- closure of sets

$A \subset X$ $\quad x \in X$ is adherent to A
if each $\mathcal{U} \in \mathcal{T}$ $x \in \mathcal{U}$ contains at least some pt of A (which may be x if $x \in A$)

$\bar{A}$ = all adherence pts of A = $\{x \in X \mid \mathcal{U} \cap A \neq \emptyset$ for all $\mathcal{U} \in \mathcal{T}$ with $x \in \mathcal{U}\}$

always $A \subseteq \bar{A}$ and $A = \bar{A}$ iff A is closed

if A closed, A^c open so $\forall x \in A^c$ $\exists\, \mathcal{U} \in \mathcal{T}$ $x \in \mathcal{U} \subseteq A^c$
i.e. $\mathcal{U} \cap A = \emptyset$, so $\forall x \in A^c$ $x \notin$ adherence pts of A
$\Rightarrow A^c \subseteq (\bar{A})^c$ hence $\bar{A} \subseteq A$, but also $A \subseteq \bar{A}$ so $=$

if $A = \bar{A}$ then $x \notin A$ means $x \notin \bar{A}$ so
$\exists\, \mathcal{U} \in \mathcal{T}$ $x \in \mathcal{U} \subseteq \bar{A}^c$ so $\bar{A}^c$ open

Properties:

$A \subseteq B \Rightarrow \bar{A} \subseteq \bar{B}$

$\bar{\bar{A}} = \bar{A}$ (the set $\bar{A}$ is closed)

$\overline{A \cup B} = \bar{A} \cup \bar{B}$

$\bar{\emptyset} = \emptyset$

and $\bar{A}$ is intersection of all closed sets that contain A