

# Spectral Triples and Supersymmetry

Matilde Marcolli

Ma148b Spring 2016  
Topics in Mathematical Physics

## Reference:

- Wim Beenakker, Thijs van den Broek, Walter van Suijlekom, *Supersymmetry and Noncommutative Geometry*, Springer, 2015

(also all images in this lecture slides are from this book)

# Minimally Supersymmetric Standard Model

| Superfield                     | Spin  |                 |               | Representation             |
|--------------------------------|-------|-----------------|---------------|----------------------------|
|                                | 0     | $\frac{1}{2}$   | 1             |                            |
| Left-handed (s)quark           | $Q_L$ | $\tilde{q}_L$   | $q_L$         | $-(1/6, 2, 3)$             |
| Up-type (s)quark               | $U_R$ | $\tilde{u}_R$   | $u_R$         | $-(2/3, 1, 3)$             |
| Down-type (s)quark             | $D_R$ | $\tilde{d}_R$   | $d_R$         | $-(1/3, 1, 3)$             |
| Left-handed (s)lepton          | $L_L$ | $\tilde{l}_L$   | $l_L$         | $-(1/2, 2, 1)$             |
| Up-type (s)lepton              | $N_R$ | $\tilde{\nu}_R$ | $\nu_R$       | $(0, 1, 1)$                |
| Down-type (s)lepton            | $E_R$ | $\tilde{e}_R$   | $e_R$         | $-(1, 1, 1)$               |
| Gluon, gluino                  | $V$   | $-$             | $g$           | $g_\mu (0, 1, 8)$          |
| $SU(2)$ gauge bosons, gauginos | $W$   | $-$             | $\lambda$     | $\mathbf{W}_\mu (0, 3, 1)$ |
| $B$ -boson, bino               | $B$   | $-$             | $\lambda_0$   | $B_\mu (0, 1, 1)$          |
| Up-type Higgs(ino)             | $H_u$ | $h_u$           | $\tilde{h}_u$ | $-(1/2, 2, 1)$             |
| Down-type Higgs(ino)           | $H_d$ | $h_d$           | $\tilde{h}_d$ | $-(1/2, 2, 1)$             |

Numbers:  $U(1)$ -hypercharge; dim of  $SU(2)$ -representation (1=singlet, 2=fundamental, 3=adjoint); dim of  $SU(3)$ -representation (1,3,8)

## R-parity

$$R = (-1)^{2S+3B+L}$$

$S$  = spin;  $B$  = baryon number;  $L$  = lepton number

All SM particles (and the additional Higgs doublet) has  $R = +1$ ;  
all superpartners have  $R = -1$

## Advantages of MSSM

- Higgs mass more stable under loop-corrections (contributions from superpartners compensate)
- $R$ -symmetry preserved: lightest  $R = -1$  particle cannot decay: cold dark matter?
- RGE for coupling constant form much smaller triangle: unification?

## Real even spectral triples

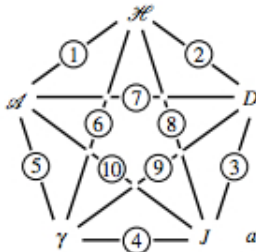
$$\textcircled{1} \quad \mathcal{A} : \mathcal{H} \rightarrow \mathcal{H}$$

$$\textcircled{2} \quad D^* = D, \text{ comp. res.}$$

$$\textcircled{3} \quad JD = \epsilon' DJ$$

$$\textcircled{4} \quad J\gamma = \epsilon'' \gamma J$$

$$\textcircled{5} \quad [\gamma, \mathcal{A}] = 0$$



$$\gamma^* = \gamma, \gamma^2 = \text{id}_{\mathcal{H}} \quad \textcircled{6}$$

$$[a, D] \in \mathcal{B}(\mathcal{H}) \quad \textcircled{7}$$

$$J^2 = \epsilon \text{id}_{\mathcal{H}} \quad \textcircled{8}$$

$$\gamma D = -D\gamma \quad \textcircled{9}$$

$$a^o = Ja^*J^*, [a, b^o] = 0 \quad \textcircled{10}$$

All previously discussed relations between the elements

$$(\mathcal{A}, \mathcal{H}, D, J, \gamma)$$

of a real even spectral triple

## SUSY: $R$ -extended real even spectral triples

- additional  $\mathbb{Z}/2\mathbb{Z}$ -grading  $R : \mathcal{H} \rightarrow \mathcal{H}$

$$\mathcal{H} = \mathcal{H}_{R=+1} \oplus \mathcal{H}_{R=-1}$$

projectors  $(1 \pm R)/2$

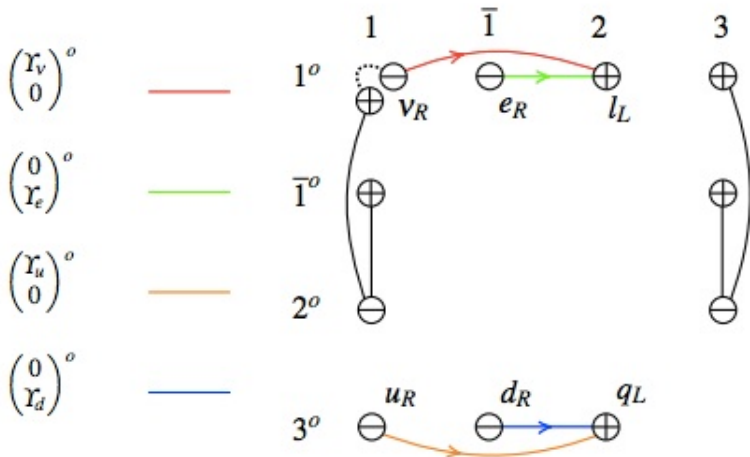
- compatibility:  $\forall a \in \mathcal{A}$

$$[R, \gamma] = [R, J] = [R, \pi(a)] = 0$$

Dirac splits:  $D = D_+ + D_-$  with  $\{D_-, R\} = [D_+, R] = 0$

**Krajewski diagrams:** color vertices black/white for  $R = \mp 1$

Reminder: the finite spectral triple of the  $\nu$ MSM



Dirac operator components: Yukawa matrices  $Y$  and (dotted line)  
Majorana mass matrix

## SUSY finite spectral triples: building blocks

- Adjoint representations
- Non-adjoint representations
- Extra interactions
- Majorana terms
- Other mass terms
- Interactions between blocks



## Adjoint representation

- case of a single component  $\mathcal{A} = M_{N_j}(\mathbb{C})$
- bimodule  $\mathbb{C}^{N_j} \otimes \mathbb{C}^{N_j^o} \simeq M_{N_j}(\mathbb{C})$ , notation:  $\mathbf{N}_j \otimes \mathbf{N}_j^o$
- KO-dim 6:  $\{J, \gamma\} = 0$  (compatible with SM part): two copies with  $J$  exchanging them (and takes adjoint)

$$J(m, n) = (n^*, m^*)$$

so grading  $\gamma$  with opposite values on the two copies

- $\mathcal{B}_j = M_{N_j, L}(\mathbb{C}) \oplus M_{N_j, R}(\mathbb{C})$  plus other stuff in  $End(\mathcal{H}_F)$  (adj rep)
- $R$ -parity:  $R|_{M_{N_j}(\mathbb{C})} = -1$

These building blocks give a **supersymmetric Yang–Mills action**

$$N_j$$
$$N_j^o$$
$$+$$
$$-$$

- matching degrees of freedom for gauginos and gauge bosons
  - discard trace part of the fermion
  - add a non-propagating auxiliary field  $G_j$  (fixes the mismatch of degrees of freedom for off-shell theory)
  - on-shell agrees with the spectral action result

**Action** for this type of block

$$(\lambda'_{j,L}, \lambda'_{j,R}) \in L^2(\mathbb{S}_+ \otimes M_{N_{j,L}}(\mathbb{C})) \otimes L^2(\mathbb{S}_- \otimes M_{N_{j,R}}(\mathbb{C}))$$

$$S(\lambda, \mathbb{A}) = \langle J_M \lambda'_{j,R}, \not{D}_{\mathbb{A}} \lambda'_{j,L} \rangle - \frac{f(0)}{24\pi^2} \int_M \text{Tr}(F_{\mu\nu}^j F^{j,\mu\nu}) + \mathcal{O}(\Lambda^{-2})$$

- with additional term  $-\frac{1}{2} \int_M \text{Tr} G_j^2$
- scaled to have normalized kinetic term  $\lambda_j \mapsto \frac{1}{\sqrt{n_j}} \lambda_j$  with  $n_j \delta_{ab} = \text{Tr} T_j^a T_j^b$
- with explicit transformation  $\delta A_j, \delta \lambda_{j,L/R}, \delta G_j$  under which action is supersymmetric
- block only with multiplicity one, otherwise breaking supersymmetry (same number of fermionic and bosonic degrees of freedom)

## Non-adjoint representations non-gaugino fermions

- off-diagonal blocks  $\mathbf{N}_i \otimes \mathbf{N}_j^o$
- with opposite values of  $\gamma$ -grading so KO-dim 6
- to get supersymmetry: need bosonic scalar superpartner, have interaction with gauge fields so need also gaugino degrees of freedom
- can achieve by combining a block  $\mathbf{N}_i \otimes \mathbf{N}_j^o$  like this with two blocks  $\mathcal{B}_i$  and  $\mathcal{B}_j$  as before

$$\mathbf{N}_i \otimes \mathbf{N}_j^o \oplus M_{N_i,L}(\mathbb{C}) \oplus M_{N_i,R}(\mathbb{C}) \oplus M_{N_j,L}(\mathbb{C}) \oplus M_{N_j,R}(\mathbb{C}) \oplus \mathbf{N}_j \otimes \mathbf{N}_i^o$$

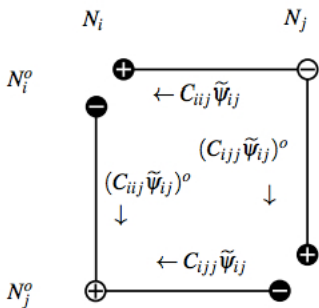
General form of Dirac operator  $D$  with  $D^* = D$ ,  $\{D, \gamma\} = 0$  and  $[D, J] = 0$ :

$$D_F = \begin{pmatrix} 0 & 0 & A & 0 & B & 0 \\ 0 & 0 & M_i & 0 & 0 & JA^*J^* \\ A^* & M_i^* & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & M_j & JB^*J^* \\ B^* & 0 & 0 & M_j^* & 0 & 0 \\ 0 & JAJ^* & 0 & JBJ^* & 0 & 0 \end{pmatrix}$$

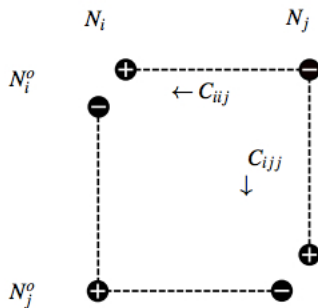
with  $A : M_{N_i}(\mathbb{C})_R \rightarrow \mathbf{N}_i \otimes \mathbf{N}_j^o$  and  $B : M_{N_j}(\mathbb{C})_R \rightarrow \mathbf{N}_i \otimes \mathbf{N}_j^o$ .

$M_i, M_j$  supersymmetry breaking gaugino masses: set  $M_i = M_j = 0$

Observation: if  $A$  and  $B$  differ by a single complex scalar  $\Rightarrow$  scalar field  $\tilde{\psi}_{ij}$  (in same rep as fermion with  $R = -1$ : sfermion)



(a) The case of an off-diagonal representation with  $R = 1$ .



(b) The case of an off-diagonal representation with  $R = -1$ .

Action for this second type of block

$$S_{ij}(\lambda'_i, \lambda'_j, \psi_L, \bar{\psi}_R, \mathbb{A}_i, \mathbb{A}_j, \tilde{\psi}, \bar{\tilde{\psi}})$$

Fermionic part

$$\begin{aligned} = & \langle J_M \bar{\psi}_R, D_A \psi_L \rangle + \langle J_M \bar{\psi}_R, \gamma^5 \lambda'_{iR} C_{ij} \tilde{\psi} \rangle + \langle J_M \bar{\psi}_R, \gamma^5 C_{ijj} \tilde{\psi} \lambda'_{jR} \rangle \\ & + \langle J_M \psi_L, \gamma^5 \bar{\tilde{\psi}} C_{ij}^* \lambda'_{iL} \rangle + \langle J_M \psi_L, \gamma^5 \lambda'_{jL} \bar{\tilde{\psi}} C_{ijj}^* \rangle, \end{aligned}$$

Bosonic part

$$\int_M |\mathcal{N}_{ij} D_\mu \tilde{\psi}|^2 + \mathcal{M}_{ij}(\tilde{\psi}, \bar{\tilde{\psi}})$$

with self-adjoint  $\mathcal{N}_{ij}$  square root of

$$\mathcal{N}_{ij}^2 = \frac{f(0)}{2\pi^2} (N_i C_{ij}^* C_{ij} + N_j C_{ijj}^* C_{ijj}) \text{ and } \mathcal{M}_{ij} \text{ given by}$$

$$\mathcal{M}_{ij}(\tilde{\psi}, \bar{\tilde{\psi}}) = \frac{f(0)}{2\pi^2} \left[ N_i |C_{ij} \tilde{\psi} \bar{\tilde{\psi}} C_{ij}^*|^2 + N_j |\bar{\tilde{\psi}} C_{ijj}^* C_{ijj} \tilde{\psi}|^2 + 2 |C_{ij} \tilde{\psi}|^2 |C_{ijj} \tilde{\psi}|^2 \right]$$

- now trace parts remain; give rise to  $u(1)$  fields
- matching degrees of freedom: identify these  $u(1)$  fields in pairs
- add off-shell  $G_j$  fields as before
- rescale  $\tilde{\psi}_{ij}$  for normalization of kinetic terms
- explicit supersymmetry transformation of the action



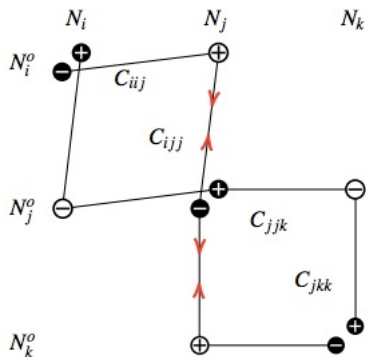
## More general $\mathcal{A}$

- algebra  $\mathcal{A} = M_{N_i}(\mathbb{C}) \oplus M_{N_j}(\mathbb{C})$
- only certain combinations of blocks will have supersymmetry: two disjoint blocks  $\mathcal{B}_i$  and  $\mathcal{B}_j$  of first kind
- same for  $\mathcal{A} = M_{N_i}(\mathbb{F}) \oplus M_{N_j}(\mathbb{F})$  with  $\mathbb{F} = \mathbb{R}, \mathbb{C}, \mathbb{H}$
- $\mathcal{A}$  with three blocks  $M_{N_{ijk}}(\mathbb{C})$
- if only building blocks of the second type not supersymmetric
- need to add **third type building block**  $\mathcal{B}_{ijk}$  to restore supersymmetry: accounts for the components of Dirac

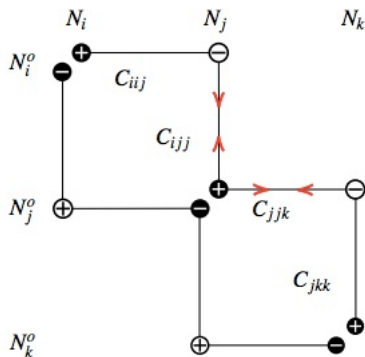
$$D_{ij}^{kj} : \mathbf{N}_k \otimes \mathbf{N}_j^o \rightarrow \mathbf{N}_i \otimes \mathbf{N}_j^o$$

and similar  $D_{jk}^{ik}$  and  $D_{ij}^{ik}$

- these  $D_{ij}^{kj}$  generate by inner fluctuations scalar field  $\tilde{\psi}_{ik}$  need corresponding fermions to restore supersymmetry: provided by  $\mathcal{B}_{ijk}$

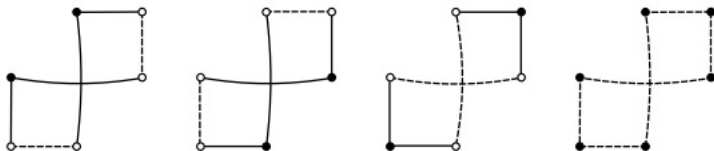
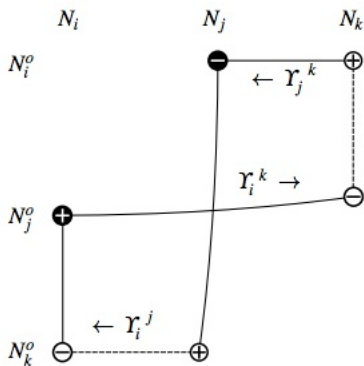


(a) Contributions when the gradings of the building blocks are different.



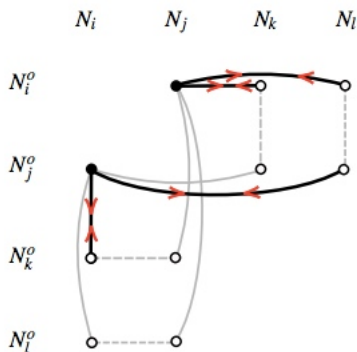
(b) Contributions when the gradings of the building blocks are the same.

Krajewski diagrams for two blocks of second type

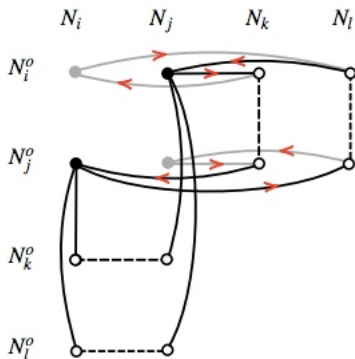


Krajewski diagrams for a block of third type and possible  $R$ -parity

Blocks of third type and second type  $\mathcal{B}_{ijk}, \mathcal{B}_{ijl}, \mathcal{B}_{ik}, \mathcal{B}_{il}$

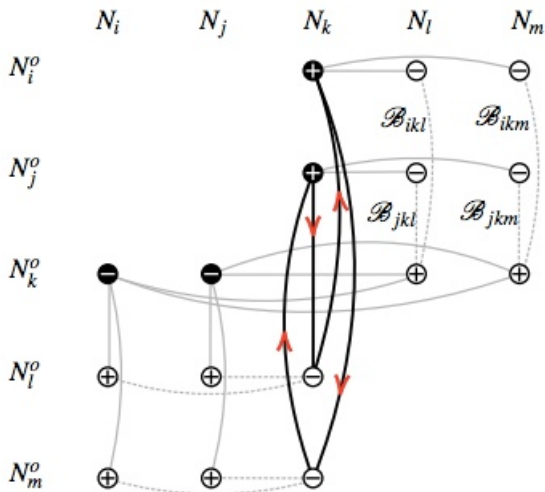


(a) Contributions corresponding to paths of which all four edges are from the building blocks  $\mathcal{B}_{ijk}$  and  $\mathcal{B}_{ijl}$  of the third type.



(b) Contributions corresponding to paths of which two edges are from building blocks  $\mathcal{B}_{ik}$  and  $\mathcal{B}_{il}$  of the second type.

Blocks  $\mathcal{B}_{ikl}, \mathcal{B}_{ikm}, \mathcal{B}_{jkl}, \mathcal{B}_{jkm}$  and (for supersymmetry)  $\mathcal{B}_{ij}$  or  $\mathcal{B}_{im}$



**Note:** no need for building blocks with more than three indices because data of finite spectral triples are

- components of the algebra (one index  $N_j$ ) and adjoint representation  $\mathbf{N}_j \otimes \mathbf{N}_j^o$
- non-adjoint representations  $\mathbf{N}_i \otimes \mathbf{N}_j^o$  (two indices)
- components of Dirac operator with order-one condition  $D_{ij}^{kj}$  (three indices)

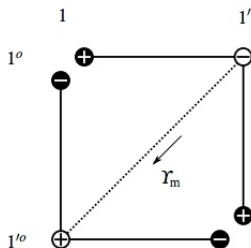
everything else (eg multiple components) is expressible in terms of these basic building blocks

## Majorana mass terms

- in the SM case: extension  $\nu$ MSM of the minimal standard model with right handed neutrinos and Majorana mass terms: also expect these terms in further supersymmetric extensions
- representations  $\mathbf{1} \otimes \mathbf{1}'^o \oplus \mathbf{1}' \otimes \mathbf{1}^o$  that are each other's antiparticles ( $(\mathbb{C} \oplus \mathbb{C})^{\oplus M}$  multiplicity  $M$ ), with the same component  $\mathbb{C}$  of algebra acting on both

$$D_{1'1}{}^{11'} : \mathbf{1} \otimes \mathbf{1}'^o \rightarrow \mathbf{1}' \otimes \mathbf{1}^o$$

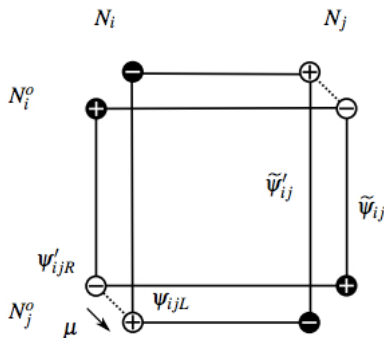
- dotted line in Krajewski diagram:



## Other mass terms

- two building blocks of second type with same indices and different grading
- on this part Dirac with self-adjoint, order-one, commuting with  $J$  gives a mass term in the action if it acts nontrivially only on generations (on the  $M$  copies in

$$((\mathbf{N}_i \otimes \mathbf{N}_j^o)_L \oplus (\mathbf{N}_j \otimes \mathbf{N}_i^o)_R \oplus \mathbf{N}_i \otimes \mathbf{N}_j^o)_R \oplus (\mathbf{N}_j \otimes \mathbf{N}_i^o)_L)^{\oplus M}$$





## Supersymmetry and the Spectral Action

- the construction of finite spectral triples with these building blocks ensures field content is supersymmetric
- it does *not* directly follow that the action is supersymmetric
- some problems:
  - a single building block first type  $\mathcal{B}_i$  with  $N_i = 1$  has vanishing bosonic interactions
  - a single building block second type  $\mathcal{B}_{ij}$  with  $R = -1$  has two interacting  $u(1)$ -fields (while gauginos don't)
  - if at least three complex matrix components in  $\mathcal{A}$ , and building blocks  $\mathcal{B}_{ij}$ ,  $\mathcal{B}_{ik}$  then two interacting  $u(1)$ -fields (while gauginos don't)
- approach: rewrite the four-scalar interactions generated by the spectral action as off-shell action via auxiliary fields
- general issue: new contributions for which one needs to obtain off-shell counterparts so that the action remains supersymmetric

## Supersymmetry Breaking

- if supersymmetry were exact in nature superpartners would be of equal mass as corresponding SM particles: obviously not the case
- so supersymmetric extensions of SM need a mechanism that breaks supersymmetry
- in MSSM one should have a SUSY-breaking Higgs that gives mass to SM particles and achieves electroweak symmetry breaking
- two possibilities: spontaneous SUSY breaking (disfavored phenomenologically: would have one slepton/squark lighter than corresponding SM fermion); or SUSY breaking Lagrangian
- SUSY breaking terms in Lagrangian should be *soft*:
  - couplings of positive mass dimension
  - no quadratically divergent loop corrections (hierarchy problem solution of SUSY: Higgs mass very sensitive to perturbative corrections, stability problem)

## Soft SUSY breaking

$$\begin{aligned}\mathcal{L}_{\text{soft}}^{\text{E}} = & \tilde{\psi}_{\alpha}^{*}(m^2)_{\alpha\beta}\tilde{\psi}_{\beta} - \left( \frac{1}{3!}A_{\alpha\beta\gamma}\tilde{\psi}_{\alpha}\tilde{\psi}_{\beta}\tilde{\psi}_{\gamma} - \frac{1}{2}B_{\alpha\beta}\tilde{\psi}_{\alpha}\tilde{\psi}_{\beta} + C_{\alpha}\tilde{\psi}_{\alpha} + h.c. \right) \\ & + \frac{1}{2}(M\lambda_a\lambda_a + h.c.).\end{aligned}$$

- self-adjoint mass term for scalar bosons  $\tilde{\psi}_{\alpha}$
- symmetric tensor  $A_{\alpha\beta\gamma}$  mass dim 1
- matrix  $B_{\alpha\beta}$  mass dim 2
- gauge singlet linear coupling  $C_{\alpha} \in \mathbb{C}$  mass dim 3
- gaugino mass terms,  $M \in \mathbb{C}$

**Question:** soft supersymmetry breaking in the Spectral Action?

## Soft SUSY breaking from $\Lambda^2$ terms in the Spectral Action

- scalar fields  $\tilde{\psi}_{ij} \in \mathcal{C}^\infty(M, \mathbf{N}_i \otimes \mathbf{N}_j^\circ)$
- fermions  $\psi_{ij} \in \mathcal{C}^\infty(M, \mathbb{S} \otimes \mathbf{N}_i \otimes \mathbf{N}_j^\circ)$
- gauginos  $\lambda_i \in L^2(M, \mathbb{S} \otimes M_{N_i}(\mathbb{C}))$  reduced to  $su(N_i)$  after eliminating trace degrees of freedom
- all possible terms arising from the Spectral Action:
  - scalar masses
  - gaugino masses
  - linear couplings
  - bilinear couplings
  - trilinear couplings

## Scalar masses (Higgs masses)

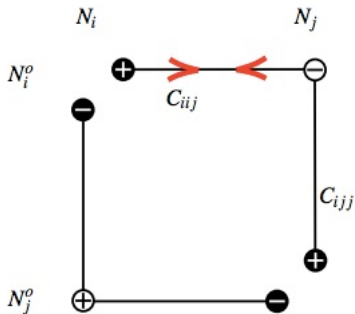


Fig. 3.1: A building block of the second type that defines a fermion–sfermion pair  $(\psi_{ij}, \tilde{\psi}_{ij})$ . Contributions to the mass term of the sfermion correspond to paths going back and forth on an edge, as is depicted on the top edge.

## Gaugino masses

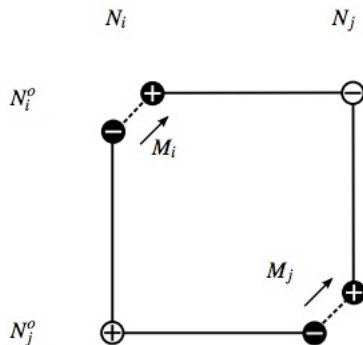


Fig. 3.2: A building block of the second type that defines a fermion–sfermion pair  $(\psi_{ij}, \tilde{\psi}_{ij})$ , dressed with mass terms for the corresponding gauginos (dashed edges, labeled by  $M_{i,j}$ ).

## Linear Couplings

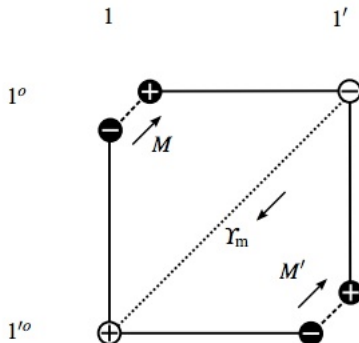
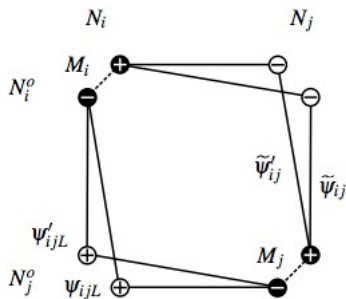
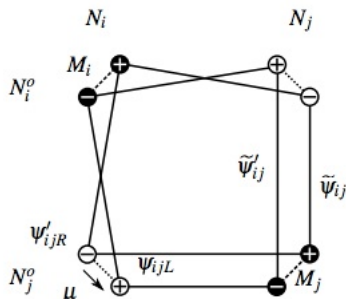


Fig. 3.3: A building block of the second type that defines a gauge singlet fermion–sfermion pair  $(\psi_{\text{sin}}, \tilde{\psi}_{\text{sin}})$ . Moreover, a Majorana mass term  $\Upsilon_m$  is possible.

# Bilinear Couplings



(a) When the gradings of the representations are equal.



(b) When the gradings of the representations differ.

Fig. 3.4: Two building blocks of the second type defining two fermion-sfermion pairs  $(\psi_{ij}, \tilde{\psi}_{ij})$  and  $(\psi'_{ij}, \tilde{\psi}'_{ij})$  in the same representation.



## Trilinear Couplings

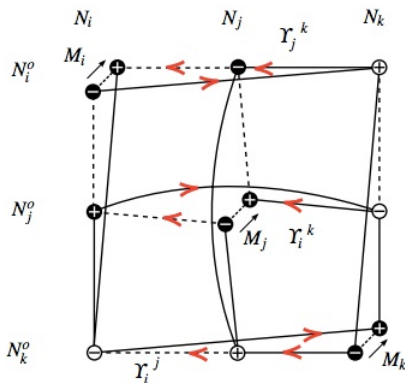


Fig. 3.5: A situation in which there are three building blocks  $\mathcal{B}_{i,j,k}$  of the first type (black vertices), three building blocks  $\mathcal{B}_{ij,jk,ik}$  of the second type and a building block  $\mathcal{B}_{ijk}$  of the third type. Adding gaugino masses (dashed edges) gives rise to trilinear interactions, corresponding to the paths in the diagram marked by arrows.

## Building the MSSM

- finite spectral triple of the MSSM:
  - should contain SM
  - gauge group same as for SM (up to finite groups)

$$\mathcal{A} = \mathcal{A}_{SM} = \mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$$

inequivalent reps **1**,  $\bar{\mathbf{1}}$ , **2**, **3** with  $\bar{\mathbf{1}}$  real rep  $\pi(\lambda)\nu = \bar{\lambda}\nu$

$$G = U(1) \times SU(2) \times SU(3)$$

- superpartners  $R = -1$  building blocks of first type

$$\mathcal{B}_1, \quad \mathcal{B}_{1_R}, \quad \mathcal{B}_{\bar{1}_R}, \quad \mathcal{B}_{2_L}, \quad \mathcal{B}_3$$

**Note:** in reducing  $\mathbb{C} \oplus \mathbb{H}_L \oplus \mathbb{H}_R \oplus M_3(\mathbb{C})$  to  $\mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$  the  $\mathbb{C} \hookrightarrow \mathbb{H}_R$  acts as  $(\lambda, \bar{\lambda})$  and  $\mathbf{2}_R$  breaks to  $\mathbf{1}_R \oplus \bar{\mathbf{1}}_R$

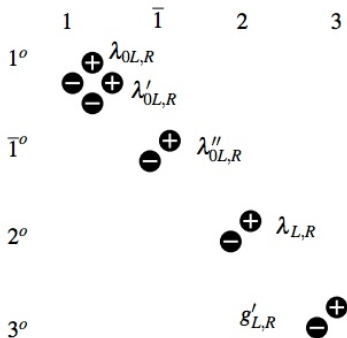
- at this point too many fermionic degrees of freedom: will need identifications
- for each SM fermion a building blocks of second type  $R = +1$  ( $M$  copies for generations):

$$\begin{aligned} \mathcal{B}_{1R1}^- &: (v_R, \tilde{v}_R), & \mathcal{B}_{\bar{1}R1}^- &: (e_R, \tilde{e}_R), & \mathcal{B}_{2L1}^+ &: (l_L, \tilde{l}_L), \\ \mathcal{B}_{1R3}^- &: (u_R, \tilde{u}_R), & \mathcal{B}_{\bar{1}R3}^- &: (d_R, \tilde{d}_R), & \mathcal{B}_{2L3}^+ &: (q_L, \tilde{q}_L) \end{aligned}$$

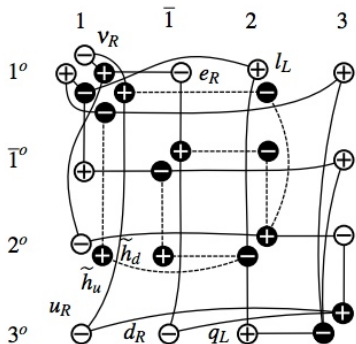
- SM Higgs: building blocks second type  $R = -1$  (one copy)

$$\mathcal{B}_{1R2L} : (h_u, \tilde{h}_u), \quad \mathcal{B}_{\bar{1}R2L} : (h_d, \tilde{h}_d),$$

Higgs/higgsino building blocks



(a) Blocks of the first type.



(b) Blocks of the second type. Each white off-diagonal node corresponds to a SM (anti)particle.

- these blocks determine the finite dimensional Hilbert space of the finite spectral triple

$$\mathcal{H}_F = \mathcal{H}_{F,R=+1} \oplus \mathcal{H}_{F,R=-1}$$

$$\begin{aligned}\mathcal{H}_{F,R=+} &= (\mathcal{E} \oplus \mathcal{E}^o)^{\oplus M}, & \mathcal{E} &= (\mathbf{2}_L \oplus \mathbf{1}_R \oplus \bar{\mathbf{1}}_R) \otimes (\mathbf{1} \oplus \mathbf{3})^o \\ \mathcal{H}_{F,R=-} &= \mathcal{F} \oplus \mathcal{F}^o, & \mathcal{F} &= (\mathbf{1} \otimes \mathbf{1}^o)^{\oplus 2} \oplus \bar{\mathbf{1}} \otimes \bar{\mathbf{1}}^o \oplus \mathbf{2} \otimes \mathbf{2}^o \\ & & & \oplus \mathbf{3} \otimes \mathbf{3}^o \oplus (\mathbf{1}_R \oplus \bar{\mathbf{1}}_R) \otimes \mathbf{2}_L^o.\end{aligned}$$

- correct properties to allow supersymmetric action
- element of  $\mathbb{C} \oplus \mathbb{H}_L \oplus \mathbb{H}_R \oplus M_3(\mathbb{C})$

$$R = -(+, -, -, +) \otimes (+, -, -, +)^o$$

has  $R = +1$  on all SM fermions and  $R = -1$  on higgsino

## Yukawa couplings of fermions and Higgs

- building blocks of third type  $\mathcal{B}_{ijk}$  should contain Higgs interaction of SM but with different up and down Higgses
- four possible building blocks

$$\mathcal{B}_{11_R 2_L},$$

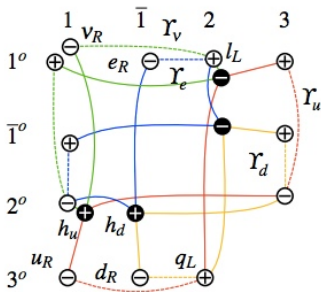
$$\mathcal{B}_{1\bar{1}_R 2_L},$$

$$\mathcal{B}_{1_R 2_L 3},$$

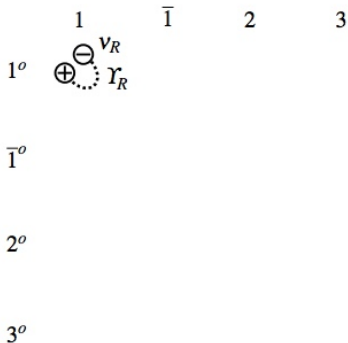
$$\mathcal{B}_{\bar{1}_R 2_L 3}.$$

**Massless photon condition:** now require for  $D_+$  part  $[D_+, \mathbb{C}_F] = 0$  with  $\mathbb{C}_F = \{(\lambda, \bar{\lambda}, 0) \in \mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})\}$

**Majorana masses** for right handed neutrinos as in  $\nu$ MSM



(c) Blocks of the third type, parametrized by the Yukawa matrices  $\gamma_{v,e,u,d}$ .



(d) The block of the fourth type, representing a Majorana mass for the right-handed neutrino.

up/down Higgs/higgsino interaction in MSSM  $\mu H_u \cdot H_d$

- need additional building blocks

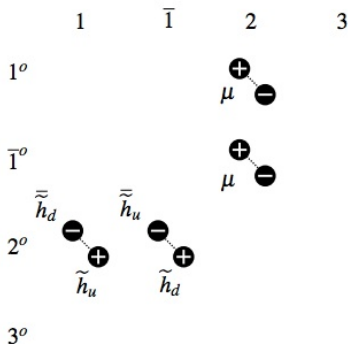


Fig. 4.2: The extra building blocks of the second type featuring a Higgs/higgsino-pair and the building blocks of the fifth type that are consequently possible.



## Sparticles and Hypercharges

- gauginos in  $\mathbf{1} \otimes \mathbf{1}^o$ ,  $\mathbf{2} \otimes \mathbf{2}^o$ ,  $\mathbf{3} \otimes \mathbf{3}^o$  with zero hypercharges
- higgsino in  $\mathbf{1}_R \otimes \mathbf{2}_L^o$  and  $\bar{\mathbf{1}}_R \otimes \mathbf{2}_L^o$  hypercharges  $+1$  and  $-1$
- hypercharges of sfermions

$$\begin{array}{lll} \tilde{q}_L: & \frac{1}{3}, & \tilde{u}_R: & \frac{4}{3}, & \tilde{d}_R: & -\frac{2}{3}, \\ \tilde{l}_L: & -1, & \tilde{\nu}_R: & 0, & \tilde{e}_R: & -2. \end{array}$$

## Correct identifications with sparticles of MSSM

- unimodularity (eliminate trace modes) for correct matching of fermion/boson degrees of freedom

## Spectral Action of MSSM

- can derive a set of relations between parameters for supersymmetry of the action to hold
- **Problem**: they cannot be satisfied for  $M \in \mathbb{N}$  (generations)
- **Conclusion** the spectral action

$$S = \text{Tr}(f(D_A/\Lambda)) + \frac{1}{2} \langle J\xi, D_A\xi \rangle$$

on an almost-commutative spectral triple  $M \times F$ , with  $F$  the finite spectral triple of the MSSM is itself *not* supersymmetric

- *then what?*
  - further extensions (beyond MSSM) may add more blocks and correct the problem?
  - is this the only way to approach spectral triples with supersymmetry?

**General comment**: in the history of this field, beware of any *no-go* result claiming that something cannot be done: it usually only means “it cannot be done *in this way*”