The Higgs Mass in the NCG Standard Model

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Topics in Mathematical Physics
References

What’s missing in the model? to get correct Higgs mass

A list of possible culprits...

- The model is very constrained on field content... but is there room for other fields that alter the RGE and the Higgs mass value?
- Is the RGE flow the correct one? νMSM not MSM... effect of Majorana mass terms
- Gravitational terms: usually negligible effect, but are there acceptable boundary conditions in the model that make the terms large at high energies and affect the RGE?

focus on first and third possibility... re-discuss second in context of Early Universe Models

... other issues: “big desert” problem, other new physics: supersymmetry, GUTs
Scalar fields in the NCG Standard Model


- Action involving Higgs and scalar field:

\[-\frac{2}{\pi} f_2 \Lambda^2 \int d^4 x \sqrt{g} \left( \frac{1}{2} a \bar{H} H + \frac{1}{4} c \sigma^2 \right) \]

\[+ \frac{f_0}{2\pi^2} \int d^4 x \sqrt{g} \left( b (\bar{H} H)^2 + a |\nabla_\mu H_\sigma|^2 + 2 c \bar{H} H \sigma^2 + \frac{1}{2} d \sigma^4 + \frac{1}{2} c (\partial_\mu \sigma)^2 \right) \]

\(H = \) Higgs doublet, \(\sigma = \) scalar field

- Higgs-singlet potential (after rescaling of fields)

\[V = \frac{1}{4} (\lambda_h \bar{h}^4 + 2 \lambda_{h\sigma} \bar{h}^2 \bar{\sigma}^2 + \lambda_{\sigma} \bar{\sigma}^4) - \frac{2g^2}{\pi^2} f_2 \Lambda^2 (\bar{h}^2 + \bar{\sigma}^2) \]
• $\bar{h} = |k^u|h$, $\bar{\sigma} = |k^\nu R|\sigma$

• $\bar{h} \mapsto \bar{h} g \sqrt{2/(n+3)}$, $\bar{\sigma} \mapsto \bar{\sigma} 2g$

$\lambda_\sigma = 8g^2$

$$\lambda_h = \frac{n^2 + 3}{(n+3)^2} 4g^2, \quad \lambda_{h\sigma} = \frac{2n}{n+3} 4g^2$$

• Parameters: unification energy $u$ and parameter $n$ with

$$k_t(u) = g \sqrt{\frac{4}{n + 3}}, \quad k_\nu = \sqrt{n} k_t$$
Resulting RGE top quark, neutrino, Higgs and singlet quartic couplings

\[
\frac{d}{d\mu} k_t = \frac{k_t}{32\pi^2} \left( - \left( \frac{17}{6} g_1^2 + \frac{9}{2} g_2^2 + 16 g_3^2 \right) + 9 k_t^2 + 2 k_\nu^2 \right)
\]

\[
\frac{d}{d\mu} k_\nu = \frac{k_\nu}{32\pi^2} \left( - \left( \frac{3}{2} g_1^2 + \frac{9}{2} g_2^2 \right) + 6 k_t^2 + 5 k_\nu^2 \right)
\]

\[
\frac{d}{d\mu} \lambda_h = \frac{1}{16\pi^2} \left( (12 k_t^2 + 4 k_\nu^2 - (3 g_1^2 + 9 g_2^2)) \lambda_h + 2 \left( 12 \lambda_h^2 + \lambda_h^2 + \frac{3}{16} (g_1^4 + 2 g_1^2 g_2^2 + 3 g_2^4) - 3 k_t^4 - k_\nu^4 \right) \right)
\]

\[
\frac{d}{d\mu} \lambda_{h\sigma} = \frac{\lambda_{h\sigma}}{16\pi^2} \left( \frac{1}{2} (12 k_t^2 + 4 k_\nu^2 - 3 g_1^2 - 9 g_2^2) + 4 \left( 3 \lambda_h + \frac{3}{2} \lambda_\sigma + 2 \lambda_{h\sigma} \right) \right)
\]

\[
\frac{d}{d\mu} \lambda_\sigma = \frac{1}{16\pi^2} \left( 8 \lambda_{h\sigma}^2 + 18 \lambda_\sigma^2 \right)
\]
Again separately run the equations for the coupling constants $g_i$ (decoupled at one-loop) then get running of $\lambda_h$, $\lambda_{h\sigma}$, $\lambda_\sigma$
Expand scalar fields around vev: $\tilde{h} = \bar{v} + \bar{\phi}$ and $\tilde{\sigma} = \bar{w} + \bar{\tau}$

$$V \sim (-\frac{1}{4} \bar{v}^4 \lambda_h - \frac{1}{2} \bar{v}^2 \bar{w}^2 \lambda_{h\sigma} - \frac{1}{4} \bar{w}^4 \lambda_{\sigma})$$

$$\quad + \bar{v}^2 \bar{\phi}^2 \lambda_h + 2 \bar{v} \bar{w} \bar{\tau} \bar{\phi} \lambda_{h\sigma} + \bar{w}^2 \bar{\tau}^2 \lambda_{\sigma}$$

Expansion gives mass terms for $\bar{\phi}$ and $\bar{\tau}$

$$\frac{1}{2} (\bar{\phi} \quad \bar{\tau}) M^2 (\bar{\phi} \quad \bar{\tau})$$

$$M^2 = 2 \begin{pmatrix} \lambda_h \bar{v}^2 & \lambda_{h\sigma} \bar{v} \bar{w} \\ \lambda_{h\sigma} \bar{v} \bar{w} & \lambda_{\sigma} \bar{w}^2 \end{pmatrix}$$

Eigenvalues:

$$m_{\pm}^2 = \lambda_h \bar{v}^2 + \lambda_{\sigma} \bar{w}^2 \pm \sqrt{\left( \lambda_h \bar{v}^2 - \lambda_{\sigma} \bar{w}^2 \right)^2 + 4 \lambda_{h\sigma}^2 \bar{v}^2 \bar{w}^2}$$

Approximation:

$$m_{+}^2 \sim 2 \lambda_{\sigma} \bar{w}^2 + 2 \frac{\lambda_{h\sigma}^2}{\lambda_{\sigma}} \bar{v}^2$$

$$m_{-}^2 \sim 2 \lambda_h \bar{v}^2 \left( 1 - \frac{\lambda_{h\sigma}^2}{\lambda_h \lambda_{\sigma}} \right)$$
Higgs mass reduced by a factor of \( \sqrt{1 - \frac{\lambda^2_{h\sigma}}{\lambda_h \lambda_\sigma}} \) around 0.78 low energy:

\[
m_t(0) = k_t(0) \frac{246}{\sqrt{2}}, \quad m_h(0) = 246 \sqrt{2\lambda_h(0)(1 - \frac{\lambda^2_{h\sigma}(0)}{\lambda_h(0)\lambda_\sigma(0)})}
\]

stable Higgs mass for \( \lambda^2_{h\sigma} < \lambda_h \lambda_\sigma \)
Parameter space \((u, n)\) and Higgs mass 125.5 GeV curve

Asymptotic Safety and Anomalous Dimensions


Based on “asymptotic safety” idea of:

Also using results of:
• Gravitational terms introduce corrections to the RGE flow in the form of “anomalous dimensions” terms

$$\partial_t x_j = \beta_j^{\text{SM}} + \beta_j^{\text{grav}}$$

$$\beta_j^{\text{grav}} = \frac{a_j}{8\pi} \frac{\Lambda^2}{M_P^2(\Lambda)} x_j$$

$a_j$ are the anomalous dimensions

$$M_P^2(\Lambda) = M_P^2 + 2\rho_0 \Lambda^2$$

scale dependence of Newton constant estimated $\rho_0 \sim 0.024$

• Examine boundary conditions at unification compatible with NCG so that effect on the Higgs running
Coupling constants running without anomalous dimensions

- ODE with exact solutions

\[ u'(t) = A u(t)^3, \quad u(0) = B \]

\[ u(t) = \pm \frac{1}{\sqrt{\frac{1-2AB^2t}{B^2}}} \]

\( A \) determined by \( \beta \) function; \( B \) by values: at \( \Lambda = M_Z \)

\( g_1(0) = 0.3575, \quad g_2(0) = 0.6514, \quad g_3(0) = 1.221 \)
Coupling constants running with anomalous dimensions

- \( a_1 = a_2 = a_3 = a_g \), with \(|a_g| \sim 1\) and negative sign

\[
    u'(t) = -a \, u(t) + A \, u(t)^3, \quad u(0) = B
\]

\[
    a = \frac{|a_g|}{16 \pi \rho_0} \sim \frac{1}{16 \pi \rho_0}
\]

Exact solutions

\[
    u(t) = \pm \sqrt{a} \sqrt{A + \exp \left( 2a \left( t + \frac{\log(-A + \frac{a}{B^2})}{2a} \right) \right)}
\]
Asymptotic safety effect

Running of the coupling constants with anomalous dimensions
Running of top Yukawa coupling without anomalous dimensions
MSM approximation as before

\[
\beta_y = \frac{1}{16\pi^2} \left( \frac{9}{2} y^3 - 8 g_3^2 y - \frac{9}{4} g_2^2 y - \frac{17}{12} g_1^2 y \right)
\]
Running of top Yukawa coupling with anomalous dimensions

\[ \partial_t y_t = -\frac{|a_y|}{16\pi \rho_0} y_t + \frac{1}{16\pi^2} \frac{9}{2} y_t^3 \]

Exact solutions of form similar to \( u(t) \) above but parameters

\[ a = \frac{|a_y|}{16\pi \rho_0}, \quad A = \frac{9}{32\pi^2} \]

anomalous dim makes the Yukawa couplings asymptotically free
Anomalous dimensions and the Higgs self-coupling

• if top Yukawa coupling contribution to beta function of Higgs self-coupling is dominant over gauge contribution further simplification

\[
\partial_t \lambda = \frac{a_\lambda}{16\pi\rho_0} \lambda + \frac{1}{16\pi^2} (24\lambda^2 + 12\lambda y^2 - 6y^4)
\]

• Riccati equation

\[
\lambda' = q_0(t) + q_1(t)\lambda + q_2(t)\lambda^2
\]

\[
q_0(t) = \frac{-3y^4(t)}{8\pi^2}, \quad q_1(t) = \frac{a_\lambda}{16\pi\rho_0} + \frac{3y^2(t)}{4\pi^2}, \quad q_2(t) = \frac{3}{2\pi^2}
\]
• Change of variables for Riccati:

\[- \frac{u'}{u} = \lambda q_2\]

\[u'' - \left( q_1(t) + \frac{q'_2(t)}{q_2(t)} \right) u' + q_2(t)q_0(t) u = 0\]

• use general form of solution for \(y_t\)

\[y(t)^2 = \frac{a}{Ce^{2at} + A}\]

parameters \(a\), \(A\), and \(C\) determined by coefficients of RGE and initial condition

• get second order linear equation

\[\partial_t^2 u(t) - \left( \frac{a\lambda}{16\pi \rho_0} + \frac{3}{4\pi^2} \frac{a}{Ce^{2at} + A} \right) \partial_t u - \frac{9}{16\pi^4} \left( \frac{a}{Ce^{2at} + A} \right)^2 u(t) = 0\]
• change variables again $x = e^{2at}$, $\nu(x) = \nu(e^{2at}) = u(t)$

$$(2ax)^2 \partial_x^2 \nu(x) + \left(2a - \frac{a_\lambda}{16\pi \rho_0} - \frac{3}{4\pi^2} \frac{a}{Cx + A}\right)(2ax \partial_x \nu(x))$$

$$- \frac{9}{16\pi^4} \left(\frac{a}{Cx + A}\right)^2 \nu(x) = 0$$

• General setting: equation

$$y'' = \lambda_0 y' + s_0 y$$

$$\lambda_0 = - \frac{3a}{4\pi^2} + \left(2a - \frac{a_\lambda}{16\pi \rho_0}\right) (A + Cx)$$

$$s_0 = - \frac{9}{64\pi^2 x^2 (A + Cx)^2}$$
• Auxiliary functions $f_1(x) = (A + Cx)^{\alpha - 3}$,

$$\theta = \frac{\sqrt{9 + 4\pi^2 (-9 + 4A (3 + 4A\pi^2))}}{16A\pi^2}$$

$$\alpha = \frac{3}{2} + \frac{3}{16A\pi^2} + \theta$$

$$f_2(x) = x^{\eta} \left( \frac{Cx}{A} \right)^{1-\eta} \ \ \text{with} \ \ \eta = \frac{9}{64A^2\pi^2}$$

$$\beta = \alpha - \eta - 1 - \frac{3}{8A\pi^2}, \quad \gamma(x) = -\frac{Cx}{A}$$

• get general solution of equation $y''' = \lambda_0 y' + s_0 y$ above

$$v(x) = C_1 f_2(x) f_1(x) \ 2F_1 (\alpha, \beta, 2 - \eta, \gamma(x))$$

$$+ \ C_2 \ x^{\eta} f_1(x) \ 2F_1 (\alpha - \left(2 + \frac{3}{8A\pi^2}\right), \beta + (2 + \frac{8A}{3})\eta, \eta \gamma(x)),$$

with $2F_1(a, b, c, z)$ is the Gauss hypergeometric function
• corresponding solutions of original Riccati equation that gives RGE of Higgs self coupling:

\[
\lambda(t) = -\frac{2\pi^2}{3} \frac{u'(t)}{u(t)}, \quad \text{with} \quad u(t) = v(e^{2at}).
\]

solution with \(a_\lambda = 5.08\) and \(A = 9/(32\pi^2)\), compatible top quark mass \(m_t \sim 171.3\) GeV and Higgs mass \(m_H \sim 125.4\)

**Problem:** sensitive dependence of RGE equations on initial conditions: fine tuning problem