(1)

if X connected then

exactly 2 functions (contin) $f: X \to \{0,1\}$

$\{0,1\} \simeq * \sqcup *$ coprod. of two single pts

So $\mathrm{Mor}_{\mathrm{Top}}(X, * \sqcup *) \cong \mathrm{Mor}_{\mathrm{Top}}(X, *) \underset{\underline{\mathrm{Sets}}}{\sqcup} \mathrm{Mor}_{\mathrm{Top}}(X, *)$

but if X not connected then $\mathrm{Mor}_{\mathrm{Top}}(X, -)$ does not preserve coproducts

e.g. $X = \{0,1\} \cup \{2,3\}$

$Y = \{0,1\}$

$\mathrm{Mor}_{\mathrm{Top}}(X, Y) = \{4 \text{ functions}\}$

$\neq \mathrm{Mor}_{\mathrm{Top}}(X, \{0\}) \underset{\mathrm{Sets}}{\sqcup} \mathrm{Mor}_{\mathrm{Top}}(X, \{1\})$

More generally;

$\mathrm{Mor}_{\mathrm{Top}}(X, Y \sqcup Z) \cong$

$\mathrm{Mor}_{\mathrm{Top}}(X, Y) \underset{\underline{\mathrm{Sets}}}{\sqcup} \mathrm{Mor}_{\mathrm{Top}}(X, Z)$

In a category $\mathcal{C}$ an object $X \in \mathrm{Obj}(\mathcal{C})$ is <u>connected</u> if $\mathrm{Mor}_{\mathcal{C}}(X, -)$ preserves coproducts

$\mathrm{Mor}_{\mathcal{C}}(X, Y \sqcup Z) \underset{\underline{\mathrm{Sets}}}{\cong} \mathrm{Mor}_{\mathcal{C}}(X, Y) \underset{\underline{\mathrm{Sets}}}{\sqcup} \mathrm{Mor}_{\mathcal{C}}(X, Z)$

Connected components functor

$\pi_0 X =$ set of connected components of X (or do w/ path connected comp.)

$\pi_0 : \mathrm{Top} \to \mathrm{Sets}$ is a functor

$f : X \to Y$ continuous

$C \subseteq X$ connected component $\Rightarrow f(C)$ connected

$\exists\, C'$ conn. comp of Y

$f(C) \subseteq C'$

$\pi_0(f) : C \in \pi_0 X \longmapsto \underset{\pi_0 Y}{\underset{\Uparrow}{C'}} \supseteq f(C)$
