

Ma 2a P: Midterm

Friday Oct 29

1. Solve the following differential equations:

- $ty' = (1 - y^2)^{1/2}$
- $y' + \frac{1}{t}y = 3 \cos(2t)$, with $t > 0$
- $y + (2xy - e^{-2y})\frac{dy}{dx} = 0$ (find an integrating factor)
- $y'' + 8y' - 9y = 0$, with $y(1) = 1$ and $y'(1) = 0$
- $y'' - y' - 2y = 2e^{-t}$

2. Consider the differential equation

$$\frac{dy}{dt} = ry \log\left(\frac{K}{y}\right),$$

with two positive constants $r > 0$ and $K > 0$.

- Find the equilibrium solutions
- Draw the graph of the function $f(y) = ry \log(K/y)$
- Describe qualitatively the behavior of the other solutions
- Are the equilibrium solutions stable or unstable?