Variants of Equivariant Seiberg–Witten Floer Homology

Matilde Marcolli and Bai-Ling Wang

ABSTRACT. For a rational homology 3-sphere $Y$ with a Spin$^c$ structure $s$, we show that simple algebraic manipulations of our construction of equivariant Seiberg–Witten Floer homology in [5] lead to a collection of variants, which are all topological invariants. We establish a long exact sequence relating them and we show that they satisfy a duality under orientation reversal. We explain their relation to the equivariant Seiberg–Witten Floer (co)homologies introduced in [5]. We conjecture the equivalence of these versions of equivariant Seiberg–Witten Floer homology with the Heegaard Floer invariants introduced by Ozsváth and Szabó.

1. Introduction

For any rational homology 3-sphere $Y$ with a Spin$^c$ structure $s$, we constructed in [5] an equivariant Seiberg–Witten Floer homology $HF^SW_{*,U(1)}(Y,s)$, which is a topological invariant. In this paper, we will generalize this construction to provide a collection of equivariant Seiberg–Witten Floer homologies, which we denote $HF^-_{*,U(1)}(Y,s), HF^SW_{*,U(1)}(Y,s), HF^SW_{*,U(1)}(Y,s), HF^SW_{*,U(1)}(Y,s)$ and $HF^SW_{red,s}(Y,s)$. All of them are topological invariants, with $HF^SW_{*,U(1)}(Y,s)$ isomorphic to the equivariant Seiberg–Witten Floer homology $HF^SW_{*,U(1)}(Y,s)$ constructed in [5]. The construction utilizes the $U(1)$-invariant forms on $U(1)$-manifolds twisted with coefficients in the Laurent polynomial algebra over integers.

In analogy to Austin and Braam’s construction of equivariant instanton Floer homology in [1], the equivariant Seiberg–Witten Floer homology $HF^SW_{*,U(1)}(Y,s)$ is the homology of the complex $(CF^SW_{*,U(1)}(Y,s), D)$, where $CF^SW_{*,U(1)}(Y,s)$ is generated by equivariant de Rham forms over all $U(1)$-orbits of the solutions of 3-dimensional Seiberg–Witten equations on $(Y,s)$ modulo based gauge transformations (cf.[5]).

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More specifically,

\[ CF_{SW, U(1)}^*(Y, s) = \bigoplus_{a \in M_Y(s)} \mathbb{Z}[\Omega] \otimes (\mathbb{Z}_{\eta_a} \oplus \mathbb{Z}_{1_a}) \oplus \mathbb{Z}[\Omega] \otimes \mathbb{Z}_{1_\theta}, \]

where \( M_Y(s) = M_Y^r(s) \cup \{ \theta \} \) is the equivalence classes of solutions to the Seiberg-Witten equations for a good pair of metric and perturbations, consists of the irreducible monopoles \( M_Y^r(s) \) and the unique reducible monopole \( \theta \). We used the notation \( \eta_a \) to denote a 1-form on \( O_a \cong S^1 \), such that the cohomology class \( [\eta_a] \) is an integral generator of \( H^1(O_a) \). Similarly, we denote by \( 1_a \) the 0-form given by the constant function.

Each generator is endowed with a grading such that, for any \( k \geq 0 \),

\[ gr(\Omega^k \otimes \eta_a) = 2k + gr(a), \quad gr(\Omega^k \otimes 1_a) = 2k + gr(a) + 1, \quad \text{and} \quad gr(\Omega^k \otimes 1_\theta) = 2k, \]

where \( gr : M_Y^r(s) \to \mathbb{Z} \) is the relative grading with respect to the reducible monopole \( \theta \). This corresponds to grading equivariant de Rham forms on each orbit \( O_a \) by codimension (cf.\[5\] §5 for details).

The differential operator \( D \) can be expressed explicitly in components as the form:

\[ D(\Omega^k \otimes \eta_a) = \sum_{b \in M_Y^r(s), \, gr(a) - gr(b) = 1} n_{ab} \Omega^k \otimes \eta_b + \sum_{c \in M_Y^r(s), \, gr(a) - gr(c) = 2} m_{ac} \Omega^k \otimes 1_c - \Omega^{k-1} \otimes 1_a + n_{a0} \Omega^k \otimes 1_\theta \text{ (if } gr(a) = 1); \]

\[ D(\Omega^k \otimes 1_a) = - \sum_{b \in M_Y^r(s), \, gr(a) - gr(b) = 1} n_{ab} \Omega^k \otimes 1_b; \]

\[ D(\Omega^k \otimes 1_\theta) = \sum_{d \in M_Y^r(s), \, gr(d) = -2} n_{\theta d} \Omega^k \otimes 1_d. \]

where \( n_{ab}, n_{a0} \) and \( n_{\theta d} \) is the counting of flow lines from \( a \) to \( b \), if \( gr(a) - gr(b) = 1 \), or from \( a \) to \( \theta \), if \( gr(a) = 1 \), or from \( \theta \) to \( d \), if \( gr(d) = -2 \). The coefficient \( m_{ac} \), for \( gr(a) - gr(c) = 2 \), is described as a relative Euler number associated to the 2-dimensional moduli space of flow lines from \( a \) to \( c \) (cf. Lemma 5.7 of \[5\]).

In the next section, we shall briefly review the construction and various relations among the coefficients, as established in \[5\]. These identities ensure that \( D^2 = 0 \). Notice that, in the complex \( CF_{*,U(1)}^{SW}(Y, s) \) and in the expression of the differential operator, only terms with non-negative powers of \( \Omega \) are considered. We modify the construction as follows.

**Definition 1.1.** Let \( CF_{*,U(1)}^{SW, \infty}(Y, s) \) be the graded complex generated by

\[ \{ \Omega^k \otimes \eta_a, \Omega^k \otimes 1_a, \Omega^k \otimes 1_\theta : a \in M_Y^r(s), k \in \mathbb{Z} \}, \]

with the grading \( gr \) and the differential operator \( D \) given by (2) and (3) respectively. Let \( CF_{*,U(1)}^-(Y, s) \) be the subcomplex of \( CF_{*,U(1)}^{SW, \infty}(Y, s) \), generated by those generators with negative power of \( \Omega \). The quotient complex is denoted by \( CF_{*,U(1)}^{SW, +}(Y, s) \). Their homologies are denoted by \( HF_{*,U(1)}^{SW, \infty}(Y, s) \), \( HF_{*,U(1)}^{SW, -}(Y, s) \) and \( HF_{*,U(1)}^{SW, +}(Y, s) \) respectively.
The main results in this paper relate these homologies to the equivariant Seiberg–Witten Floer homology $HF^*_{U(1)}(Y, s)$ and cohomology $HF^{SW,*}_{U(1)}(Y, s)$ constructed in [5] and establish some of their main properties.

**Theorem 1.2.** For any rational homology 3-sphere $Y$ with a Spin$^c$ structure $s \in \text{Spin}^c(Y)$, these homologies satisfy the following properties:

1. $HF^*_{U(1)}(Y, s) \cong \mathbb{Z}[\Omega, \Omega^{-1}]$.
2. $HF^*_{U(1)}(Y, s) \cong HF^*_{U(1)}(Y, s)$, where $HF^*_{U(1)}(Y, s)$ is the equivariant Seiberg–Witten Floer homology for $(Y, s)$ constructed in [5].
3. $HF^*_{U(1)}(Y, s) \cong HF^*_{U(1)}(-Y, s)$, where $HF^*_{U(1)}(-Y, s)$ is the equivariant Seiberg–Witten Floer cohomology for $(-Y, s)$ constructed in [5].
4. There exists a long exact sequence

\[ \cdots \to HF^*_{U(1)}(Y, s) \xrightarrow{t} HF^*_{U(1)}(Y, s) \xrightarrow{\partial} HF^*_{U(1)}(Y, s) \xrightarrow{\partial} HF^*_{U(1)}(Y, s) \to \cdots \]

Moreover, the homologies $HF^*_{U(1)}(Y, s)$, $HF^*_{U(1)}(Y, s)$, $HF^*_{U(1)}(Y, s)$, and $HF^*_{U(1)}(Y, s)$ are all topological invariants of $(Y, s)$.
5. There is an action on $HF^*_{U(1)}(Y, s)$, $HF^*_{U(1)}(Y, s)$, and $HF^*_{U(1)}(Y, s)$ respectively which decreases the degree by two, and is related to the cutting down moduli spaces of flow lines by a geometric representative of a degree 2 characteristic form. The long exact sequence (4) is a long exact sequence of $\mathbb{Z}[u]$-modules.
6. There is a homology group $\overline{HF}^*_{U(1)}(Y, s)$, which is also a topological invariant of $(Y, s)$, such that the following sequence is exact:

\[ \cdots \to \overline{HF}^*_{U(1)}(Y, s) \to HF^*_{U(1)}(Y, s) \to HF^*_{U(1)}(Y, s) \to \cdots \]

and that $\overline{HF}^*_{U(1)}(Y, s)$ is non-trivial if and only if $HF^*_{U(1)}(Y, s)$ is non-trivial.

The $u$-action in the main theorem is induced from a $u$-action on the chain complex

\[ u : CF^*_{SW, U(1)} \to CF^*_{SW, U(1)}, \]

which decreases the degree by 2. We will show that this $u$-action is homotopic to the obvious $\Omega^{-1}$ action on the chain complex $CF^*_{SW, U(1)}$. Thus, the induced $u$-action on $HF^*_{U(1)}(Y, s)$ endows them with $\mathbb{Z}[u]$-module structures.

Let $\overline{CF}^*_{SW}(Y, s)$ be the subcomplex of $CF^*_{SW, U(1)}(Y, s)$ such that the following sequence is a short exact sequence of chain complexes:

\[ 0 \to \overline{CF}^*_{SW}(Y, s) \to CF^*_{SW, U(1)}(Y, s) \to HF^*_{U(1)}(Y, s) \]

We can define $\overline{HF}^*_{SW}(Y, s)$ to be the homology of $\overline{CF}^*_{SW}(Y, s)$.

In recent work [7] [8], Ozsváth and Szabó introduced Heegaard Floer invariants $HF^*_{SW}(Y, s)$, $HF^*_{U(1)}(Y, s)$, $\overline{HF}^*_{SW}(Y, s)$, and $HF^*_{red, U(1)}(Y, s)$, with exact sequences relating them. In view of their construction, the result of Theorem 1.2, together with the identification of our $HF^*_{U(1)}(Y, s)$ and the $HF^*_{U(1)}(Y, s)$ of Ozsváth and Szabó, suggest the following conjecture.
CONJECTURE 1.3. For any rational homology 3-sphere $Y$ with a $\text{Spin}^{c}$ structure $s \in \text{Spin}^{c}(Y)$, there are isomorphisms

$$HF_{*, U(1)}^{SW+}(Y,s) \cong HF_{*, U(1)}^{+}(Y,s), \quad HF_{*, U(1)}^{SW-}(Y,s) \cong HF_{*, U(1)}^{-}(Y,s);$$

$$\overline{HF}_{*, U(1)}^{SW}(Y,s) \cong \overline{HF}_{*, U(1)}^{+}(Y,s), \quad HF_{\text{red},*}^{SW}(Y,s) \cong HF_{\text{red},*}^{+}(Y,s).$$

2. Review of equivariant Seiberg–Witten Floer homology

In this section, we recall some of basic ingredients in the definition of the equivariant Seiberg–Witten Floer homology from [5] (See [5] for all the details).

Let $(Y,s)$ be a rational homology 3-sphere $Y$ with a $\text{Spin}^{c}$ structure $s \in \text{Spin}^{c}(Y)$. For a good pair $(g, \nu)$ of a metric and a perturbation by a co-closed imaginary-valued 1-form $\nu$ on $Y$, the 3-dimensional Seiberg–Witten equations on $(Y, s)$ (cf. [2] [3] [4] [5])

$$
\begin{cases}
*F_A = \sigma(\psi, \psi) + \nu \\
\theta_A \psi = 0,
\end{cases}
$$

for a pair of $\text{Spin}^{c}$ connection $A$ and a spinor $\psi$, have only finitely many irreducible solutions modulo gauge transformations. We denote by $M_Y^{\text{irr}}(s)$ the set of equivalence classes of irreducible gauge classes of solutions of (6), and by $\theta$ the unique reducible solution modulo gauge. We write $M_Y(s) = M_Y^{\text{irr}}(s) \cup \{\theta\}$.

Gauge classes of finite energy solutions to the 4-dimensional Seiberg–Witten equations, perturbed as in [2] [3] [5], can be regarded as moduli spaces of flow lines of the Chern-Simons-Dirac functional on the gauge equivalence classes of $\text{Spin}^{c}$ connections and spinors for $(Y,s)$. These can be partitioned into moduli spaces of flow lines between pairs of critical points from $M_Y(s)$. Each is a smooth oriented manifold which can be compactified to a smooth manifold with corners by adding broken flow lines that split through intermediate critical points.

The spectral flow of the Hessian operator of the Chern-Simons-Dirac functional defines a relative grading on $M_Y(s)$:

$$gr(\cdot, \cdot) : M_Y(s) \times M_Y(s) \to \mathbb{Z}.$$

In particular, using the unique reducible point $\theta$ in $M_Y(s)$, there is a $\mathbb{Z}$-lifting of the relative grading given by $gr(a) = gr(a, \theta)$.

Let $a$ be an irreducible monopole in $M_Y(s)$. Then, for any $b \neq a$ in $M_Y(s)$, the moduli space of flow lines from $a$ to $b$, denoted by $M(a, b)$, has dimension $gr(a) - gr(b) > 0$, if non-empty. The moduli space of flow lines from $\theta$ to $d \in M_Y^*(s)$, denoted by $M(\theta, d)$ has dimension $-gr(d) - 1 > 0$, if non-empty. Notice that all these moduli spaces of flow lines admit an $\mathbb{R}$-action given by the reparameterization by translations: the corresponding quotient spaces are denoted by $\overline{M}(a, b)$ and $\overline{M}(\theta, d)$, respectively.

For any irreducible critical points $a$ and $c$ in $M_Y(s)$ with $gr(a) - gr(c) = 2$, we can construct a canonical complex line bundle over $M(a, c)$ and a canonical section as follows (see section 5.3 in [5]). Choose a base point $(y_0, t_0)$ on $Y \times \mathbb{R}$, and a complex line $\ell_{y_0}$ in the fiber $W_{y_0}$ of the spinor bundle $W$ over $y_0 \in Y$. We choose $\ell_{y_0}$ so that it does not contain the spinor part $\psi$ of any irreducible critical point. Since there are only finitely many critical points we can guarantee such choice exists. Denote the based moduli space of $M(a, c)$ by $M(O_a, O_c)$ as in [5], where $O_a$ and
$O_c$ are the $U(1)$-orbits of monopoles on the based configuration space. We consider the line bundle
\begin{equation}
\mathcal{L}_{ac} = \mathcal{M}(O_a, O_c) \times_{U(1)} (W_{y_0}/\ell_{y_0}) \to \mathcal{M}(a, c)
\end{equation}
with a section given by
\begin{equation}
\sigma([A, \Psi]) = ([A, \Psi], \Psi(y_0, t_0)).
\end{equation}
For a generic choice of $(y_0, t_0)$ and $\ell_{y_0}$, the section $s$ of (8) has no zeroes on the boundary strata of the compactification of $\mathcal{M}(a, c)$. This determines a trivialization of $\mathcal{L}_{ac}$ away from a compact set in $\mathcal{M}(a, c)$. The line bundle $\mathcal{L}_{ac}$ over $\mathcal{M}(a, c)$, with the trivialization $\varphi$ specified above, has a well-defined relative Euler class (cf. Lemma 5.7 in [5]).

**DEFINITION 2.1.** (1) For any two irreducible critical points $a$ and $b$ in $\mathcal{M}_Y(s)$ with $\text{gr}(a) - \text{gr}(b) = 1$, we define $n_{ab} := \#(\mathcal{M}(a, b))$, the number of flow lines in $\mathcal{M}(a, b)$ counted with orientations. Similarly, for any $a \in \mathcal{M}_Y(s)$ with $\text{gr}(a) = 1$ and any $d \in \mathcal{M}_Y(s)$ with $\text{gr}(d) = -2$, we define $n_{ad} := \#(\mathcal{M}(a, d))$ and $n_{d} := \#(\mathcal{M}(d, b))$, respectively.

(2) For any two irreducible critical points $a$ and $c$ in $\mathcal{M}_Y(s)$ with $\text{gr}(a) - \text{gr}(c) = 1$, we define $m_{ac}$ to be the relative Euler number of the canonical line bundle $\mathcal{L}_{ac}$ of (7), with the canonical trivialization given by (8).

The following proposition states various relations satisfied by the integers defined in Definition 2.1, whose proof can be found in Remark 5.8 of [5].

**PROPOSITION 2.2.** (1) For any irreducible critical point $a$ in $\mathcal{M}_Y(s)$ and any critical point $c$ in $\mathcal{M}_Y(s)$ with $\text{gr}(a) - \text{gr}(c) = 2$, we have the identity
\[
\sum_{b \in \mathcal{M}_Y(s), \text{gr}(a) - \text{gr}(b) = 1} n_{ab} n_{bc} = 0.
\]

(2) Let $a$ and $d$ be two irreducible critical points with $\text{gr}(a) - \text{gr}(d) = 3$. Assume that all the critical points $c$ with $\text{gr}(a) > \text{gr}(c) > \text{gr}(d)$ are irreducible. Then we have the identity
\[
\sum_{c_1: \text{gr}(a) - \text{gr}(c_1) = 1} n_{a, c_1} m_{c_1, d} - \sum_{c_2: \text{gr}(c_2) - \text{gr}(d) = 1} m_{a, c_2} n_{c_2, d} = 0.
\]

When $\text{gr}(a) = 1$ and $\text{gr}(d) = -2$, we have the identity
\[
\sum_{c_1: \text{gr}(c_1) = 0} n_{a, c_1} m_{c_1, d} + n_{a \theta} n_{\theta d} - \sum_{c_2: \text{gr}(c_2) = -1} m_{a, c_2} n_{c_2, d} = 0.
\]

With the help of this Proposition, we can check that the equivariant Seiberg-Witten Floer complex $CF^{SW}_{*, U(1)}(Y, s)$ as given in (1), with the grading and the differential operator given by (2) and (3), is well-defined, and we denote its homology by $HF^{SW}_{*, U(1)}(Y, s)$. The equivariant Seiberg-Witten–Floer cohomology, denoted by $HF^{SW,*}_{U(1)}(Y, s)$, is the homology of the dual complex $\text{Hom}(CF^{SW}_{*, U(1)}(Y, s), \mathbb{Z})$. The main result in [5] shows that the equivariant Seiberg-Witten Floer homology $HF^{SW}_{*, U(1)}(Y, s)$ and cohomology $HF^{SW,*}_{U(1)}(Y, s)$ are topological invariants of $(Y, s)$.
3. Variants of equivariant Seiberg–Witten Floer homology

As mentioned in the introduction, we will generalize the construction of the equivariant Seiberg–Witten Floer homology in several ways.

First, we denote by $CF_{*,U(1)}^{SW,\infty}(Y, s)$ the graded complex generated by

$$\{\Omega^k \otimes \eta_a, \Omega^k \otimes 1_a, \Omega^k \otimes 1_\theta : a \in M^*_r(s), k \in \mathbb{Z}\}$$

More precisely, for any irreducible critical orbits $O_a$, we set

$$C^\infty_{*, U(1)}(O_a) = \mathbb{Z}[\Omega, \Omega^{-1}] \otimes \Omega^*_0(O_a) := \bigoplus_{k \in \mathbb{Z}} (\mathbb{Z} \Omega^k \otimes \eta_a + \mathbb{Z} \Omega^k \otimes 1_a)$$

with the grading $gr(\Omega^k \otimes \eta_a) = 2k + gr(a)$ and $gr(\Omega^k \otimes 1_a) = 2k + gr(a) + 1$, and we set

$$C^\infty_{*, U(1)}(\theta) = \bigoplus_{k \in \mathbb{Z}} \mathbb{Z} \Omega^k \otimes 1_\theta$$

with the grading $gr(\Omega^k \otimes 1_\theta) = 2k$.

We then consider

$$(9) \quad CF_{*, U(1)}^{SW,\infty}(Y, s) = \bigoplus_{a \in M_Y(s)} \mathbb{Z}[\Omega, \Omega^{-1}] \otimes \Omega^*_0^{-\dim(O_a)}(O_a),$$

with the grading and the differential operator given by (2) and (3) respectively. That is, $CF_{*, U(1)}^{SW,\infty}(Y, s)$ is given by

$$\bigoplus_{a \in M_Y(s)} C^\infty_{*, U(1)}(O_a) = \bigoplus_{a \in M_Y(s)} C^\infty_{*, U(1)}(O_a) \oplus C^\infty_{*, U(1)}(\theta).$$

**Theorem 3.1.** Define $HF_{*, U(1)}^{SW,\infty}(Y, s)$ to be the homology of $(CF_{*, U(1)}^{SW,\infty}(Y, s), D)$. Then we have

$$HF_{*, U(1)}^{SW,\infty}(Y, s) \cong \mathbb{Z}[\Omega, \Omega^{-1}].$$

**Proof.** Consider the filtration of $CF_{*, U(1)}^{SW,\infty}(Y, s)$ according to the grading of the critical points,

$$F_n C^\infty_{*, U(1)} = \bigoplus_{gr(a) \leq n} C^\infty_{*, U(1)}(O_a),$$

and the corresponding spectral sequence $E^{kt}_k$. The filtration is exhaustive, that is,

$$CF_{*, U(1)}^{SW,\infty}(Y, s) = \bigcup_n F_n C^\infty_{*, U(1)},$$

and

$$\cdots \subset F_{n-1} C^\infty_{*, U(1)} \subset F_n C^\infty_{*, U(1)} \subset F_{n+1} C^\infty_{*, U(1)} \subset \cdots \subset CF_{*, U(1)}^{SW,\infty}(Y, s).$$

Moreover, by the compactness of the moduli space of critical orbits, the set of indices $gr(a)$ is bounded from above and below, hence the filtration is bounded. Thus, the spectral sequence converges to $HF_{*, U(1)}^{SW,\infty}(Y, s)$. 

We compute the $E^0$-term:

$$E^0_{kl} = \mathcal{F}_k C_{k+l-1, U(1)}^\infty / \mathcal{F}_{k-1} C_{k+l, U(1)}^\infty$$

$$= \bigoplus_{a \in M_Y(s); gr(a) = i < k} C_{k+i-1, U(1)}^\infty (O_a) / \bigoplus_{a \in M_Y(s); gr(a) = i \leq k-1} C_{k+i, U(1)}^\infty (O_a)$$

$$= \bigoplus_{a \in M_Y(s); gr(a) = k} C_{l, U(1)}^\infty (O_a).$$

For $k \neq 0$ this complex is just the direct sum of the separate complexes

$$(C_{*, U(1)}^\infty (O_a), \partial_{U(1)})$$

on each orbit $O_a$ with $gr(a) = k$:

$$\cdots \to \mathbb{Z} \Omega \otimes \mathbb{1}_a \to 0 \to \mathbb{Z} \partial \otimes \mathbb{1}_a \to 0 \to \mathbb{Z} \partial^{-1} \otimes \mathbb{1}_a \to \cdots$$

In the case $k = 0$ we have

$$E^0_{0,l} = C_{l, U(1)}^\infty (\theta) \oplus \bigoplus_{a \in M_Y(s); gr(a) = 0} C_{l, U(1)}^\infty (O_a),$$

which again is a direct sum of the complexes $(C_{*, U(1)}^\infty (O_a), \partial_{U(1)})$, here $\partial_{U(1)}$ is the equivariant de Rham differential, and of the complex with generators $\partial \otimes 1_\theta$ in degree $l = 2r$ and trivial differentials.

We then compute the $E^1_{pq}$ term directly: we have

$$E^1_{k+1} = H_{k+l}(E_{k,*}^0) = \begin{cases} 
\mathbb{Z} \partial \otimes 1_\theta & k = 0, l = 2r \\
0 & k \neq 0,
\end{cases}$$

since each complex (10) is acyclic. Thus, the only non-trivial $E^1$-terms are of the form $E^1_{0l} = \mathbb{Z} \partial \otimes 1_\theta$, $l = 2r$, with trivial differentials, so that the spectral sequence collapses and we obtain the result. \(\Box\)

### 3.1. Long exact sequence.

**Definition 3.2.** Let $CF_{U(1)}^{SW,-}(Y, s)$ be the subcomplex of $CF_{U(1)}^{SW,\infty}(Y, s)$, generated by

$$\{ \partial^{-k} \otimes \eta_a, \partial^{-k} \otimes 1_a, \partial^{-k} \otimes 1_\theta : a \in M^*_Y(s), k \in \mathbb{Z} \text{ and } k < 0 \},$$

whose homology groups are denoted by $HF_{U(1)}^{SW,-}(Y, s)$. The quotient complex is denoted by $CF_{U(1)}^{SW,+}(Y, s)$, with the homology groups denoted by $HF_{U(1)}^{SW,+}(Y, s)$.

**Theorem 3.3.**

1. $HF_{U(1)}^{SW,+}(Y, s) \cong HF_{U(1)}^{SW,-}(Y, s)$, where $HF_{U(1)}^{SW}(Y, s)$ is the equivariant Seiberg–Witten–Floer homology defined in [5].

2. There is an exact sequence of $\mathbb{Z}$-modules which relates these variants of equivariant Seiberg–Witten–Floer homologies:

$$\cdots \to HF_{U(1)}^{SW,-}(Y, s) \to HF_{U(1)}^{SW,\infty}(Y, s) \to HF_{U(1)}^{SW,+}(Y, s) \to HF_{U(1)}^{SW,-}(Y, s) \to \cdots$$

**Proof.** It is easy to see that $CF_{U(1)}^{SW,+}(Y, s) = CF_{U(1)}^{SW,-}(Y, s)$, with the same grading and differentials, hence $HF_{U(1)}^{SW,+}(Y, s) \cong HF_{U(1)}^{SW,-}(Y, s)$. The long exact sequence in homology is induced by the short exact sequence of chain complexes:

$$0 \to CF_{U(1)}^{SW,-}(Y, s) \to CF_{U(1)}^{SW,\infty}(Y, s) \to CF_{U(1)}^{SW,+}(Y, s) \to 0.$$
From the above long exact sequence, we can define
\[
HF^{SW}_{red,*}(Y, s) = \text{Coker}(\pi_*) \cong HF^{SW,+}_{*,U(1)}(Y, s) / \text{Ker}(\delta_*)
\cong \text{Im}(\delta_*) \cong \text{Ker}(l_{*-1}).
\]

3.2. The spectral sequence for $HF^{SW,+}_{*,U(1)}(Y, s)$. We consider again the filtration by index of critical orbits,
\[
\mathcal{F}_n C^{+,*}_{*,U(1)} := \bigoplus_{\text{gr}(a) \leq n} C^{+,*}_{*,U(1)}(O_a),
\]
for
\[
C^{+,*}_{*,U(1)}(O_a) = \mathbb{Z}[\Omega] \otimes \Omega^{*-\dim(O_a)}(O_a).
\]
We have
\[
E^{0}_{kl} = \mathcal{F}_k C^{+,*+l}_{k+l,U(1)}/\mathcal{F}_{k-1} C^{+,*+l}_{k+l,U(1)} = \bigoplus_{\text{gr}(a) = k} C^{+,*}_{l,U(1)}(O_a).
\]
This is a direct sum of the complexes
\[
\cdots \rightarrow \mathbb{Z}, \Omega \otimes 1_a \rightarrow \mathbb{Z}, \Omega \otimes \eta_a \rightarrow \mathbb{Z}, 1 \otimes 1_a \rightarrow \mathbb{Z}, 1 \otimes \eta_a \rightarrow 0,
\]
over each orbit $O_a \cong S^1$ and, in the case $k = 0$, the complex with generators $\Omega^r \otimes 1_\theta$ in degree $l = 2r$, and trivial differentials.
Thus, we obtain that $E^{1}_{pq} = H_{p+q}(E^{0}_{p,*})$ is of the form
\[
E^{1}_{pq} = \begin{cases} 
0 & q > 0 \\
\mathbb{Z}, 1 \otimes \eta_a & q = 0, \text{gr}(a) = p
\end{cases}
\]
for $p \neq 0$, and
\[
E^{1}_{1q} = \begin{cases} 
\mathbb{Z}, \Omega^r \otimes 1_\theta & q = 2r > 0 \\
\mathbb{Z}, 1 \otimes \eta_a \oplus \mathbb{Z}, 1 \otimes 1_\theta & q = 0, \text{gr}(a) = 0.
\end{cases}
\]
The differential $d^1 : E^{1}_{p,q} \rightarrow E^{1}_{p-1,q}$ is of the form
\[
d^1(1 \otimes \eta_a) = n_{ab} 1 \otimes \eta_b + n_{a\theta} 1 \otimes 1_\theta \quad (\text{if } \text{gr}(a) = 1)
\]
Thus, we obtain
\[
E^{2}_{pq} = \begin{cases} 
HF^{SW}_p(Y, s) & p \neq 0, q = 0 \\
\text{Ker}(\Delta_1) & p = 1, q = 0 \\
HF^{SW}_0(Y, s) \oplus T_0 & p = 0, q = 0 \\
\mathbb{Z}, \Omega^r \otimes 1_\theta & p = 0, q = 2r > 0.
\end{cases}
\]
Here $HF^{SW}_*(Y, s)$ denotes the non-equivariant (metric and perturbation dependent) Seiberg–Witten Floer homology. This is the homology of the complex with generators $1 \otimes \eta_a$ in degree $\text{gr}(a)$ and boundary coefficients $n_{ab}$ for $\text{gr}(a) - \text{gr}(b) = 1$. We also denoted by $\Delta_1$ the map
\[
\Delta_1 : HF^{SW}_1(Y, s) \rightarrow \mathbb{Z}, 1 \otimes 1_\theta,
\]
\[ \Delta_1(\sum x_a 1 \otimes \eta_a) = \sum x_a n_{a \theta} 1 \otimes 1_\theta, \]

where the coefficients \( x_a \) satisfy \( \sum x_a n_{ab} = 0 \). Finally, the term \( T_0 \) denotes the term

\[ T_0 = \mathbb{Z}.1 \otimes 1_\theta / \text{Im}(\Delta_1). \]

Notice then that the boundary \( d^2 : E^2_{p,q} \to E^2_{p-2,q+1} \) is trivial, hence the \( E^3_{p,q} \) terms are disposed as in the diagram:

\[
\begin{array}{ccccccc}
\cdots & 0 & 0 & 0 & 0 & \mathbb{Z}\Omega^2 \otimes 1_\theta & 0 & 0 & \cdots \\
\cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\
\cdots & 0 & 0 & 0 & 0 & \mathbb{Z}\Omega \otimes 1_\theta & 0 & 0 & \cdots \\
\cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\
\cdots & H^SW_4 & H^SW_3 & H^SW_2 & \text{Ker}(\Delta_1) & H^SW_0 \oplus T_0 & H^SW_{-1} & H^SW_{-2} & \cdots \\
\end{array}
\]

The differential \( d^3 : E^3_{p,q} \to E^3_{p-3,q+2} \) is given by the expression

\[ d^3(\sum x_a 1 \otimes \eta_a) = \sum x_a m_{a c} n_{c \theta} \Omega \otimes 1_\theta, \]

for \( \text{gr}(a) - \text{gr}(c) = 2 \). The expression is obtained by considering the unique choice of a representative of the class \( \sum x_a 1 \otimes \eta_a \) in \( E^3_{p,q} \) whose boundary (3) defines a class in \( E^3_{p-3,q+2} \).

The differential \( d^4 : E^4_{p,q} \to E^4_{p-4,q+3} \) is again trivial, and we obtain the \( E^5_{pq} \) of the form

\[
\begin{array}{ccccccc}
\cdots & 0 & 0 & 0 & 0 & \mathbb{Z}\Omega^2 \otimes 1_\theta & 0 & 0 & \cdots \\
\cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\
\cdots & 0 & 0 & 0 & 0 & T_1 & 0 & 0 & \cdots \\
\cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\
\cdots & H^SW_5 & H^SW_4 & H^SW_2 & \text{Ker}(\Delta_3) & H^SW_0 \oplus T_0 & H^SW_{-1} & H^SW_{-2} & \cdots \\
\end{array}
\]

where again we denote by \( T_1 \) the term

\[ T_1 = \mathbb{Z}\Omega \otimes 1_\theta / \text{Im}(\Delta_3). \]

Thus, by iterating the process, we observe that all the differentials \( d^{2k} : E^{2k}_{p,q} \to E^{2k}_{p-2k,q+2k+1} \) are trivial and the differentials \( d^{2k+1} : E^{2k+1}_{p,q} \to E^{2k+1}_{p-2k-1,q+2k} \) consist of one map for \( p = 2k + 1, q = 0 \):

\[ \Delta_{2k+1} : H^SW_{2k+1} \to \mathbb{Z}\Omega^k \otimes 1_\theta, \]

induced by

\[ \Delta_{2k+1}(\sum x_a 1 \otimes \eta_a) = \sum x_a m_{a a_{2k-1}} m_{a_{2k-1} a_{2k-3}} \cdots m_{a_{3} a_1} n_{a_1 \theta} \Omega^k \otimes 1_\theta. \]

Here we have \( \text{gr}(a) = 2k + 1 \) and \( \text{gr}(a_r) = r \). Notice that these maps agree with the morphism \( \Delta_* \), which is obtained in [5] as the connecting homomorphism in the
long exact sequence relating equivariant and non-equivariant Seiberg–Witten Floer homologies.

We thus obtain the following structure theorem for equivariant Seiberg–Witten Floer homology.

**Theorem 3.4.** The equivariant Seiberg–Witten Floer homology $H_{*;U(1)}^{SW, +}(Y, s)$ has the form

$$H_{*;U(1)}^{SW, +}(Y, s) = \begin{cases} 
\text{Ker}(\Delta_{2k+1}) & * = 2k + 1 > 0 \\
H_{2k}^{SW}(Y, s) \oplus T_k & * = 2k \geq 0 \\
H_{*}^{SW}(Y, s) & * < 0
\end{cases}$$

where $T_k$ is the term

$$T_k = \mathbb{Z}.\mathbb{Q}^k \otimes 1_\theta / \text{Im}(\Delta_{2k+1}).$$

This result refines the long exact sequence obtained in [5]:

$$HF_{*;U(1)}^{SW}(Y, s) \xrightarrow{i} HF_{*}^{SW}(Y, s, g, \nu) \xrightarrow{\Delta} \mathbb{Z} [\Omega].$$

Similar results can be obtained for $HF_{*;U(1)}^{SW, -}(Y, s)$.

**3.3. Topological invariance.** Notice that the definition of these homologies depends on the Seiberg–Witten equations, which use the metric and perturbation on $(Y, s)$. By the result of [5], we know that $H_{*;U(1)}^{SW, +}(Y, s) \cong H_{*;U(1)}^{SW}(Y, s)$ is a topological invariant of $(Y, s)$, we first recall this topological invariance as stated in Theorem 6.1 [5].

**Theorem 3.5.** (Theorem 6.1 [5]) Let $(Y, s)$ be a rational homology sphere with a Spin$^c$ structure. Suppose given two metrics $g_0$ and $g_1$ on $Y$ and perturbations $\nu_0$ and $\nu_1$ such that $\text{Ker}(\Phi_{\nu_0}^{g_0}) = \text{Ker}(\Phi_{\nu_1}^{g_1}) = 0$, so that the corresponding monopole moduli spaces $M_Y(s, g_0, \nu_0)$ and $M_Y(s, g_1, \nu_1)$ consist of finitely many isolated points. Then there exists an isomorphism between the equivariant Seiberg–Witten Floer homologies $HF_{*;U(1)}^{SW}(Y, s, g_0, \nu_0)$ and $HF_{*;U(1)}^{SW}(Y, s, g_1, \nu_1)$, with a degree shift given by the spectral flow of the Dirac operator $\Phi_{\nu_1}^{g_1}$ along a path of metrics and perturbations connecting $(g_0, \nu_0)$ and $(g_1, \nu_1)$. That is, if the complex spectral flow along the path $(g_t, \nu_t)$ is denoted by $SF_C(\Phi_{\nu_t}^{g_t})$, then for any $k \in \mathbb{Z}$,

$$HF_{k;U(1)}^{SW}(Y, s, g_0, \nu_0) \cong HF_{k+2SF_C(\Phi_{\nu_1}^{g_1})\cdot U(1)}^{SW}(Y, s, g_1, \nu_1).$$

From Theorem 3.1, we know that

$$HF_{*, U(1)}^{SW, \infty}(Y, s) \cong \mathbb{Z}[\Omega, \Omega^{-1}]$$

is independent of $(Y, s)$, up to a degree shift as given in Theorem 3.5. Thus, applying the five lemma to the long exact sequence in Theorem 3.3, we obtain that $HF_{*, U(1)}^{SW, -}(Y, s)$ and $HF_{red, U(1)}^{SW}(Y, s)$ are also topological invariants of $(Y, s)$. 


Theorem 3.6. $HF^*_{\text{SW}}(Y, s)$ and $HF^*_{\text{red, SW}}(Y, s)$ are topological invariants of $(Y, s)$, in the sense that, given any two metrics $g_0$ and $g_1$ on $Y$ and perturbations $\nu_0$ and $\nu_1$, with $\text{Ker}(\mathcal{D}_0) = \text{Ker}(\mathcal{D}_1) = 0$, there exist isomorphisms

$$HF^*_{\text{SW}}(Y, s, g_0, \nu_0) \cong HF^*_{\text{SW}}(Y, s, g_1, \nu_1)$$

$$HF^*_{\text{red, SW}}(Y, s, g_0, \nu_0) \cong HF^*_{\text{red, SW}}(Y, s, g_1, \nu_1).$$

Here $SF_C(\mathcal{D}_1)$ denotes the complex spectral flow of the Dirac operator $\mathcal{D}_1$ along the path $(g_t, \nu_t)$.

4. Properties of equivariant Seiberg–Witten Floer homologies

In this section, we briefly discuss some of the algebraic structures and properties of the equivariant Seiberg–Witten Floer homologies defined in the previous section.

Notice that, for any irreducible critical points $a$ and $b$ in $\mathcal{M}_a$, the associated integer $m_{abc}$ is the counting of points in the geometric representative of the relative first Chern class of the canonical line bundle (7) over $\mathcal{M}(a, c)$. We can apply this fact to define a $u$-action on the chain complex $CF^*_{\text{SW, inf}}(Y, s)$

$$u : CF^*_{\text{SW, inf}}(Y, s) \rightarrow CF^*_{\text{SW, inf}}(Y, s),$$

which decreases the grading by two. The action is given in terms of its actions on generators as

$$u(\Omega^n \otimes \eta_a) = \sum_{c \in \mathcal{M}_a(Y, s), \sigma^r(a) - \sigma^r(c) = 2} m_{abc} \Omega^n \otimes \eta_c$$

if $\sigma^r(a) \neq 1$

$$u(\Omega^n \otimes 1_a) = \begin{cases} 
\sum_{c \in \mathcal{M}_a(Y, s), \sigma^r(a) - \sigma^r(c) = 2} m_{abc} \Omega^n \otimes 1_c + n_{a\theta} \Omega^n \otimes 1_{\theta} & \text{if } \sigma^r(a) = 1 
\end{cases}$$

$$u(\Omega^n \otimes 1_{\theta}) = \sum_{d \in \mathcal{M}_{\theta}(s), \sigma^r(d) = -2} \eta_{\theta d} \Omega^n \times \eta_{\theta} + \Omega^{n-1} \otimes 1_{\theta}.$$

**Proposition 4.1.** The $u$-action (14) on the chain complex $CF^*_{\text{SW, inf}}(Y, s)$ is homotopic to the $\Omega^{-1}$-action acting on $CF^*_{\text{SW, inf}}(Y, s)$. The induced action on $CF^*_{\text{SW, inf}}(Y, s)$ defines $\mathbb{Z}[u]$-module structures on $HF^*_{\text{SW, inf}}(Y, s)$.

**Proof.** Define $H : CF^*_{\text{SW, inf}}(Y, s) \rightarrow CF^*_{\text{SW, inf}}(Y, s)$ by its actions on the generators as follows:

$$H(\Omega^n \otimes \eta_a) = 0,$$

$$H(\Omega^n \otimes 1_a) = \Omega^n \otimes \eta_a,$$

$$H(\Omega^n \otimes 1_{\theta}) = 0.$$
Then it is a direct calculation to show that we have:

\[(u - \Omega^{-1})(\Omega^k \otimes \eta_a) = \sigma_{a\sigma}\Omega^k \otimes \eta_a - \Omega^{k-1} \otimes \eta_a\]

\[= (DH + HD)(\Omega^k \otimes \eta_a)\]

\[(u - \Omega^{-1})(\Omega^k \otimes 1_a) = \sigma_{a\sigma}\Omega^k \otimes 1_a - \Omega^{n-1} \otimes 1_a (+\sigma_{a\sigma}\Omega^n \otimes 1_{\sigma}) \text{ if } \text{gr}(a) = 1\]

\[= (DH + HD)(\Omega^k \otimes 1_a),\]

\[(u - \Omega^{-1})(\Omega^k \otimes 1_{\sigma}) = \sigma_{\sigma\sigma}\Omega^n \otimes \eta_{\sigma} = (DH + HD)(\Omega^k \otimes 1_{\sigma}).\]

Thus the claim follows using the chain homotopy \(u - \Omega^{-1} = D \circ H + H \circ D\).

Thus, at the homological level, we can identify the \(u\)-action with the induced \(\Omega^{-1}\) action on various homologies. In particular, we see that there is a subcomplex \(\widehat{CF}^*_{SW}(Y, s)\) of \(CF^*_{SW,+}(Y, s)\), such that the following short exact sequence of chain complexes holds:

\[
0 \rightarrow \widehat{CF}^*_{SW}(Y, s) \rightarrow CF^*_{SW,+}(Y, s) \rightarrow \Omega^{-1} \rightarrow CF^*_{SW,+}(Y, s) \rightarrow 0.
\]

**Proposition 4.2.** Let \(\widehat{HF}_{SW}(Y, s)\) be the homology of \(\widehat{CF}^*_{SW}(Y, s)\), then the homology \(\widehat{HF}^*_{SW}(Y, s)\) is also a topological invariant of \((Y, s)\), and it is determined by the long exact sequence

\[
\cdots \rightarrow \widehat{HF}^*_{SW}(Y, s) \rightarrow HF^*_{SW,+}(Y, s) \rightarrow HF^*_{SW,+}(Y, s) \rightarrow HF^*_{SW,-}(Y, s) \rightarrow \cdots.
\]

Moreover, \(\widehat{HF}_{SW}(Y, s)\) is non-trivial if and only if \(HF^*_{SW,+}(Y, s)\) is non-trivial.

**Proof.** The long exact sequence follows from the short exact sequence of chain complexes (15) and Proposition 4.1. This long exact sequence implies that the homology \(\widehat{HF}^*_{SW}(Y, s)\) is also a topological invariant of \((Y, s)\).

Note that, from the compactness of \(\mathcal{M}_Y(s)\), we see that each element in \(HF^*_{SW,+}(Y, s)\) can be annihilated by a sufficiently large power of \(\Omega^{-1}\). Hence, \(u\) is an isomorphism on \(HF^*_{SW,+}(Y, s)\) if and only if \(HF^*_{SW,+}(Y, s)\) is trivial. Then the last claim follows from this observation and the long exact sequence.

If we think of the set of \(\operatorname{Spin}^c\) structures on \(Y\) as the set of equivalence classes of nowhere vanishing vector fields on \(Y\) (cf.[9]), then there is a natural bijection between \(\operatorname{Spin}^c(Y)\) and \(\operatorname{Spin}^c(-Y)\) where \(-Y\) is the same \(Y\) with the opposite orientation.

**Theorem 4.3.** Let \((Y, s)\) be a rational homology 3-sphere with a \(\operatorname{Spin}^c\) structure \(s\), and \((-Y, s)\) denote \(Y\) with the opposite orientation and the corresponding \(\operatorname{Spin}^c\) structure. Then there is a natural isomorphism

\[HF^*_{SW,+}(Y, s) \cong HF^*_{SW,+}(-Y, s)\]

where \(HF^*_{SW,+}(Y, s)\) is the equivariant Seiberg–Witten Floer cohomology defined in [5].
Proof. Notice that $HF_{U(1)}^{SW*}(Y,s)$ is the homology of the dual complex

$$\text{Hom}(CF_{*,U(1)}^{SW*}(Y,s), \mathbb{Z}).$$

We start to construct a natural pairing

$$(\cdot, \cdot): CF_{*,U(1)}^{SW}(Y,s) \times CF_{*,U(1)}^{SW}(-Y,s) \to \mathbb{Z}$$

which satisfies

$$(17) \quad \langle D_Y(\xi_1), \xi_2 \rangle = \langle \xi_1, D_Y^{-1}(\xi_2) \rangle, \quad \langle \Omega^{-1}(\xi_1), \xi_2 \rangle = \langle \xi_1, \Omega^{-1}(\xi_2) \rangle.$$ 

for any element $\xi_1 \in CF_{*,U(1)}^{SW}(Y,s)$ and any element $\xi_2 \in CF_{*,U(1)}^{SW}(-Y,s)$.

Then we will show that the above pairing is non-degenerate when restricted to $CF_{*,U(1)}^{SW*}(Y,s) \times CF_{*,U(1)}^{SW*}(-Y,s)$.

From the nature of the Seiberg-Witten equations, we see that there is an identification

$$\mathcal{M}_Y(s) \to \mathcal{M}_{-Y}(s)$$

for a good pair of metric and perturbation on $(Y,s)$ and the corresponding metric and perturbation on $(-Y,s)$. Then the relative gradings with respect to the unique reducible monopole in $\mathcal{M}_Y(s)$ and $\mathcal{M}_{-Y}(s)$ respectively, satisfies

$$gr_{-Y}(a^-) = gr_Y(a) - 1,$$

where $a^-$ is the element in $\mathcal{M}_{-Y}(s)$ corresponding to $a \in \mathcal{M}_Y(s)$, we assume that $gr_Y(\theta) = gr_{-Y}(\theta^-)$. Moreover, there is an natural identification between the moduli spaces of flow lines for $(Y,s)$ and $(-Y,s)$, that is,

$$\mathcal{M}_{Y \times \mathbb{R}}(a, b) \cong \mathcal{M}_{-Y \times \mathbb{R}}(b^-, a^-).$$

Now we define the pairing on $CF_{*,U(1)}^{SW}(Y,s) \times CF_{*,U(1)}^{SW}(-Y,s)$ such that the following pairings are the only non-trivial pairings:

$$(18) \quad \langle \Omega^n \otimes \eta_a, \Omega^{-n-1} \otimes 1_{a^-} \rangle = 1$$

$$(19) \quad \langle \Omega^n \otimes 1_a, \Omega^{-n-1} \otimes \eta_{a^-} \rangle = 1$$

$$(20) \quad \langle \Omega^n \otimes 1_\theta, \Omega^{-n-1} \otimes 1_{\theta^-} \rangle = 1.$$ 

It is a direct calculation to show that this pairing satisfies the relation (17) and the restriction of this pairing to $CF_{*,U(1)}^{SW}(Y,s) \times CF_{*,U(1)}^{SW}(-Y,s)$ is non-degenerate. Then the claim follows from the definition. \qed

Let $\widehat{HF}_{U(1)}^{SW*}(Y,s)$ and $HF_{\pm,U(1)}^{SW*}(Y,s)$ denote the homology groups of the dual complexes $\text{Hom}(\overline{CF}_{*,U(1)}^{SW}(Y,s), \mathbb{Z})$ and $\text{Hom}(CF_{*,U(1)}^{SW}(Y,s), \mathbb{Z})$ of $\overline{CF}_{*,U(1)}^{SW}(Y,s)$ and $CF_{*,U(1)}^{SW}(Y,s)$ respectively. From the proof of Theorem 4.3 above, we actually establish the following duality between these homologies.

Theorem 4.4. For any rational homology 3-sphere $Y$ with a Spin$^c$ structure $s$, there exist natural isomorphisms

$$(18) \quad \widehat{HF}_{U(1)}^{SW*}(Y,s) \cong \widehat{HF}_{*,U(1)}^{SW}(-Y,s), \quad HF_{\pm,U(1)}^{SW*}(Y,s) \cong HF_{*,U(1)}^{SW*}(-Y,s).$$
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M. MARCOLLI: MAX–PLANCK INSTITUT FÜR MATHEMATIK, VIVATSGASSE 7, D-53111 BONN GERMANY
E-mail address: marcolli@mpim-bonn.mpg.de

B.L. WANG: INSTITUT FÜR MATHEMATIK, UNIVERSITÄT ZÜRICH, WINTERTHURERSTRASSE 190, CH-8057 ZÜRICH SWITZERLAND
E-mail address: bwang@math.unizh.ch

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