The Higgs Mass in the NCG Standard Model

Matilde Marcolli

Ma148b Spring 2016
Topics in Mathematical Physics

References

- A.H. Chamseddine, A. Connes, *Resilience of the Spectral Standard Model*, JHEP 1209 (2012) 104
- C. Estrada, M. Marcolli, *Asymptotic safety, hypergeometric functions and the Higgs mass in spectral action models*, Int. J. Geom. Meth. Mod. Phys., Vol.10 (2013) N.7, 1350036

What's missing in the model? to get correct Higgs mass

A list of possible culprits...

- The model is very constrained on field content... but is there room for other fields that alter the RGE and the Higgs mass value?
- Is the RGE flow the correct one? νMSM not MSM... effect of Majorana mass terms
- Gravitational terms: usually negligible effect, but are there acceptable boundary conditions in the model that make the terms large at high energies and affect the RGE?

focus on first and third possibility... re-discuss second in context of Early Universe Models

... other issues: "big desert" problem, other new physics: supersymmetry, GUTs

Scalar fields in the NCG Standard Model

- A.H. Chamseddine, A. Connes, *Resilience of the Spectral Standard Model*, JHEP 1209 (2012) 104

- Action involving Higgs and scalar field:

$$-\frac{2}{\pi} f_2 \Lambda^2 \int d^4x \sqrt{g} \left(\frac{1}{2} a \bar{H} H + \frac{1}{4} c \sigma^2 \right)$$

$$+\frac{f_0}{2\pi^2} \int d^4x \sqrt{g} \left(\mathfrak{b} (\bar{H} H)^2 + \mathfrak{a} |\nabla_\mu H_a|^2 + 2\mathfrak{e} \bar{H} H \sigma^2 + \frac{1}{2} d \sigma^4 + \frac{1}{2} \mathfrak{c} (\partial_\mu \sigma)^2 \right)$$

$H =$ Higgs dublet, $\sigma =$ scalar field

- Higgs-singlet potential (after rescaling of fields)

$$V = \frac{1}{4} (\lambda_h \bar{h}^4 + 2\lambda_{h\sigma} \bar{h}^2 \bar{\sigma}^2 + \lambda_\sigma \bar{\sigma}^4) - \frac{2g^2}{\pi^2} f_2 \Lambda^2 (\bar{h}^2 + \bar{\sigma}^2)$$

- $\bar{h} = |k^u| h$, $\bar{\sigma} = |k^{\nu_R}| \sigma$
- $\bar{h} \mapsto \bar{h} g \sqrt{2/(n+3)}$, $\bar{\sigma} \mapsto \bar{\sigma} 2g$

$\lambda_\sigma = 8g^2$

$$\lambda_h = \frac{n^2+3}{(n+3)^2} 4g^2, \quad \lambda_{h\sigma} = \frac{2n}{n+3} 4g^2$$

- Parameters: unification energy u and parameter n with

$$k_t(u) = g\sqrt{\frac{4}{n+3}}, \quad k_\nu = \sqrt{n}\, k_t$$

Resulting RGE top quark, neutrino, Higgs and singlet quartic couplings

$$\frac{d}{d\mu}k_t = \frac{k_t}{32\pi^2}\left(-\left(\frac{17}{6}g_1^2 + \frac{9}{2}g_2^2 + 16g_3^2\right) + 9{k_t}^2 + 2{k_\nu}^2\right)$$

$$\frac{d}{d\mu}k_\nu = \frac{k_\nu}{32\pi^2}\left(-\left(\frac{3}{2}g_1^2 + \frac{9}{2}g_2^2\right) + 6{k_t}^2 + 5{k_\nu}^2\right)$$

$$\frac{d}{d\mu}\lambda_h = \frac{1}{16\pi^2}\left(\left(12{k_t}^2 + 4{k_\nu}^2 - (3g_1^2 + 9g_2^2)\right)\lambda_h\right.$$

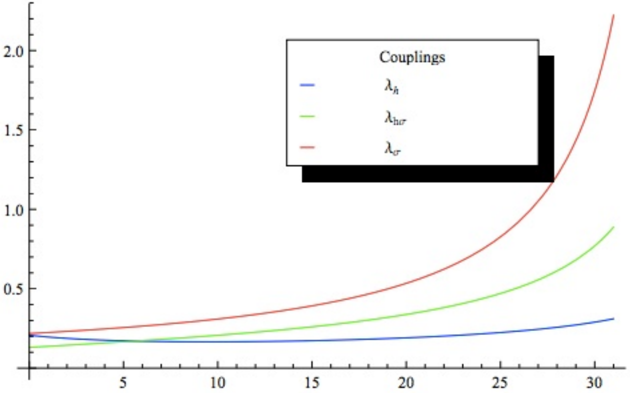
$$\left. + 2\left(12\lambda_h^2 + \lambda_{h\sigma}^2 + \frac{3}{16}(g_1^4 + 2g_1^2g_2^2 + 3g_2^4) - 3{k_t}^4 - {k_\nu}^4\right)\right)$$

$$\frac{d}{d\mu}\lambda_{h\sigma} = \frac{\lambda_{h\sigma}}{16\pi^2}\left(\frac{1}{2}\left(12{k_t}^2 + 4{k_\nu}^2 - 3g_1^2 - 9g_2^2\right)\right.$$

$$\left. +4\left(3\lambda_h + \frac{3}{2}\lambda_\sigma + 2\lambda_{h\sigma}\right)\right)$$

$$\frac{d}{d\mu}\lambda_\sigma = \frac{1}{16\pi^2}\left(8\lambda_{h\sigma}^2 + 18\lambda_\sigma^2\right)$$

Again separately run the equations for the coupling constants g_i (decoupled at one-loop) then get running of λ_h, $\lambda_{h\sigma}$, λ_σ



Expand scalar fields around vev: $\bar{h} = \bar{v} + \bar{\phi}$ and $\bar{\sigma} = \bar{w} + \bar{\tau}$

$$V \sim (-\frac{1}{4}\bar{v}^4\lambda_h - \frac{1}{2}\bar{v}^2\bar{w}^2\lambda_{h\sigma} - \frac{1}{4}\bar{w}^4\lambda_\sigma)$$
$$+\bar{v}^2\bar{\phi}^2\lambda_h + 2\bar{v}\bar{w}\bar{\tau}\bar{\phi}\lambda_{h\sigma} + \bar{w}^2\bar{\tau}^2\lambda_\sigma$$

Expansion gives mass terms for $\bar{\phi}$ and $\bar{\tau}$

$$\frac{1}{2}\begin{pmatrix}\bar{\phi} & \bar{\tau}\end{pmatrix} M^2 \begin{pmatrix}\bar{\phi} \\ \bar{\tau}\end{pmatrix}$$

$$M^2 = 2\begin{pmatrix}\lambda_h\bar{v}^2 & \lambda h\sigma\bar{v}\bar{w} \\ \lambda h\sigma\bar{v}\bar{w} & \lambda_\sigma\bar{w}^2\end{pmatrix}$$

Eigenvalues:

$$m_\pm^2 = \lambda_h\bar{v}^2 + \lambda_\sigma\bar{w}^2 \pm \sqrt{(\lambda_h\bar{v}^2 - \lambda_\sigma\bar{w}^2)^2 + 4\lambda_{h\sigma}^2\bar{v}^2\bar{w}^2}$$

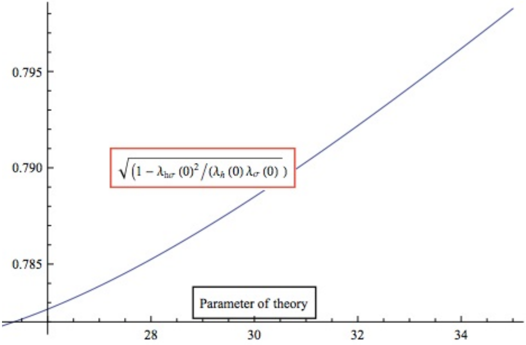
Approximation:

$$m_+^2 \sim 2\lambda_\sigma\bar{w}^2 + 2\frac{\lambda_{h\sigma}^2}{\lambda_\sigma}\bar{v}^2$$

$$m_-^2 \sim 2\lambda_h\bar{v}^2(1 - \frac{\lambda_{h\sigma}^2}{\lambda_h\lambda_\sigma})$$

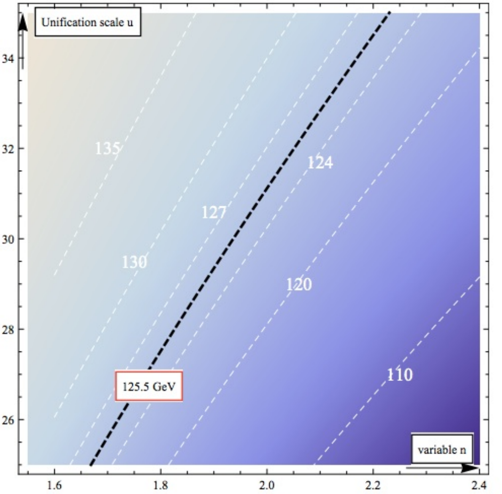
Higgs mass reduced by a factor of $\sqrt{(1-\frac{\lambda_{h\sigma}^2}{\lambda_h\lambda_\sigma})}$ around 0.78 low energy:

$$m_t(0) = k_t(0)\frac{246}{\sqrt{2}}, \quad m_h(0) = 246\sqrt{2\lambda_h(0)(1-\frac{\lambda_{h\sigma}^2(0)}{\lambda_h(0)\lambda_\sigma(0)})}$$



stable Higgs mass for $\lambda_{h\sigma}^2 < \lambda_h\lambda_\sigma$

Parameter space (u, n) and Higgs mass 125.5 GeV curve



graphics from A.H. Chamseddine, A. Connes, *Resilience of the Spectral Standard Model*, JHEP 1209 (2012) 104

Asymptotic Safety and Anomalous Dimensions

• C. Estrada, M. Marcolli, *Asymptotic safety, hypergeometric functions and the Higgs mass in spectral action models*, Int. J. Geom. Meth. Mod. Phys., Vol.10 (2013) N.7, 1350036

Based on "asymptotic safety" idea of:
- S. Weinberg, *Ultraviolet divergences in quantum theories of gravitation*, in "General Relativity: an Einstein centenary survey" Cambridge Univ. Press, 1979, pp.790–831

Also using results of:
- M. Reuter, *Nonperturbative evolution equation for quantum gravity*, Phys Rev D 57 (1998) N.2, 971–985

- M. Shaposhnikov, C. Wetterich, *Asymptotic safety of gravity and the Higgs boson mass*, Phys. Lett. B Vol.683 (2010) N.2-3, 196–200

• Gravitational terms introduce corrections to the RGE flow in the form of "anomalous dimensions" terms

$$\partial_t x_j = \beta_j^{\mathrm{SM}} + \beta_j^{\mathrm{grav}}$$

$$\beta_j^{\mathrm{grav}} = \frac{a_j}{8\pi} \frac{\Lambda^2}{M_P^2(\Lambda)} x_j$$

a_j are the *anomalous dimensions*

$$M_P^2(\Lambda) = M_P^2 + 2\rho_0 \Lambda^2$$

scale dependence of Newton constant estimated $\rho_0 \sim 0.024$

• Examine boundary conditions at unification compatible with NCG so that effect on the Higgs running

Coupling constants running without anomalous dimensions

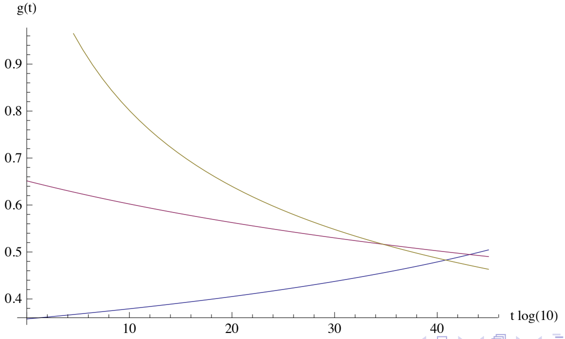
- ODE with exact solutions

$$u'(t) = A\, u(t)^3, \quad u(0) = B$$

$$u(t) = \pm \frac{1}{\sqrt{\frac{1-2AB^2 t}{B^2}}}$$

A determined by β function; B by values: at $\Lambda = M_Z$

$$g_1(0) = 0.3575, \quad g_2(0) = 0.6514, \quad g_3(0) = 1.221$$



Coupling constants running with anomalous dimensions

- $a_1 = a_2 = a_3 = a_g$, with $|a_g| \sim 1$ and negative sign

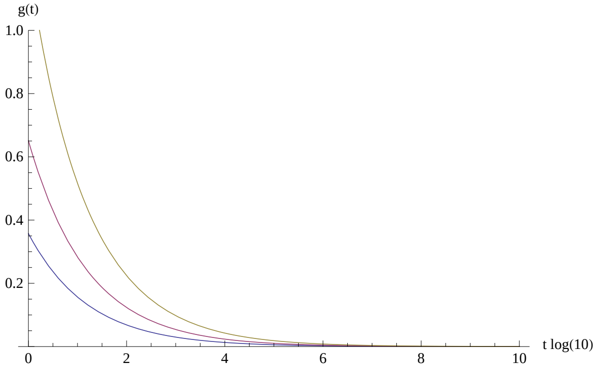
$$u'(t) = -a\, u(t) + A\, u(t)^3, \qquad u(0) = B$$

$$a = \frac{|a_g|}{16\pi\rho_0} \sim \frac{1}{16\pi\rho_0}$$

Exact solutions

$$u(t) = \frac{\pm\sqrt{a}}{\sqrt{A + \exp\left(2a\left(t + \frac{\log(-A + \frac{a}{B^2})}{2a}\right)\right)}}$$

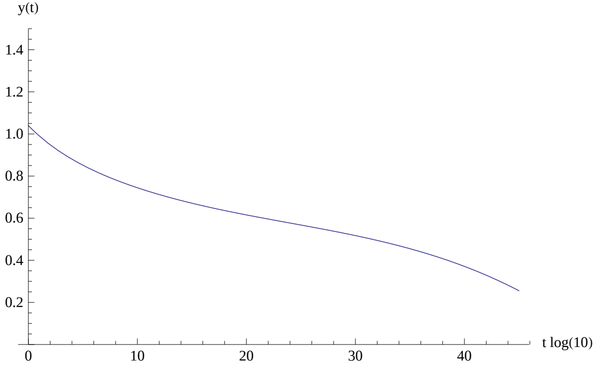
Asymptotic safety effect



Running of the coupling constants with anomalous dimensions

Running of top Yukawa coupling without anomalous dimensions

MSM approximation as before



$$\beta_y = \frac{1}{16\pi^2}\left(\frac{9}{2}y^3 - 8g_3^2 y - \frac{9}{4}g_2^2 y - \frac{17}{12}g_1^2 y\right)$$

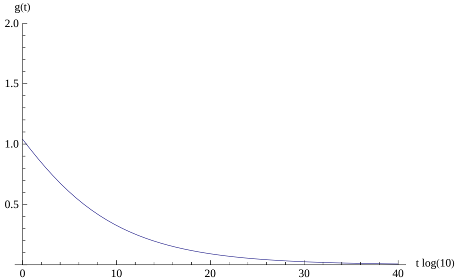
Running of top Yukawa coupling with anomalous dimensions

$$\partial_t y_t = -\frac{|a_y|}{16\pi\rho_0} y_t + \frac{1}{16\pi^2}\frac{9}{2} y_t^3$$

Exact solutions of form similar to $u(t)$ above but parameters

$$a = \frac{|a_y|}{16\pi\rho_0}, \qquad A = \frac{9}{32\pi^2}$$

anomalous dim makes the Yukawa couplings asymptotically free



Anomalous dimensions and the Higgs self-coupling

- if top Yukawa coupling contribution to beta function of Higgs self-coupling is dominant over gauge contribution further simplification

$$\partial_t \lambda = \frac{a_\lambda}{16\pi\rho_0}\lambda + \frac{1}{16\pi^2}(24\lambda^2 + 12\lambda y^2 - 6y^4)$$

- Riccati equation

$$\lambda' = q_0(t) + q_1(t)\lambda + q_2(t)\lambda^2$$

$$q_0(t) = \frac{-3y^4(t)}{8\pi^2}, \quad q_1(t) = \frac{a_\lambda}{16\pi\rho_0} + \frac{3y^2(t)}{4\pi^2}, \quad q_2(t) = \frac{3}{2\pi^2}$$

- Change of variables for Riccati:

$$-\frac{u'}{u} = \lambda\, q_2$$

$$u'' - \left(q_1(t) + \frac{q_2'(t)}{q_2(t)} \right)\, u' + q_2(t) q_0(t)\, u = 0$$

- use general form of solution for y_t

$$y(t)^2 = \frac{a}{Ce^{2at} + A}$$

parameters a, A, and C determined by coefficients of RGE and initial condition

- get second order linear equation

$$\partial_t^2 u(t) - \left(\frac{a_\lambda}{16\pi\rho_0} + \frac{3}{4\pi^2}\frac{a}{Ce^{2at} + A}\right)\partial_t u - \frac{9}{16\pi^4}\left(\frac{a}{Ce^{2at} + A}\right)^2 u(t) = 0$$

- change variables again $x = e^{2at}$, $v(x) = v(e^{2at}) = u(t)$

$$(2ax)^2 \partial_x^2 v(x) + (2a - \frac{a_\lambda}{16\pi\rho_0} - \frac{3}{4\pi^2}\frac{a}{Cx + A})(2ax\partial_x v(x))$$

$$-\frac{9}{16\pi^4}(\frac{a}{Cx + A})^2 v(x) = 0$$

- General setting: equation

$$y'' = \lambda_0 y' + s_0 y$$

$$\lambda_0 = \frac{-\frac{3a}{4\pi^2} + \left(2a - \frac{a_\lambda}{16\pi\rho_0}\right)(A + Cx)}{2ax(A + Cx)}$$

$$s_0 = -\frac{9}{64\pi^2 x^2 (A + Cx)^2}$$

- Auxiliary functions $f_1(x) = (A + Cx)^{\alpha-3}$,

$$\theta = \frac{\sqrt{9 + 4\pi^2\left(-9 + 4A\left(3 + 4A\pi^2\right)\right)}}{16A\pi^2}$$

$$\alpha = \frac{3}{2} + \frac{3}{16A\pi^2} + \theta$$

$$f_2(x) = x^\eta \left(\frac{Cx}{A}\right)^{1-\eta} \quad \text{with} \quad \eta = \frac{9}{64\, A^2\pi^2}$$

$$\beta = \alpha - \eta - 1 - \frac{3}{8A\pi^2}, \quad \gamma(x) = -\frac{Cx}{A}$$

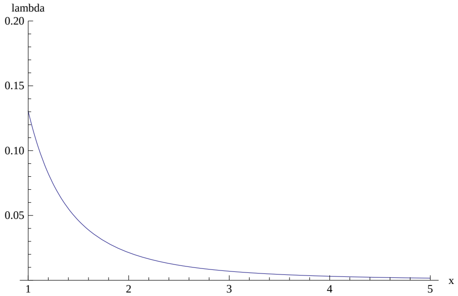
- get general solution of equation $y'' = \lambda_0 y' + s_0 y$ above

$$\begin{aligned} v(x) = \quad & C_1\ f_2(x)\, f_1(x)\ {}_2F_1\left(\alpha, \beta, 2-\eta, \gamma(x)\right) \\ + \quad & C_2\ x^\eta\, f_1(x)\ {}_2F_1\left(\alpha - (2 + \tfrac{3}{8A\pi^2}), \beta + (2 + \tfrac{8A}{3})\eta, \eta\gamma(x)\right), \end{aligned}$$

with ${}_2F_1(a, b, c, z)$ is the Gauss hypergeometric function

• corresponding solutions of original Riccati equation that gives RGE of Higgs self coupling:

$$\lambda(t) = -\frac{2\pi^2}{3}\frac{u'(t)}{u(t)}, \qquad \text{with} \qquad u(t) = v(e^{2at}).$$



solution with $a_\lambda = 5.08$ and $A = 9/(32\pi^2)$, compatible top quark mass $m_t \sim 171.3$ GeV and Higgs mass $m_H \sim 125.4$

Problem: sensitive dependence of RGE equations on initial conditions: fine tuning problem