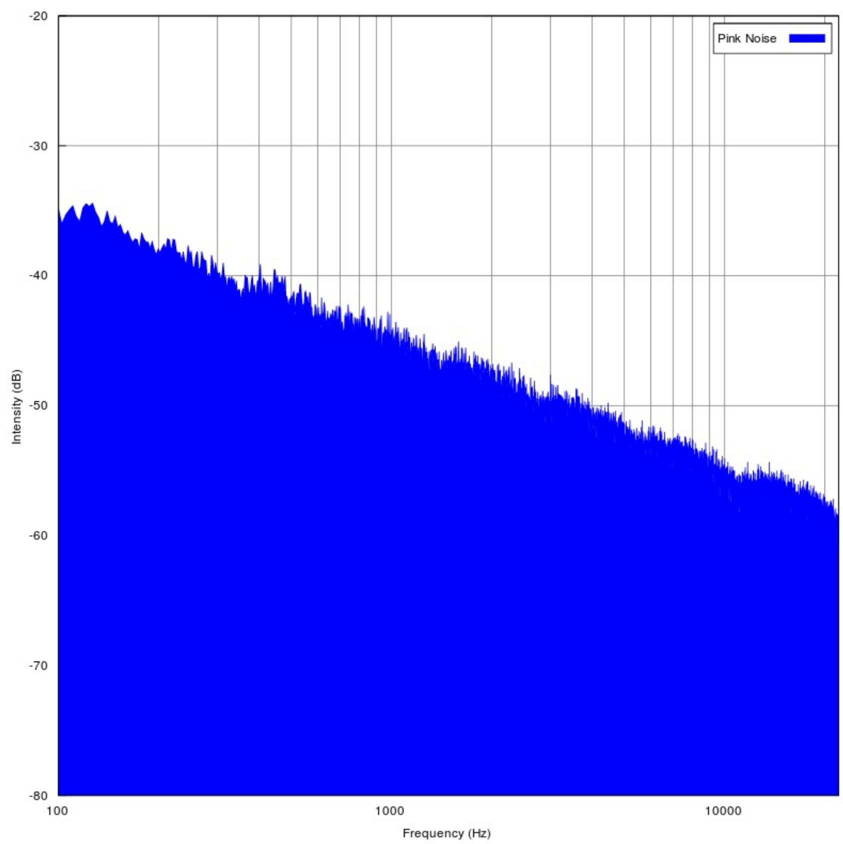
$\Rightarrow$ Power spectral density (expectation value)

Power-law noise: spectral density $\sim 1/\omega^\alpha$

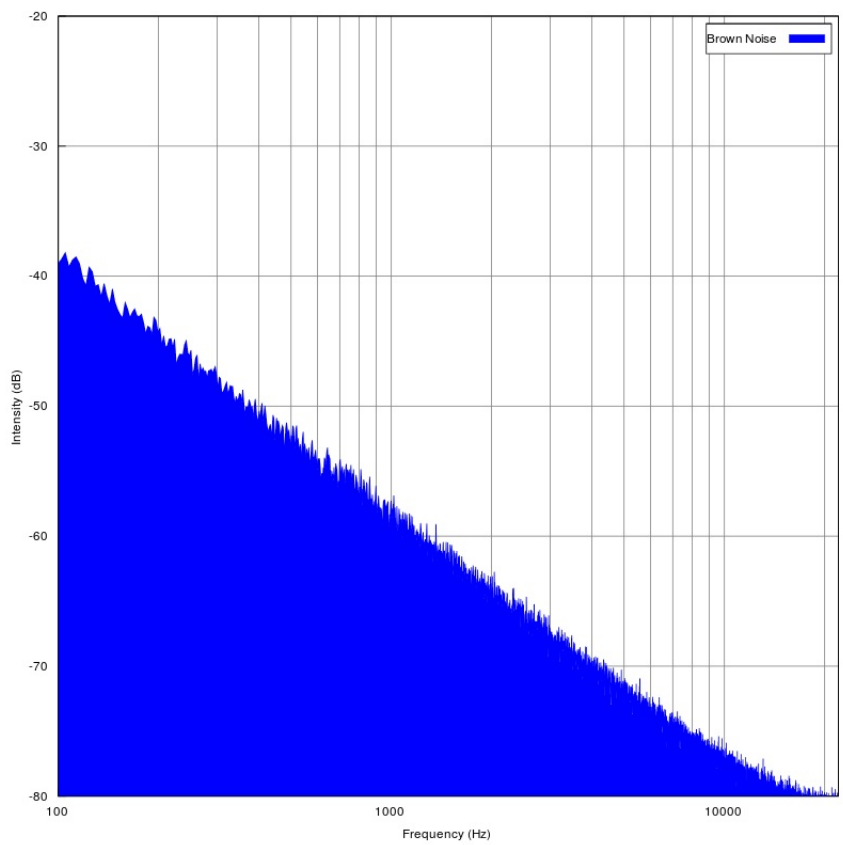
- White noise $\quad \alpha = 0$



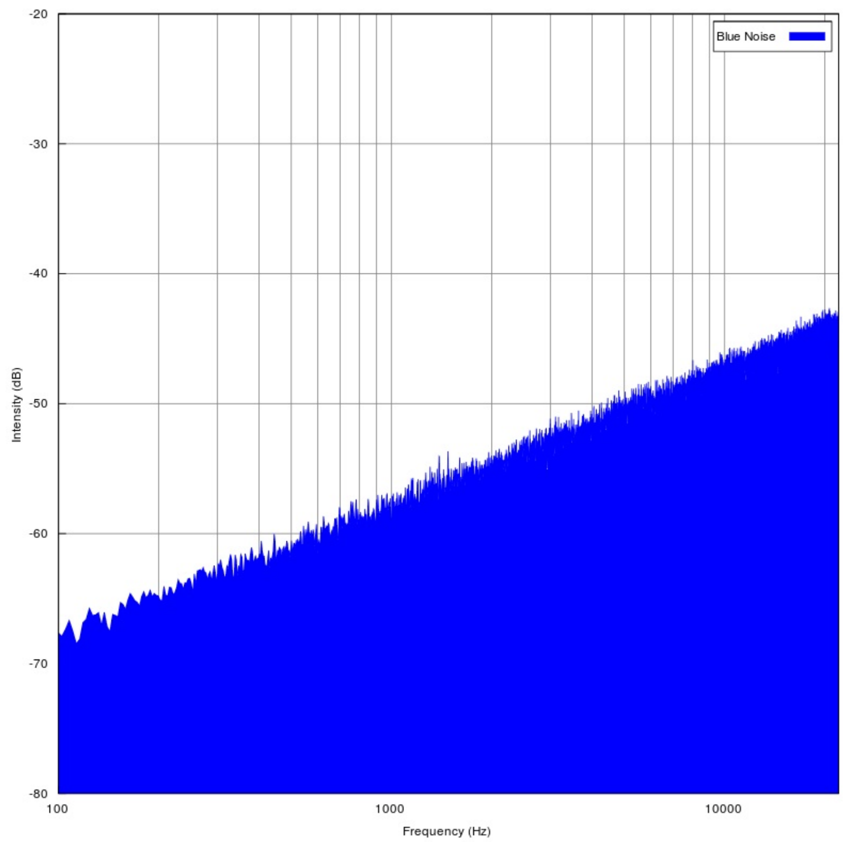
• Pink noise $\alpha = 1$



- Brown noise (Brownian) $\alpha = 2$



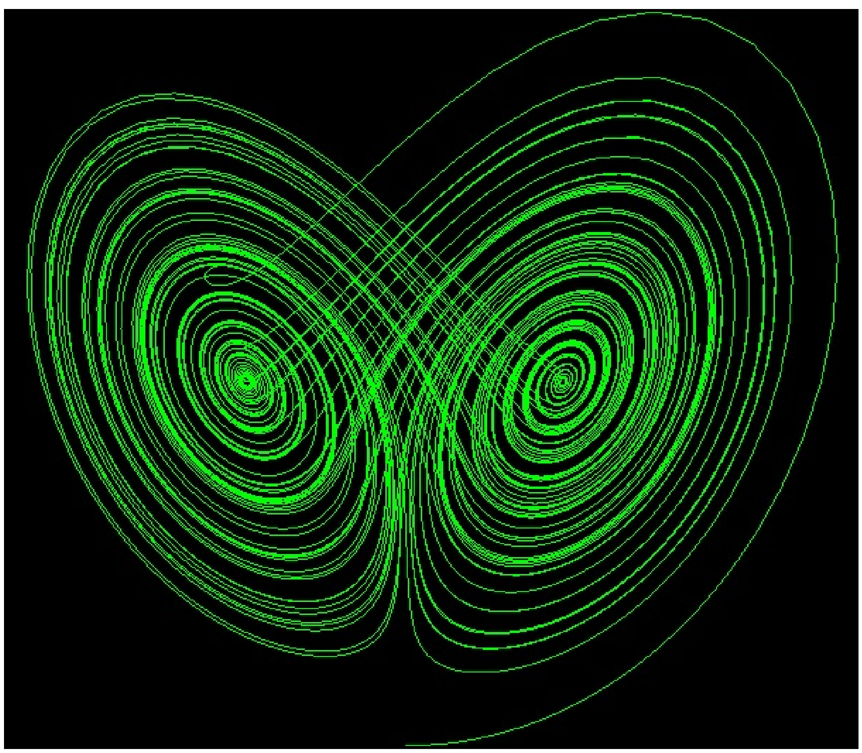
• Blue noise $\alpha = -1$



Chaos versus randomness

• Chaos theory developed from early ideas of Poincaré (1880's) on non-periodic orbits of dynamical systems

• Came to prominence in the 1960's: Edward Lorenz's *strange attractor*

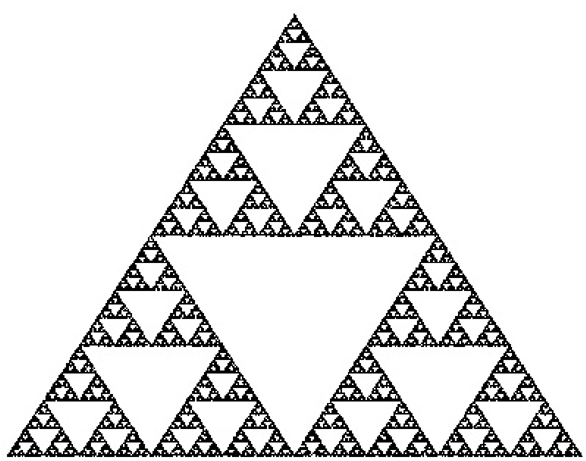


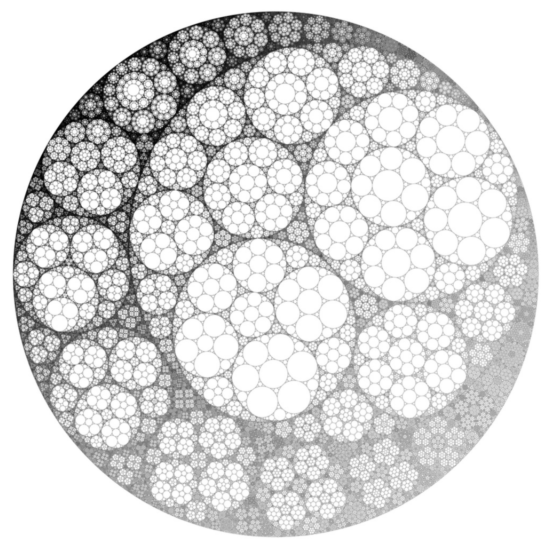
... but Chaos is *not* randomness, it is deterministic! ... sensitive dependence on initial conditions

Chaos and Fractals

• The strange attractors in chaotic dynamical systems are typically *fractals*

• Fractals are spaces of non-integer dimension: curves of dimension between one and two, surfaces of dimension between two and three...





• Fractal dimension: how volume scales with size

• Self-similarity

The fractal dimension of Pollock's drip paintings

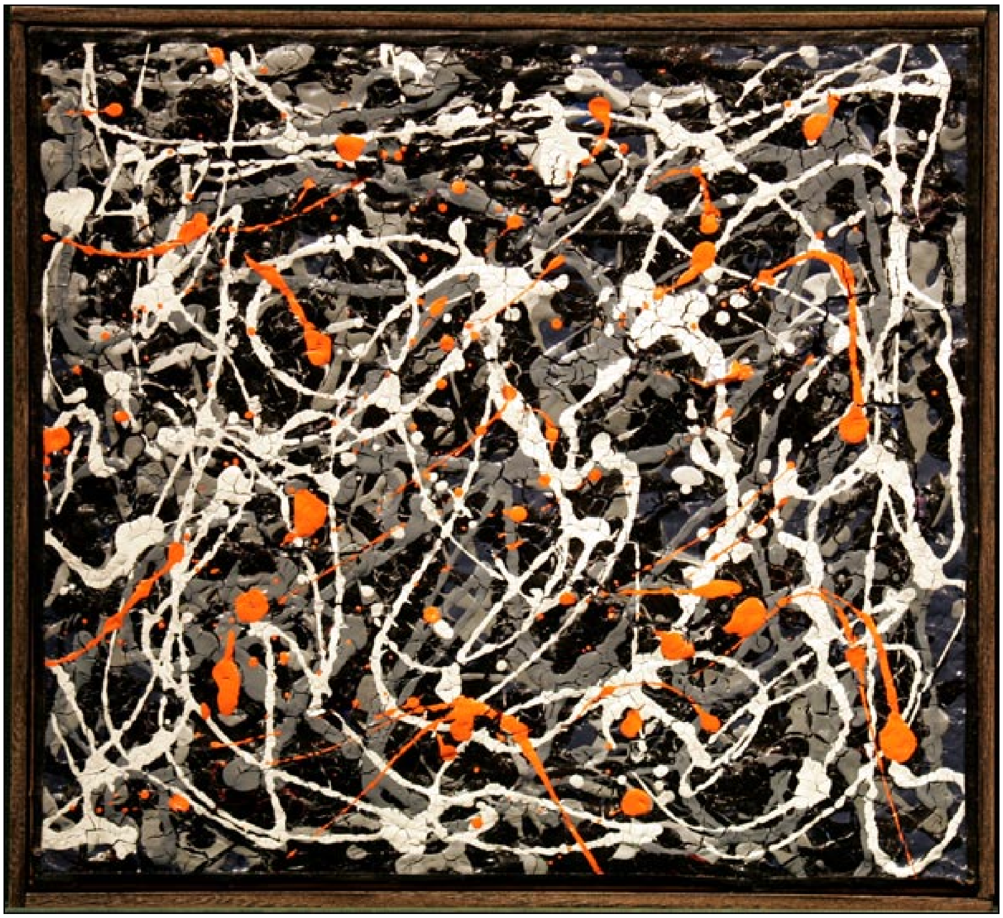
Jackson Pollock, One: N.31, 1950

- the fractal dimension of Pollock's paintings increased from $D \sim 1$ in 1943 to $D = 1.72$ in 1952 (increased complexity of paintings)

Richard P. Taylor, Adam Micolich and David Jonas, *Fractal analysis of Pollock's drip paintings*, Nature, Vol. 399 (1999) 422-423

- reliable method to date Pollock's paintings
- can also be used to detect fakes !?

The fake Pollocks controversy



• in 2003 Taylor claims 24 putative Pollock painting are fakes, based on fractal analysis
• two other physicists criticize the methodology used by Taylor

Kate Jones-Smith and Harsh Mathur, *Fractal Analysis: Revisiting Pollock's drip paintings*, Nature, Vol. 444 (2006) E9

Pollock's entropy dimension

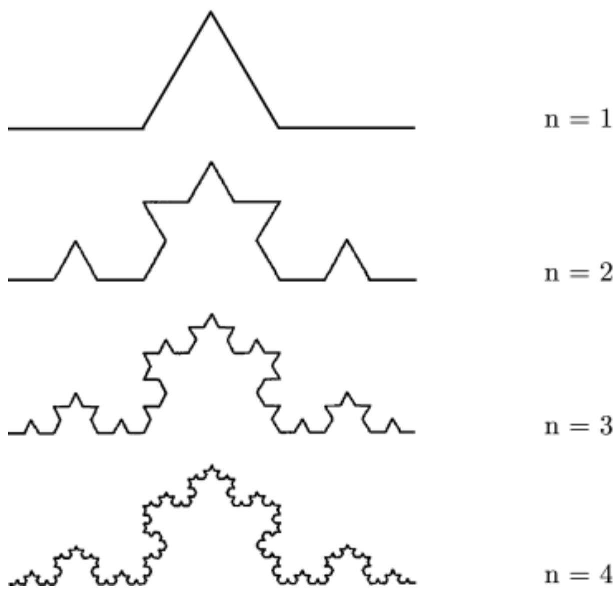
• in 2008 a new mathematical measure of "structured randomness" of Pollock paintings introduced: *entropy dimension*
• confirmed Taylor's fake claim

Jim Coddington, John Elton, Daniel Rockmore, Yang Wang, *Multifractal analysis and authentication of Jackson Pollock paintings*, MOMA

Deterministic fractals are very homogeneous (same fractal dimension everywhere; different notions of fractal dimension agree); *Random* fractals have varying local dimension (stratification by subfractals: multifractal)

D.-J. Feng and Y. Wang, *A class of self-affine measures*, J. Fourier Anal. and Appl. 11, (2005) 107–124

A deterministic fractal:



A random fractal:

local dimension (entropy dimension)

Jackson Pollock, One: N.31, 1950, detail

Pollock's fluid dynamics

• More mathematical and physical methods: *fluid dynamics* (how patterns fluids form as they fall depend on viscosity and speed)

Andrzej Herczyński, Claude Cernuschi, L. Mahadevan, *Painting with drops, jets, and sheets*, Phys. Today, Vol.64 N.6 (2011) 31–40

- is there a conclusion? ... mathematical analysis alone is not sufficient to authenticate Pollock's paintings, but it provides interesting methods for studying them

The Philosophy of Randomness

Stanislaw Lem, *Filozofia Przypadku*, 1968

Philosophy of Chance