Geometry of Neuroscience

Spring 2017

Matilde Marcolli & Doris Tsao

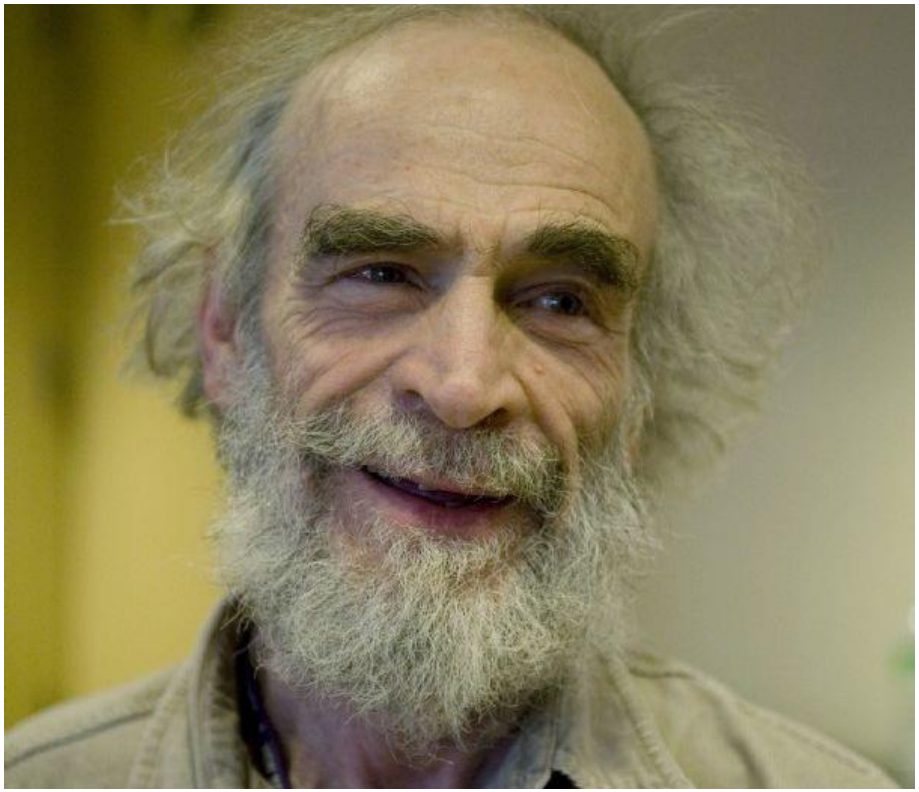
http://www.its.caltech.edu/~matilde/GeomNeuroClass.html

Overview

This class will cover topics in geometry and topology applied to neuroscience, with particular focus on vision and language. Topics covered will include:

- Geometry and topology of the visual cortex
- Geometry of segmentation and invariance
- Neural codes and neural rings
- Deep learning neural networks: mathematical aspects, and applications to vision and language
- Mathematics and neuroscience of language and syntax

Structures, Learning, and Ergosystems (2011, 2013)



Mikhail Gromov

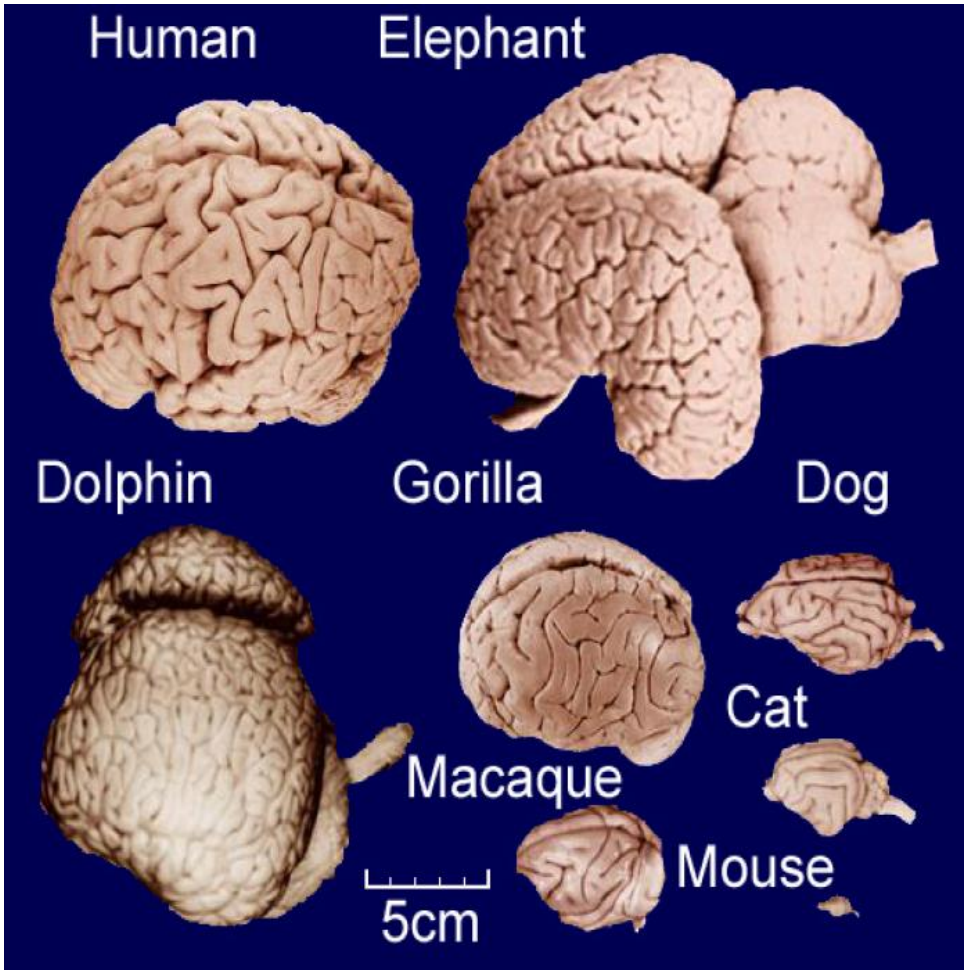
“Nature shows us only the tail of the lion. But there is no doubt in my mind that the lion belongs with it even if he cannot reveal himself to the eye all at once because of his huge dimension.”

-Albert Einstein

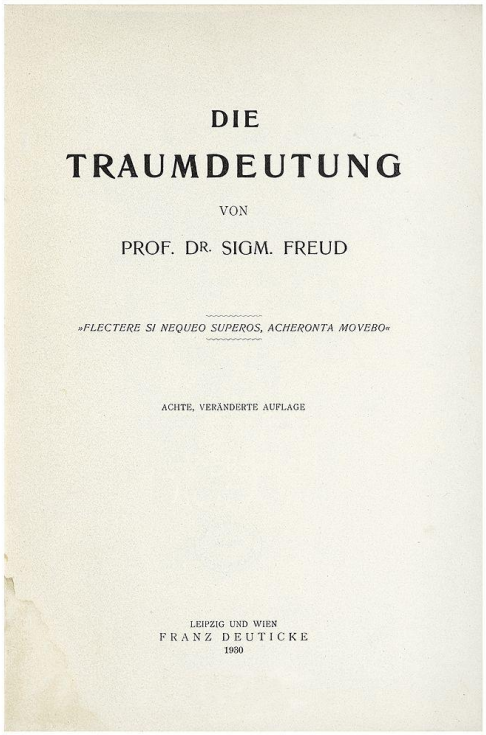
“Nature shows us only the tail of the lion. But there is no doubt in my mind that the lion belongs with it even if he cannot reveal himself to the eye all at once because of his huge dimension.”

-Albert Einstein

Consciousness



Precursors


DIE

TRAUMDEUTUNG

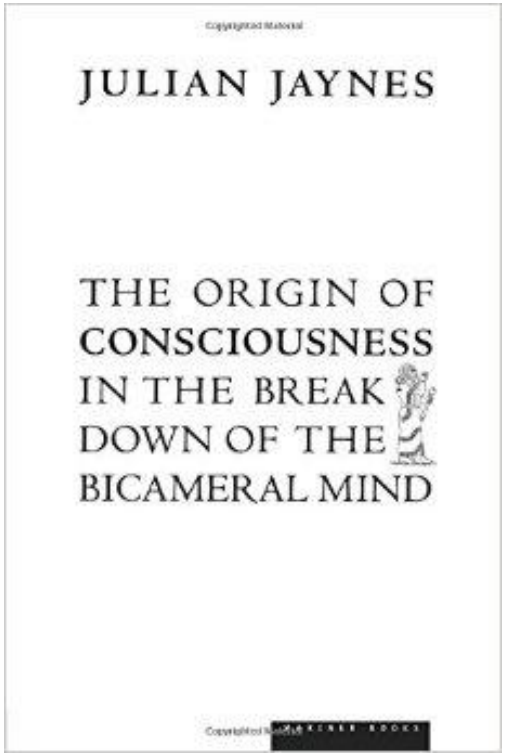
VON

PROF. DR. SIGM. FREUD

»FLECTERE SI NEQUEO SUPEROS, ACHERONTA MOVEBO«

ACHTE, VERÄNDERTE AUFLAGE

LEIPZIG UND WIEN
FRANZ DEUTICKE
1930


JULIAN JAYNES

THE ORIGIN OF CONSCIOUSNESS IN THE BREAK DOWN OF THE BICAMERAL MIND

- Freud: Dreams, slips of tongue, jokes, etc. -> a theory of subconscious mental processes & psychoanalysis as means to treat mental disorders caused by these processes

- Jaynes: Iliad, hypnosis, schizophrenia, anatomy of language pathways, inordinate power of music -> a theory of how modern consciousness arose from an early pre-conscious state in which the left brain obeyed orders given by the right brain.

Organization: Five chapters

Ch1: free-form speculation about why ergosystem should exist & what its nature should be

Ch2: mathematical promenade

Ch3: unwritten

Ch4: language as ergosystem

Ch5: postmodern AI-inspired short story

Organization: Five chapters

Ch1: free-form speculation about why ergosystem should exist & what its nature should be

Ch2: mathematical promenade

Ch3: unwritten

Ch4: language as ergosystem

Ch5: postmodern AI-inspired short story

Structures, Learning, and Ergosystems

- The existence of Ramunujan, the activity of mathematics, children learning natural languages, play of Dholes, the evolution of peacock feathers, the icosahedral shape of viruses, … -> A hypothesis that the interface between the brain and the mind must consist of a yet unknown mathematical entity, the ergosystem, capable of goal-free structure learning.

With so many implications…

- A deep critique of the short-sightedness of our society's focus on the ego (*"It takes 5-10 mental regurgitations over a several months time interval to assimilate an entirely new non-trivial idea, be it a mathematician's or non-mathematician's ergo brain, while keeping your smart ego-mind at bay.")*

- A deep critique of our educational system (*"Several months old babies start playing with: "PA PA BA BA". Their auditory system records these "PA PA BA BA" with an amazing correlation between the sounds and the somatosensory and tactual perceptions. Deaf children start "PA PA BA BA" at the same age but stop doing it sooner: there is less to be amazed with. Eventually, babies get bored with this and start producing more interesting/meaningful sequences of sounds, unless taught that BLAH-BLAH-BLAH is an acceptable adult speech."*)

- A glimpse into a future in which humans co-exist with intelligent robots (attempt to understand last chapter…)

Structures, Learning, and Ergosystems

- Transformation of signals by the brain: Incoming signals -> outgoing signals
- Structure mediating this transformation: “ergosystem”
- Ergosystem is a machine for extracting *structure* from incoming signals, through a process of *goal-free learning (*contrast w/ reinforcement learning*)*
- A “dynamical reduction (quotient)” of neuro-brain
- Contrast w/ ego-mind: pragmatic mental processes serving survival needs

Why “ergosystem”?

Cogito ergo sum

I see therefore I am

I envy therefore I am

I love therefore I am

I dream therefore I am

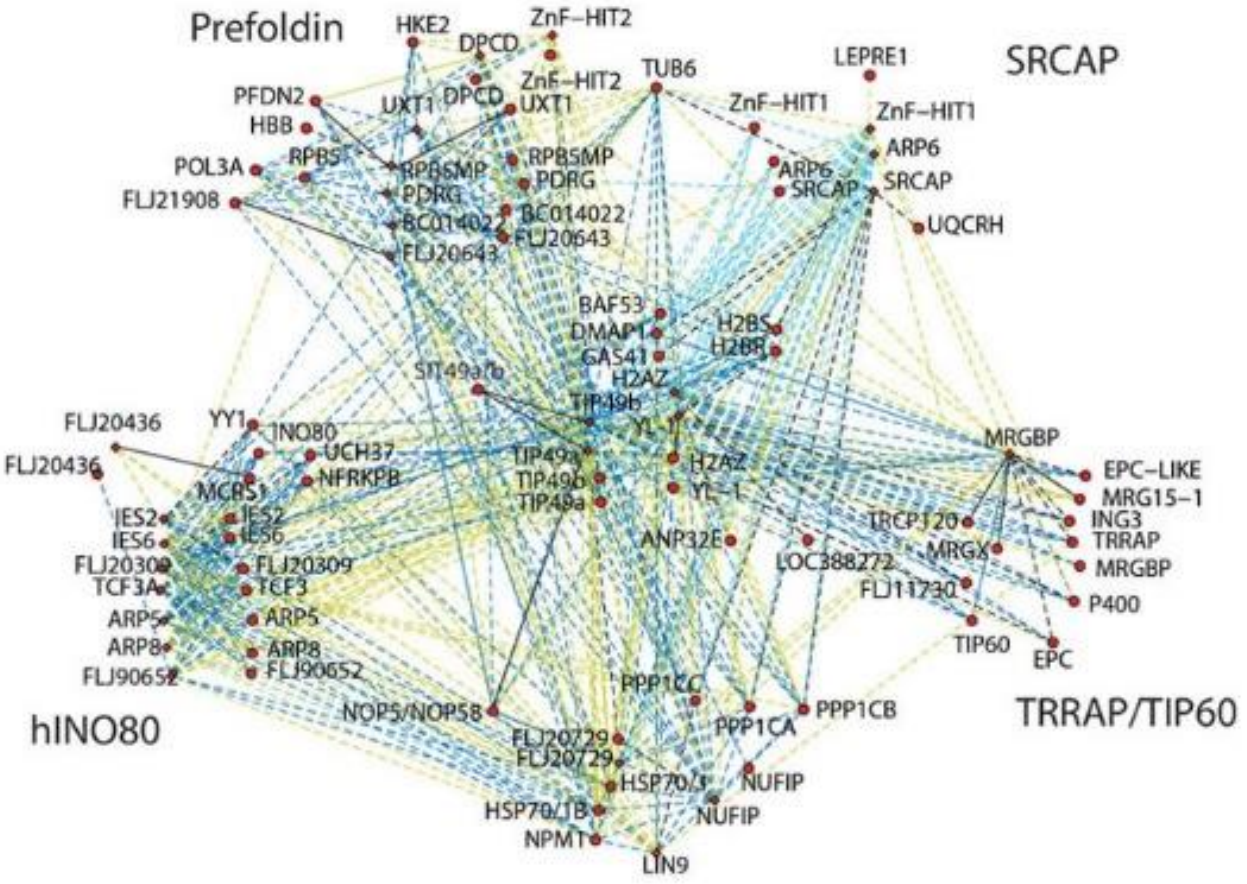
“Therefore” = hope to understand consciousness through reasoning/mathematics

What is the nature of the ergosystem?

Analogy to cell

- Interaction with environment: “ego-activity” of cell (cytotologist/psychologist)
- Selection of beneficial DNA mutations: neurophysiology of learning (molecular geneticist/neurophysiologist)
- What is missing: biochemistry of proteins

Protein interaction network



What is the architecture of the ergobrain? An analogy to viruses

"In the end of the day, the symmetry of viruses depends on the structural constrains imposed by the geometry of the physical space which allows the existence of such improbable objects as icosahedra.

Similarly, we expect that the constrains of the mathematical space (which we have not defined yet) where ergosystems reside will have a strong structural imprint on a possible architecture of the ergobrain. For example, we want to understand what in the (ergo)brain is responsible for the unreasonable power of our visual system which allowed a discovery of icosahedra by humans (at least) as early as –400."

The power of the ergobrain: existence of Ramanujan

$$\frac{1}{\pi} = \frac{2\sqrt{2}}{9801} \sum_{k=0}^{\infty} \frac{(4k)!(1103 + 26390k)}{(k!)^4 396^{4k}} = \frac{2\sqrt{2}}{9801} \left(1103 + \frac{24 \cdot 27493}{396^4} + \ldots\right)$$

What makes a star (of a given mass) a supernova is (essentially) the value of a single parameter, the age of the star. What made Ramanujan, I believe, is a minor (on the brain scale) increment of something like the "conductivity parameter" of the communication line from his (ergo)brain to his conscious mind or a decrement in translucency of the "brain/mind window".

Listening to Bach

Crab fugue as Mobius strip: https://vimeo.com/69715960

"The motto of ergosystems is that of a practicing theoretical physicist: *imaginative non-rigorous mathematics*"

(Bird flying between two trains, one traveling at 40 mph, the other at 60 mph. Trains separated by 100 km. Bird travels at 100 km/hour. What is the distance covered by bird before trains meet?)

Seeking symmetries

Is there a mathematical rule for "symmetry searching"?

- 1+1 = 2, 2+ 2 = 4, 3+3 =6, …
- Galois theory ("a mathematical framework for the study of symmetry, arbitrariness and ambiguity" based on "Galois group": permutations of roots of polynomial which preserve rational relations)
- Category theory (a very abstract way to formulate all of math)

"The ideas of the category theory show that there are certain (often non-obvious) rules for generating proper concepts."

Language as a model ergosystem

- Linguistic information entering the ergobrain does not much depend on the physical carrier of this information. This suggest a universal class of structures encoding this information; our main objective is describing these structures, which we call syntactic ergo-structures.
- Such a structure is a combinatorial object X, a kind of a network made of finitely many "atomic units = ergo-ideas". This network structure generalizes/refines that of a graph, where some patterns are similar to those found in the mathematical theory of n-categories.
- The combinatorial structure is intertwined with a geometric one; both an individual network X and the totality of these, say X, carry natural distance-like geometries which are derived from combinatorics and, at the same time, are used for constructing combinatorial relations.
- The learning process is modeled by some transformation(s) L in the space X; the resulting "educated learner" appears as an attractive fixed point x (or a class of points) under this transformation(s).
- The (ergo)learning process (transformation) L starts from a space of signals and eventually compresses (folds) the information they carry by some coclustering algorithms into our x.

Vision as a model ergosystem

- The input of the visual system amounts, roughly, to arrays of pixels changing over time.
- How does brain segment groups of pixels into objects, and learn that these objects are invariant under O(3) transformations?

"The mathematics of building/identifying the O(3)-symmetry of the visual perception field is similar to but more complicated than how Alfred Sturtevant reconstructed in 1913 linearity of the gene arrangements on the basis of distributions of phenotype linkages long before the advent of the molecular biology and discovery of DNA."

Organization: Five chapters

Ch1: free-form speculation about why ergosystem should exist & what its nature should be

Ch2: mathematical promenade

Ch3: unwritten

Ch4: language as ergosystem

Ch5: postmodern AI-inspired short story