Ma 2a P: Homework N.2

due Tuesday Oct 19, 12 noon

1. For the following differential equations describe the equilibrium solutions and the asymptotic behavior of the other solutions, for different choices of the initial condition $y(0) = y_0$:

   - $\dfrac{dy}{dt} = e^y - 1$, with initial conditions $-\infty < y_0 < \infty$;
   - $\dfrac{dy}{dt} = y^2(1-y)^2$, with initial conditions $-\infty < y_0 < \infty$;
   - $\dfrac{dy}{dt} = y(a - y^2)$, for different possible values of the parameter $a > 0$, $a = 0$, or $a < 0$, and with initial conditions $-\infty < y_0 < \infty$.

2. Check if the following equations of the form $M(x,y) + N(x,y)\frac{dy}{dx}$ satisfy the condition $M_y = N_x$. If so, find a function $\psi(x,y)$ such that $\psi_x = M$ and $\psi_y = N$.

   - $\dfrac{dy}{dx} = -\dfrac{ax + by}{bx + cy}$;
   - $\dfrac{x}{(x^2+y^2)^{3/2}} + \dfrac{y}{(x^2+y^2)^{3/2}} \dfrac{dy}{dx} = 0$;
   - $(ye^{xy}\cos(2x) - 2e^{xy}\sin(2x) + 2x) + (xe^{xy}\cos(2x) - 3)\dfrac{dy}{dx} = 0$.

3. For each of the following equations, find an integrating factor so that the resulting equations can be solved by the method of the previous problem, then write the solutions, as curves in the (x, y)-plane.

   - $1 + (\frac{x}{y} - \sin(y))\frac{dy}{dx} = 0$;
   - $y + (2xy - e^{-2y})\frac{dy}{dx} = 0$.

4. Write the following linear differential equations equivalently as integral equations, and equivalently as fixed point problems $\phi(t) = T(\phi(t))$; starting with $\psi(t) \equiv 0$, compute the iterates $T^k(\psi(t))$ and show that they converge to the solution $\phi(t) = T(\phi(t))$.

   - $y' = -y - 1$ with $y(0) = 0$;
   - $y' = y + 1 - t$ with $y(0) = 0$.