Cone and Suspension

X top. space (X, T)

$\mathscr{C}X$ cone of X = quotient space

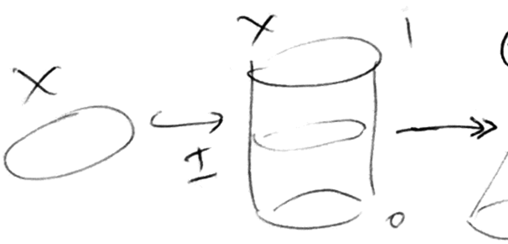
$$\mathscr{C}X = X \times I / \mathcal{R} \qquad I = [0,1]$$

$$= X \times I / X \times \{1\}$$

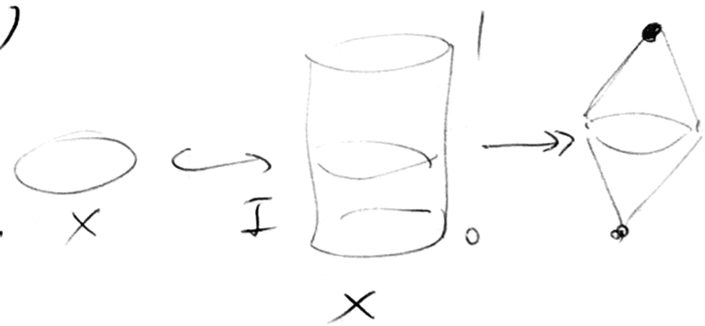
$\mathcal{R}$ = equiv. relation

$(x, 1) \sim (x_0, 1)$ identifies to a single pt all $X \times \{1\}$

(x, t) no identif. for $t \neq 1$



$\mathscr{C}(X)$



ΣX suspension of X

$$\Sigma X = X \times I / \mathcal{R}$$

$\mathcal{R}$ relation $(x, 1) \sim (x_0, 1)$

$(x, 0) \sim (x_0, 0)$

(x, t) no identif for $t \neq 0, 1$

$\boxed{\Sigma S^n \cong S^{n+1} \quad \text{spheres as suspensions}}$

in Sets injective maps $Y \xhookrightarrow{j} X$

surjective maps $Z \xrightarrow[\pi]{} X$

and bijections (= isomorphisms of sets)

both injective and surjective

in other categories $\mathcal{C}$

- $f \in Mor_{\mathcal{C}}(X,Y)$ is an <u>epimorphism</u>

  if for any $Z \in Obj(\mathcal{C})$
  and any $g_1, g_2 \in Mor_{\mathcal{C}}(Y,Z)$

  $$X \xrightarrow{f} Y \overset{g_1}{\underset{g_2}{\rightrightarrows}} Z$$

  if $g_1 \circ f = g_2 \circ f$

  then $g_1 = g_2$

(in Sets epimorphism iff surjective)

- $f \in Mor_{\mathcal{C}}(X,Y)$ is a <u>monomorphism</u>

  if for any $Z \in Obj(\mathcal{C})$ and any $g_1, g_2 \in Mor_{\mathcal{C}}(Z,X)$

  $$Z \overset{g_1}{\underset{g_2}{\rightrightarrows}} X \xrightarrow{f} Y$$

  if $f \circ g_1 = f \circ g_2$

  then $g_1 = g_2$

(in Sets monom iff injective)

- isomorphism $f \in Mor_{\mathcal{C}}(X,Y)$ if

invertible morphism i.e. $\exists\, g \in Mor_{\mathcal{C}}(Y,X)$

$$g \circ f = 1_X \qquad f \circ g = 1_Y$$

- isomorphism $\Longrightarrow$ epimorphism & monomorphism

$\not\Longleftarrow$

in general not

- $\mathcal{C}$ is a balanced category if

epimorphism & monomorphism $\Longrightarrow$ isomorphism

— Sets is balanced

— Top is not balanced

isomorphisms = homeomorphisms (continuous bijection with continuous inverse)

epimorphism & monomorphism = continuous bijections (don't nec. have continuous inverse)