Spectral Triples and Supersymmetry

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Reference:

- Wim Beenakker, Thijs van den Broek, Walter van Suijlekom, *Supersymmetry and Noncommutative Geometry*, Springer, 2015

(also all images in this lecture slides are from this book)

Minimally Supersymmetric Standard Model

Superfield		Spin 0	Spin $\frac{1}{2}$	Spin 1	Representation
Left-handed (s)quark	Q_L	$\tilde{q}_L$	q_L	–	$(1/6,2,3)$
Up-type (s)quark	U_R	$\tilde{u}_R$	u_R	–	$(2/3,1,3)$
Down-type (s)quark	D_R	$\tilde{d}_R$	d_R	–	$(-1/3,1,3)$
Left-handed (s)lepton	L_L	$\tilde{l}_L$	l_L	–	$(-1/2,2,1)$
Up-type (s)lepton	N_R	$\tilde{\nu}_R$	ν_R	–	$(0,1,1)$
Down-type (s)lepton	E_R	$\tilde{e}_R$	e_R	–	$(-1,1,1)$
Gluon, gluino	V	–	g	g_μ	$(0,1,8)$
$SU(2)$ gauge bosons, gauginos	W	–	λ	$\mathbf{W}_\mu$	$(0,3,1)$
B-boson, bino	B	–	λ_0	B_μ	$(0,1,1)$
Up-type Higgs(ino)	H_u	h_u	$\tilde{h}_u$	–	$(1/2,2,1)$
Down-type Higgs(ino)	H_d	h_d	$\tilde{h}_d$	–	$(-1/2,2,1)$

Numbers: $U(1)$-hypercharge; dim of $SU(2)$-representation (1=singlet, 2=fundamental, 3=adjoint); dim of $SU(3)$-representation (1,3,8)

R-parity

$$R = (-1)^{2S+3B+L}$$

S= spin; B= baryon number; L= lepton number

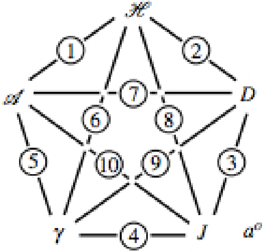
All SM particles (and the additional Higgs doublet) has $R = +1$; all superpartners have $R = -1$

Advantages of MSSM

- Higgs mass more stable under loop-corrections (contributions from superpartners compensate)
- R-symmetry preserved: lightest $R = -1$ particle cannot decay: cold dark matter?
- RGE for coupling constant form much smaller triangle: unification?

Real even spectral triples

1. $\mathscr{A} : \mathscr{H} \to \mathscr{H}$
2. $D^* = D$, comp. res.
3. $JD = \varepsilon' DJ$
4. $J\gamma = \varepsilon'' \gamma J$
5. $[\gamma, \mathscr{A}] = 0$



6. $\gamma^* = \gamma, \gamma^2 = \mathrm{id}_{\mathscr{H}}$
7. $[a, D] \in \mathscr{B}(\mathscr{H})$
8. $J^2 = \varepsilon\, \mathrm{id}_{\mathscr{H}}$
9. $\gamma D = -D\gamma$
10. $a^o = Ja^*J^*, [a, b^o] = 0$

All previously discussed relations between the elements

$$(\mathcal{A}, \mathcal{H}, D, J, \gamma)$$

of a real even spectral triple

SUSY: R-extended real even spectral triples

- additional $\mathbb{Z}/2\mathbb{Z}$-grading $R : \mathcal{H} \to \mathcal{H}$

$$\mathcal{H} = \mathcal{H}_{R=+1} \oplus \mathcal{H}_{R=-1}$$

  projectors $(1 \pm R)/2$

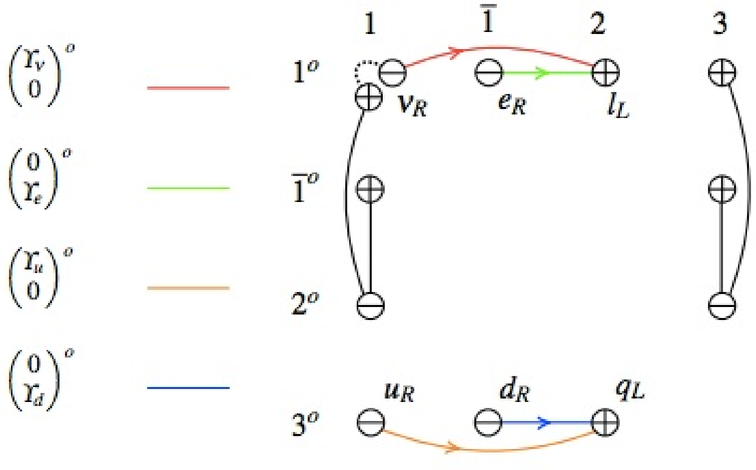
- compatibility: $\forall a \in \mathcal{A}$

$$[R, \gamma] = [R, J] = [R, \pi(a)] = 0$$

Dirac splits: $D = D_+ + D_-$ with $\{D_-, R\} = [D_+, R] = 0$

Krajewski diagrams: color vertices black/white for $R = \mp 1$

Reminder: the finite spectral triple of the νMSM



Dirac operator components: Yukawa matrices Y and (dotted line) Majorana mass matrix

SUSY finite spectral triples: building blocks

- Adjoint representations
- Non-adjoint representations
- Extra interactions
- Majorana terms
- Other mass terms
- Interactions between blocks

Adjoint representation

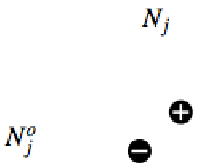
- case of a single component $\mathcal{A} = M_{N_j}(\mathbb{C})$
- bimodule $\mathbb{C}^{N_j} \otimes \mathbb{C}^{N_j \, o} \simeq M_{N_j}(\mathbb{C})$, notation: $\mathbf{N}_j \otimes \mathbf{N}_j^o$
- KO-dim 6: $\{J, \gamma\} = 0$ (compatible with SM part): two copies with J exchanging them (and takes adjoint)

$$J(m, n) = (n^*, m^*)$$

so grading γ with opposite values on the two copies

- $\mathcal{B}_j = M_{N_j,L}(\mathbb{C}) \oplus M_{N_j,R}(\mathbb{C})$ plus other stuff in $End(\mathcal{H}_F)$ (adj rep)
- R-parity: $R|_{M_{N_j}(\mathbb{C})} = -1$

These building blocks give a supersymmetric Yang–Mills action



- matching degrees of freedom for gauginos and gauge bosons
  - discard trace part of the fermion
  - add a non-propagating auxiliary field G_j (fixes the mismatch of degrees of freedom for off-shell theory)
  - on-shell agrees with the spectral action result

Action for this type of block

$$(\lambda'_{j,L}, \lambda'_{j,R}) \in L^2(\mathbb{S}_+ \otimes M_{N_j,L}(\mathbb{C})) \otimes L^2(\mathbb{S}_- \otimes M_{N_j,R}(\mathbb{C}))$$

$$S(\lambda, \mathbb{A}) = \langle J_M \lambda'_{j,R}, \partial\!\!\!/_{\mathbb{A}} \lambda'_{j,L} \rangle - \frac{f(0)}{24\pi^2} \int_M \mathrm{Tr}(F^j_{\mu\nu} F^{j,\mu\nu}) + \mathcal{O}(\Lambda^{-2})$$

• with additional term $-\frac{1}{2} \int_M \mathrm{Tr}\, G_j^2$

• scaled to have normalized kinetic term $\lambda_j \mapsto \frac{1}{\sqrt{n_j}} \lambda_j$ with $n_j \delta_{ab} = \mathrm{Tr}\, T_j^a T_j^b$

• with explicit transformation δA_j, $\delta \lambda_{j,L/R}$, δG_j under which action is supersymmetric

• block only with multiplicity one, otherwise breaking supersymmetry (same number of fermionic and bosonic degrees of freedom)

Non-adjoint representations non-gaugino fermions

• off-diagonal blocks $\mathbf{N}_i \otimes \mathbf{N}_j^o$

• with opposite values of γ-grading so KO-dim 6

• to get supersymmetry: need bosonic scalar superpartner, have interaction with gauge fields so need also gaugino degrees of freedom

• can achieve by combining a block $\mathbf{N}_i \otimes \mathbf{N}_j^o$ like this with two blocks $\mathcal{B}_i$ and $\mathcal{B}_j$ as before

$$\mathbf{N}_i \otimes \mathbf{N}_j^o \oplus M_{N_i,L}(\mathbb{C}) \oplus M_{N_i,R}(\mathbb{C}) \oplus M_{N_j,L}(\mathbb{C}) \oplus M_{N_j,R}(\mathbb{C}) \oplus \mathbf{N}_j \otimes \mathbf{N}_i^o$$

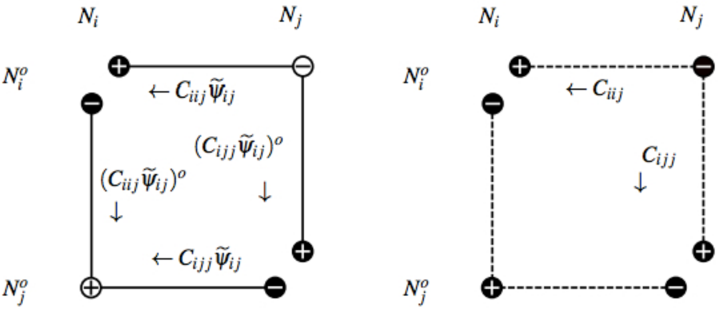
General form of Dirac operator D with $D^* = D$, $\{D, \gamma\} = 0$ and $[D, J] = 0$:

$$D_F = \begin{pmatrix} 0 & 0 & A & 0 & B & 0 \\ 0 & 0 & M_i & 0 & 0 & JA^*J^* \\ A^* & M_i^* & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & M_j & JB^*J^* \\ B^* & 0 & 0 & M_j^* & 0 & 0 \\ 0 & JAJ^* & 0 & JBJ^* & 0 & 0 \end{pmatrix}$$

with $A : M_{N_i}(\mathbb{C})_R \to \mathbf{N}_i \otimes \mathbf{N}_j^o$ *and* $B : M_{N_j}(\mathbb{C})_R \to \mathbf{N}_i \otimes \mathbf{N}_j^o$.

M_i, M_j supersymmetry breaking gaugino masses: set $M_i = M_j = 0$

Observation: if A and B differ by a single complex scalar $\Rightarrow$ scalar field $\tilde{\psi}_{ij}$ (in same rep as fermion with $R = -1$: sfermion)



(a) The case of an off-diagonal representation with $R = 1$.

(b) The case of an off-diagonal representation with $R = -1$.

Action for this second type of block

$$S_{ij}(\lambda'_i, \lambda'_j, \psi_L, \bar\psi_R, \mathbb{A}_i, \mathbb{A}_j, \tilde\psi, \bar{\tilde\psi})$$

Fermionic part

$$= \langle J_M \overline{\Psi}_R, D_A \Psi_L \rangle + \langle J_M \overline{\Psi}_R, \gamma^5 \lambda'_{iR} C_{iij} \widetilde{\Psi} \rangle + \langle J_M \overline{\Psi}_R, \gamma^5 C_{ijj} \widetilde{\Psi} \lambda'_{jR} \rangle$$
$$+ \langle J_M \Psi_L, \gamma^5 \overline{\widetilde{\Psi}} C^*_{iij} \lambda'_{iL} \rangle + \langle J_M \Psi_L, \gamma^5 \lambda'_{jL} \overline{\widetilde{\Psi}} C^*_{ijj} \rangle \rangle,$$

Bosonic part

$$\int_M |\mathcal{N}_{ij} D_\mu \tilde\psi|^2 + \mathcal{M}_{ij}(\tilde\psi, \bar{\tilde\psi})$$

with self-adjoint $\mathcal{N}_{ij}$ square root of
$\mathcal{N}_{ij}^2 = \frac{f(0)}{2\pi^2}(N_i C^*_{iij} C_{iij} + N_j C^*_{ijj} C_{ijj})$ and $\mathcal{M}_{ij}$ given by

$$\mathscr{M}_{ij}(\widetilde{\Psi}, \overline{\widetilde{\Psi}}) = \frac{f(0)}{2\pi^2}\left[N_i |C_{iij} \widetilde{\Psi} \overline{\widetilde{\Psi}} C^*_{iij}|^2 + N_j |\overline{\widetilde{\Psi}} C^*_{ijj} C_{ijj} \widetilde{\Psi}|^2 + 2|C_{iij}\widetilde{\Psi}|^2 |C_{ijj}\widetilde{\Psi}|^2 \right]$$

- now trace parts remain; give rise to $u(1)$ fields
- matching degrees of freedom: identify these $u(1)$ fields in pairs
- add off-shell G_j fields as before
- rescale $\tilde{\psi}_{ij}$ for normalization of kinetic terms
- explicit supersymmetry transformation of the action

More general $\mathcal{A}$

• algebra $\mathcal{A} = M_{N_i}(\mathbb{C}) \oplus M_{N_j}(\mathbb{C})$

• only certain combinations of blocks will have supersymmetry: two disjiont blocks $\mathcal{B}_i$ and $\mathcal{B}_j$ of first kind

• same for $\mathcal{A} = M_{N_i}(\mathbb{F}) \oplus M_{N_j}(\mathbb{F})$ with $\mathbb{F} = \mathbb{R}, \mathbb{C}, \mathbb{H}$

• $\mathcal{A}$ with three blocks $M_{N_{ijk}}(\mathbb{C})$

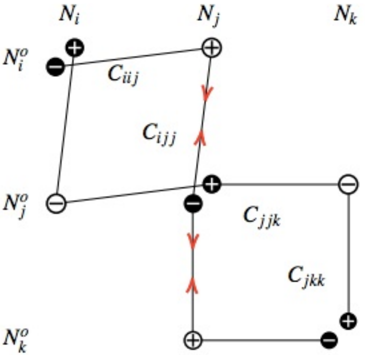
• if only building blocks of the second type not supersymmetric

• need to add third type building block $\mathcal{B}_{ijk}$ to restore supersymmetry: accounts for the components of Dirac

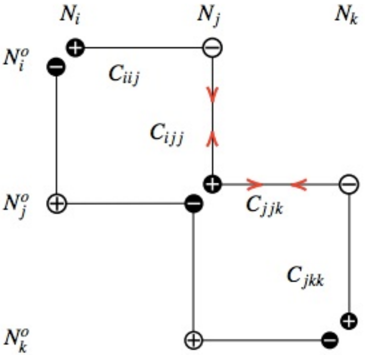
$$D_{ij}{}^{kj} : \mathbf{N}_k \otimes \mathbf{N}_j^o \to \mathbf{N}_i \otimes \mathbf{N}_j^o$$

and similar $D_{jk}{}^{ik}$ and $D_{ij}{}^{ik}$

• these $D_{ij}{}^{kj}$ generate by inner fluctuations scalar field $\tilde{\psi}_{ik}$ need corresponding fermions to restore supersymmetry: provided by $\mathcal{B}_{ijk}$

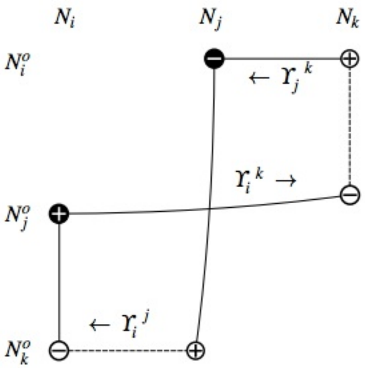


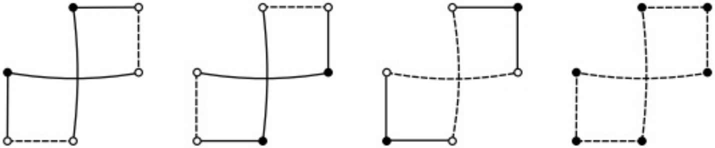
(a) Contributions when the gradings of the building blocks are different.



(b) Contributions when the gradings of the building blocks are the same.

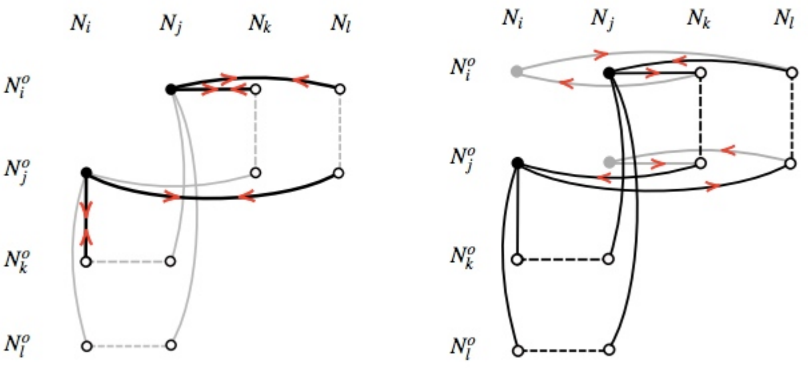
Krajewski diagrams for two blocks of second type





Krajewski diagrams for a block of third type and possible R-parity

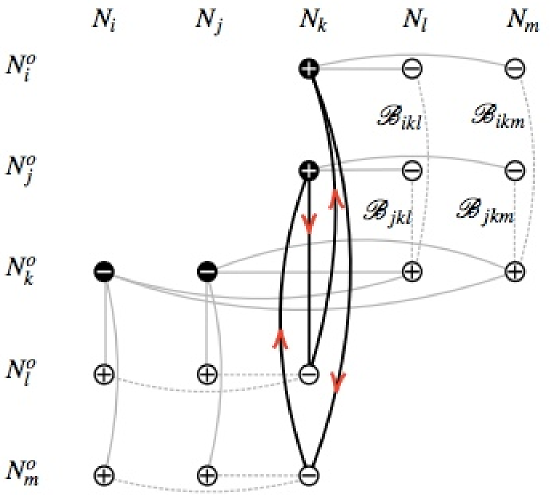
Blocks of third type and second type $\mathcal{B}_{ijk}, \mathcal{B}_{ijl}, \mathcal{B}_{ik}, \mathcal{B}_{il}$



(a) Contributions corresponding to paths of which all four edges are from the building blocks $\mathscr{B}_{ijk}$ and $\mathscr{B}_{ijl}$ of the third type.

(b) Contributions corresponding to paths of which two edges are from building blocks $\mathscr{B}_{ik}$ and $\mathscr{B}_{il}$ of the second type.

Blocks $\mathcal{B}_{ikl}, \mathcal{B}_{ikm}, \mathcal{B}_{jkl}, \mathcal{B}_{jkm}$ and (for supersymmetry) $\mathcal{B}_{ij}$ or $\mathcal{B}_{im}$



Note: no need for building blocks with more than three indices because data of finite spectral triples are

- components of the algebra (one index N_j) and adjoint representation $\mathbf{N}_j \otimes \mathbf{N}_j^o$
- non-adjoint representations $\mathbf{N}_i \otimes \mathbf{N}_j^o$ (two indices)
- components of Dirac operator with order-one condition ${D_{ij}}^{kj}$ (three indices)

everything else (eg multiple components) is expressible in terms of these basic building blocks

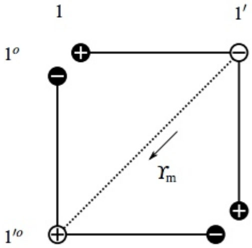
Majorana mass terms

• in the SM case: extension νMSM of the minimal standard model with right handed neutrinos and Majorana mass terms: also expect these terms in further supersymmetric extensions

• representations $\mathbf{1} \otimes \mathbf{1}'^o \oplus \mathbf{1}' \otimes \mathbf{1}^o$ that are each other's antiparticles ($(\mathbb{C} \oplus \mathbb{C})^{\oplus M}$ multiplicity M), with the same component $\mathbb{C}$ of algebra acting on both

$$D_{1'1}{}^{11'} : \mathbf{1} \otimes \mathbf{1}'^o \to \mathbf{1}' \otimes \mathbf{1}^o$$

• dotted line in Krajewski diagram:

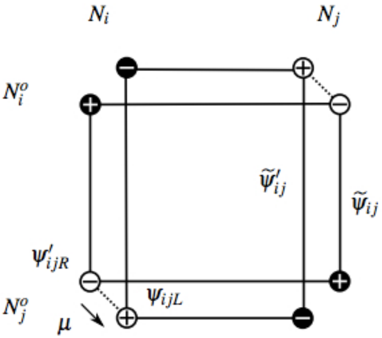


Other mass terms

• two building blocks of second type with same indices and different grading

• on this part Dirac with self-adjoint, order-one, commuting with J gives a mass term in the action if it acts nontrivially only on generations (on the M copies in

$$((\mathbf{N}_i \otimes \mathbf{N}_j^o)_L \oplus (\mathbf{N}_j \otimes \mathbf{N}_i^o)_R \oplus \mathbf{N}_i \otimes \mathbf{N}_j^o)_R \oplus (\mathbf{N}_j \otimes \mathbf{N}_i^o)_L)^{\oplus M}$$



Supersymmetry and the Spectral Action

- the construction of finite spectral triples with these building blocks ensures field content is supersymmetric
- it does *not* directly follow that the action is supersymmetric
- some problems:
  - a single building block first type $\mathcal{B}_i$ with $N_i = 1$ has vanishing bosonic interactions
  - a single building block second type $\mathcal{B}_{ij}$ with $R = -1$ has two interacting $u(1)$-fields (while gauginos don't)
  - if at least three complex matrix components in $\mathcal{A}$, and building blocks $\mathcal{B}_{ij}$, $\mathcal{B}_{ik}$ then two interacting $u(1)$-fields (while gauginos don't)
- approach: rewrite the four-scalar interactions generated by the spectral action as off-shell action via auxiliary fields
- general issue: new contributions for which one needs to obtain off-shell counterparts so that the action remains supersymmetric

Supersymmetry Breaking

• if supersymmetry were exact in nature superpartners would be of equal mass as corresponding SM particles: obviously not the case

• so supersymmetric extensions of SM need a mechanism that breaks supersymmetry

• in MSSM one should have a SUSY-breaking Higgs that gives mass to SM particles and achieves electroweak symmetry breaking

• two possibilities: spontaneous SUSY breaking (disfavored phenomenologically: would have one slepton/squark lighter than corresponding SM fermion); or SUSY breaking Lagrangian

• SUSY breaking terms in Lagrangian should be *soft*:

- couplings of positive mass dimension
- no quadratically divergent loop corrections (hierarchy problem solution of SUSY: Higgs mass very sensitive to perturbative corrections, stability problem)

Soft SUSY breaking

$$\mathcal{L}^{\mathrm{E}}_{\mathrm{soft}} = \widetilde{\psi}^*_\alpha (m^2)_{\alpha\beta} \widetilde{\psi}_\beta - \left(\frac{1}{3!} A_{\alpha\beta\gamma} \widetilde{\psi}_\alpha \widetilde{\psi}_\beta \widetilde{\psi}_\gamma - \frac{1}{2} B_{\alpha\beta} \widetilde{\psi}_\alpha \widetilde{\psi}_\beta + C_\alpha \widetilde{\psi}_\alpha + h.c. \right)$$
$$+ \frac{1}{2}(M \lambda_a \lambda_a + h.c.).$$

- self-adjoint mass term for scalar bosons $\tilde{\psi}_\alpha$
- symmetric tensor $A_{\alpha\beta\gamma}$ mass dim 1
- matrix $B_{\alpha\beta}$ mass dim 2
- gauge singlet linear coupling $C_\alpha \in \mathbb{C}$ mass dim 3
- gaugino mass terms, $M \in \mathbb{C}$

Question: soft supersymmetry breaking in the Spectral Action?

Soft SUSY breaking from Λ^2 terms in the Spectral Action

• scalar fields $\tilde{\psi}_{ij} \in \mathcal{C}^\infty(M, \mathbf{N}_i \otimes \mathbf{N}_j^o)$

• fermions $\psi_{ij} \in \mathcal{C}^\infty(M, \mathbb{S} \otimes \mathbf{N}_i \otimes \mathbf{N}_j^o)$

• gauginos $\lambda_i \in L^2(M, \mathbb{S} \otimes M_{N_i}(\mathbb{C}))$ reduced to $su(N_i)$ after eliminating trace degrees of freedom

• all possible terms arising from the Spectral Action:

- scalar masses
- gaugino masses
- linear couplings
- bilinear couplings
- trilinear couplings

Scalar masses (Higgs masses)

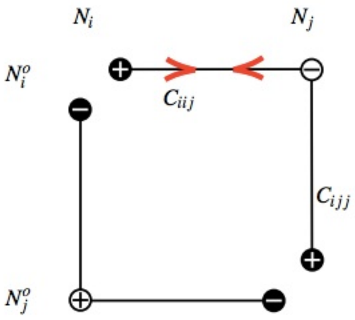


Fig. 3.1: A building block of the second type that defines a fermion–sfermion pair $(\psi_{ij}, \widetilde{\psi}_{ij})$. Contributions to the mass term of the sfermion correspond to paths going back and forth on an edge, as is depicted on the top edge.

Gaugino masses

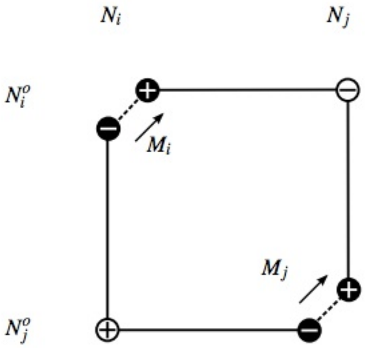


Fig. 3.2: A building block of the second type that defines a fermion–sfermion pair $(\psi_{ij}, \widetilde{\psi}_{ij})$, dressed with mass terms for the corresponding gauginos (dashed edges, labeled by $M_{i,j}$).

Linear Couplings

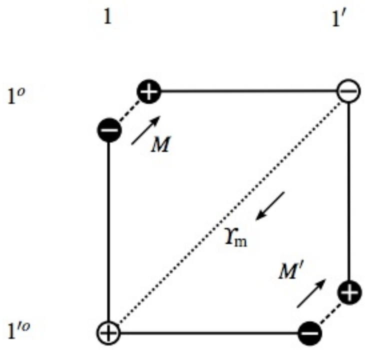
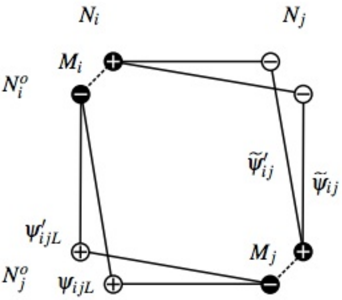
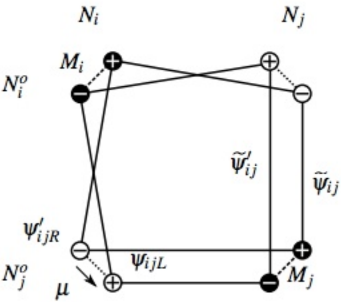


Fig. 3.3: A building block of the second type that defines a gauge singlet fermion–sfermion pair $(\psi_{\rm sin}, \widetilde{\psi}_{\rm sin})$. Moreover, a Majorana mass term Υ_m is possible.

Bilinear Couplings



(a) When the gradings of the representations are equal.



(b) When the gradings of the representations differ.

Fig. 3.4: Two building blocks of the second type defining two fermion–sfermion pairs $(\psi_{ij}, \widetilde{\psi}_{ij})$ and $(\psi'_{ij}, \widetilde{\psi}'_{ij})$ in the same representation.

Trilinear Couplings

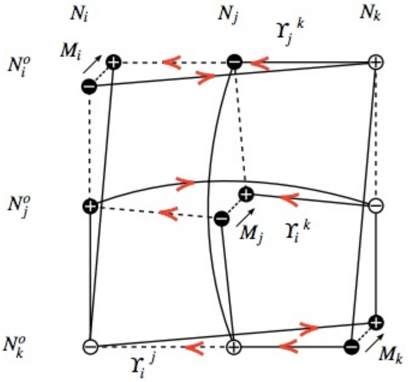


Fig. 3.5: A situation in which there are three building blocks $\mathscr{B}_{i,j,k}$ of the first type (black vertices), three building blocks $\mathscr{B}_{ij,jk,ik}$ of the second type and a building block $\mathscr{B}_{ijk}$ of the third type. Adding gaugino masses (dashed edges) gives rise to trilinear interactions, corresponding to the paths in the diagram marked by arrows.

Building the MSSM

- finite spectral triple of the MSSM:
  - should contain SM
  - gauge group same as for SM (up to finite groups)

$$\mathcal{A} = \mathcal{A}_{SM} = \mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$$

inequivalent reps $\mathbf{1}$, $\bar{\mathbf{1}}$, $\mathbf{2}$, $\mathbf{3}$ with $\bar{\mathbf{1}}$ real rep $\pi(\lambda)\nu = \bar{\lambda}\nu$

$$G = U(1) \times SU(2) \times SU(3)$$

  - superpartners $R = -1$ building blocks of first type

$$\mathcal{B}_1, \quad \mathcal{B}_{1_R}, \quad \mathcal{B}_{\bar{1}_R}, \quad \mathcal{B}_{2_L}, \quad \mathcal{B}_3$$

Note: in reducing $\mathbb{C} \oplus \mathbb{H}_L \oplus \mathbb{H}_R \oplus M_3(\mathbb{C})$ to $\mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$ the $\mathbb{C} \hookrightarrow \mathbb{H}_R$ acts as $(\lambda, \bar{\lambda})$ and $\mathbf{2}_R$ breaks to $\mathbf{1}_R \oplus \bar{\mathbf{1}}_R$

• at this point too many fermionic degrees of freedom: will need identifications

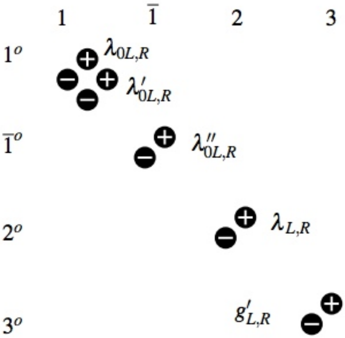
• for each SM fermion a building blocks of second type $R = +1$ (M copies for generations):

$$\mathcal{B}^-_{1_R 1} : (\nu_R, \widetilde{\nu}_R), \quad \mathcal{B}^-_{\bar{1}_R 1} : (e_R, \widetilde{e}_R), \quad \mathcal{B}^+_{2_L 1} : (l_L, \widetilde{l}_L),$$
$$\mathcal{B}^-_{1_R 3} : (u_R, \widetilde{u}_R), \quad \mathcal{B}^-_{\bar{1}_R 3} : (d_R, \widetilde{d}_R), \quad \mathcal{B}^+_{2_L 3} : (q_L, \widetilde{q}_L)$$

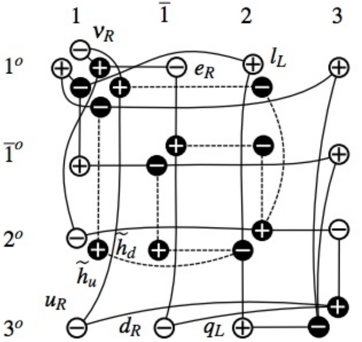
• SM Higgs: building blocks second type $R = -1$ (one copy)

$$\mathcal{B}_{1_R 2_L} : (h_u, \widetilde{h}_u), \quad \mathcal{B}_{\bar{1}_R 2_L} : (h_d, \widetilde{h}_d),$$

Higgs/higgsino building blocks



(a) Blocks of the first type.



(b) Blocks of the second type. Each white off-diagonal node corresponds to a SM (anti)particle.

• these blocks determine the finite dimensional Hilbert space of the finite spectral triple

$$\mathcal{H}_F = \mathcal{H}_{F,R=+1} \oplus \mathcal{H}_{F,R=-1}$$

$$\mathscr{H}_{F,R=+} = \left(\mathscr{E} \oplus \mathscr{E}^o\right)^{\oplus M}, \qquad \mathscr{E} = (\mathbf{2}_L \oplus \mathbf{1}_R \oplus \bar{\mathbf{1}}_R) \otimes (\mathbf{1} \oplus \mathbf{3})^o$$

$$\mathscr{H}_{F,R=-} = \mathscr{F} \oplus \mathscr{F}^o, \qquad \mathscr{F} = (\mathbf{1} \otimes \mathbf{1}^o)^{\oplus 2} \oplus \bar{\mathbf{1}} \otimes \bar{\mathbf{1}}^o \oplus \mathbf{2} \otimes \mathbf{2}^o \oplus \mathbf{3} \otimes \mathbf{3}^o \oplus (\mathbf{1}_R \oplus \bar{\mathbf{1}}_R) \otimes \mathbf{2}_L^o.$$

• correct properties to allow supersymmetric action

• element of $\mathbb{C} \oplus \mathbb{H}_L \oplus \mathbb{H}_R \oplus M_3(\mathbb{C})$

$$R = -(+,-,-,+) \otimes (+,-,-,+)^o$$

has $R = +1$ on all SM fermions and $R = -1$ on higgsino

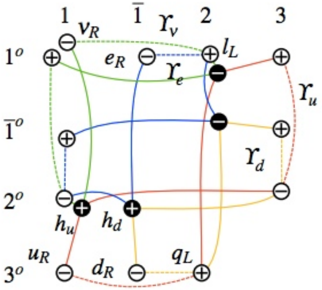
Yukawa couplings of fermions and Higgs

- building blocks of third type $\mathcal{B}_{ijk}$ should contain Higgs interaction of SM but with different up and down Higgses
- four possible building blocks

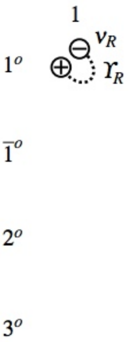
$$\mathscr{B}_{11_R2_L}, \quad \mathscr{B}_{1\bar{1}_R2_L}, \quad \mathscr{B}_{1_R2_L3}, \quad \mathscr{B}_{\bar{1}_R2_L3}.$$

Massless photon condition: now require for D_+ part $[D_+, \mathbb{C}_F] = 0$ with $\mathbb{C}_F = \{(\lambda, \bar{\lambda}, 0) \in \mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})\}$

Majorana masses for right handed neutrinos as in νMSM



(c) Blocks of the third type, parametrized by the Yukawa matrices $\Upsilon_{\nu,e,u,d}$.



(d) The block of the fourth type, representing a Majorana mass for the right-handed neutrino.

up/down Higgs/higgsino interaction in MSSM $\mu\, H_u \cdot H_d$

- need additional building blocks

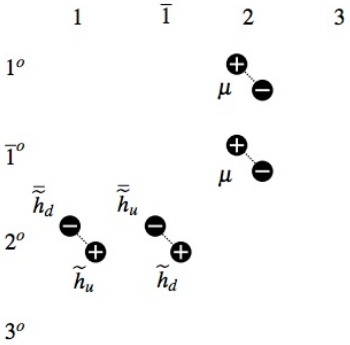


Fig. 4.2: The extra building blocks of the second type featuring a Higgs/higgsino-pair and the building blocks of the fifth type that are consequently possible.

Sparticles and Hypercharges

- gauginos in $\mathbf{1} \otimes \mathbf{1}^o$, $\mathbf{2} \otimes \mathbf{2}^o$, $\mathbf{3} \otimes \mathbf{3}^o$ with zero hypercharges
- higgsino in $\mathbf{1}_R \otimes \mathbf{2}_L^o$ and $\bar{\mathbf{1}}_R \otimes \mathbf{2}_L^o$ hypercharges $+1$ and -1
- hypercharges of sfermions

$$\widetilde{q}_L : \quad \tfrac{1}{3}, \qquad \widetilde{u}_R : \quad \tfrac{4}{3}, \qquad \widetilde{d}_R : \quad -\tfrac{2}{3},$$
$$\widetilde{l}_L : \quad -1, \qquad \widetilde{\nu}_R : \quad 0, \qquad \widetilde{e}_R : \quad -2.$$

Correct identifications with sparticles of MSSM

• unimodularity (eliminate trace modes) for correct matching of fermion/boson degrees of freedom

Spectral Action of MSSM

- can derive a set of relations between parameters for supersymmetry of the action to hold
- Problem: they cannot be satisfied for $M \in \mathbb{N}$ (generations)
- Conclusion the spectral action

$$S = \mathrm{Tr}(f(D_A/\Lambda)) + \frac{1}{2}\langle J\xi, D_A\xi\rangle$$

on an almost-commutative spectral triple $M \times F$, with F the finite spectral triple of the MSSM is itself *not* supersymmetric

- *then what?*
  - further extensions (beyond MSSM) may add more blocks and correct the problem?
  - is this the only way to approach spectral triples with supersymmetry?

General comment: in the history of this field, beware of any *no-go* result claiming that something cannot be done: it usually only means "it cannot be done *in this way*"