The Spectral Action and the Standard Model

Matilde Marcolli

M148b: Topics in Mathematical Physics, Caltech Winter 2021

References

- A.H. Chamseddine, A. Connes, M. Marcolli, *Gravity and the Standard Model with Neutrino Mixing*, Adv. Theor. Math. Phys., Vol.11 (2007) 991–1090

The spectral action functional

- Ali Chamseddine, Alain Connes, *The spectral action principle*, Comm. Math. Phys. 186 (1997), no. 3, 731–750.

A good action functional for noncommutative geometries

$$\mathrm{Tr}(f(D/\Lambda))$$

D Dirac, Λ mass scale, $f > 0$ even smooth function (cutoff approx)
Simple dimension spectrum $\Rightarrow$ expansion for $\Lambda \to \infty$

$$\mathrm{Tr}(f(D/\Lambda)) \sim \sum_k f_k \Lambda^k \int\!\!\!\!- |D|^{-k} + f(0)\,\zeta_D(0) + o(1),$$

with $f_k = \int_0^\infty f(v)\, v^{k-1}\, dv$ momenta of f
where $\mathrm{DimSp}(\mathcal{A}, \mathcal{H}, D) =$ poles of $\zeta_{b,D}(s) = \mathrm{Tr}(b|D|^{-s})$

Asymptotic expansion of the spectral action

$$\mathrm{Tr}(e^{-t\Delta}) \sim \sum a_\alpha \, t^\alpha \qquad (t \to 0)$$

and the ζ function

$$\zeta_D(s) = \mathrm{Tr}(\Delta^{-s/2})$$

- Non-zero term a_α with $\alpha < 0 \Rightarrow$ *pole* of ζ_D at -2α with

$$\mathrm{Res}_{s=-2\alpha} \, \zeta_D(s) = \frac{2\, a_\alpha}{\Gamma(-\alpha)}$$

- No $\log t$ terms $\Rightarrow$ regularity at 0 for ζ_D with $\zeta_D(0) = a_0$

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- Get first statement from

$$|D|^{-s} = \Delta^{-s/2} = \frac{1}{\Gamma\left(\frac{s}{2}\right)} \int_0^\infty e^{-t\Delta}\, t^{s/2-1}\, dt$$

with $\int_0^1 t^{\alpha+s/2-1}\, dt = (\alpha + s/2)^{-1}$.

- Second statement from

$$\frac{1}{\Gamma\left(\frac{s}{2}\right)} \sim \frac{s}{2} \qquad \text{as} \quad s \to 0$$

contrib to $\zeta_D(0)$ from pole part at $s = 0$ of

$$\int_0^\infty \mathrm{Tr}(e^{-t\Delta})\, t^{s/2-1}\, dt$$

given by $a_0 \int_0^1 t^{s/2-1}\, dt = a_0 \frac{2}{s}$

Spectral action with fermionic terms

$$S = \mathrm{Tr}(f(D_A/\Lambda)) + \frac{1}{2}\langle J\tilde{\xi}, D_A\tilde{\xi}\rangle, \quad \tilde{\xi}\in \mathcal{H}_{cl}^{+},$$

D_A = Dirac with unimodular inner fluctuations, J = real structure, $\mathcal{H}_{cl}^{+}$ = classical spinors, Grassmann variables

Fermionic terms

$$\frac{1}{2}\langle J\tilde{\xi}, D_A\tilde{\xi}\rangle$$

antisymmetric bilinear form $\mathfrak{A}(\tilde{\xi})$ on

$$\mathcal{H}_{cl}^{+} = \{\xi \in \mathcal{H}_{cl} \,|\, \gamma\xi = \xi\}$$

$\Rightarrow$ nonzero on Grassmann variables

Euclidean functional integral $\Rightarrow$ Pfaffian

$$Pf(\mathfrak{A}) = \int e^{-\frac{1}{2}\mathfrak{A}(\tilde{\xi})} D[\tilde{\xi}]$$

avoids Fermion doubling problem of previous models based on symmetric $\langle \xi, D_A\xi\rangle$ for NC space with KO-dim=0

Grassmann variables

Anticommuting variables with basic integration rule

$$\int \xi \, d\xi = 1$$

An antisymmetric bilinear form $\mathfrak{A}(\xi_1, \xi_2)$: if ordinary commuting variables $\mathfrak{A}(\xi, \xi) = 0$ but not on Grassmann variables

Example: 2-dim case $\mathfrak{A}(\xi', \xi) = a(\xi'_1 \xi_2 - \xi'_2 \xi_1)$, if ξ_1 and ξ_2 anticommute, with integration rule as above

$$\int e^{-\frac{1}{2}\mathfrak{A}(\xi,\xi)} D[\xi] = \int e^{-a\xi_1\xi_2} d\xi_1 d\xi_2 = a$$

Pfaffian as functional integral: antisymmetric quadratic form

$$Pf(\mathfrak{A}) = \int e^{-\frac{1}{2}\mathfrak{A}(\xi,\xi)} \, D[\xi]$$

Method to treat Majorana fermions in the Euclidean setting

Fermionic part of SM Lagrangian

Explicit computation of

$$\frac{1}{2}\langle J\tilde{\xi}, D_A\tilde{\xi}\rangle$$

gives part of SM Larangian with

- $\mathcal{L}_{Hf}$ = coupling of Higgs to fermions
- $\mathcal{L}_{gf}$ = coupling of gauge bosons to fermions
- $\mathcal{L}_{f}$ = fermion terms

Bosonic part of the spectral action

$$
\begin{aligned}
S = \;& \frac{1}{\pi^2}(48\, f_4\, \Lambda^4 - f_2\, \Lambda^2\, \mathfrak{c} + \frac{f_0}{4}\, \mathfrak{d}) \int \sqrt{g}\, d^4x \\
+ \;& \frac{96\, f_2\, \Lambda^2 - f_0\, \mathfrak{c}}{24\pi^2} \int R \sqrt{g}\, d^4x \\
+ \;& \frac{f_0}{10\,\pi^2} \int (\frac{11}{6}\, R^* R^* - 3\, C_{\mu\nu\rho\sigma}\, C^{\mu\nu\rho\sigma}) \sqrt{g}\, d^4x \\
+ \;& \frac{(-2\, \mathfrak{a}\, f_2\, \Lambda^2 + \mathfrak{e}\, f_0)}{\pi^2} \int |\varphi|^2 \sqrt{g}\, d^4x \\
+ \;& \frac{f_0 \mathfrak{a}}{2\,\pi^2} \int |D_\mu \varphi|^2 \sqrt{g}\, d^4x \\
- \;& \frac{f_0 \mathfrak{a}}{12\,\pi^2} \int R\, |\varphi|^2 \sqrt{g}\, d^4x \\
+ \;& \frac{f_0 \mathfrak{b}}{2\,\pi^2} \int |\varphi|^4 \sqrt{g}\, d^4x \\
+ \;& \frac{f_0}{2\,\pi^2} \int (g_3^2\, G^i_{\mu\nu}\, G^{\mu\nu i} + g_2^2\, F^\alpha_{\mu\nu}\, F^{\mu\nu\alpha} + \frac{5}{3}\, g_1^2\, B_{\mu\nu}\, B^{\mu\nu}) \sqrt{g}\, d^4x,
\end{aligned}
$$

Parameters:

- f_0, f_2, f_4 free parameters, $f_0 = f(0)$ and, for $k > 0$,

$$f_k = \int_0^\infty f(v) v^{k-1} dv.$$

- $\mathfrak{a}, \mathfrak{b}, \mathfrak{c}, \mathfrak{d}, \mathfrak{e}$ functions of Yukawa parameters of νMSM

$$\begin{aligned}
\mathfrak{a} = &\quad \mathrm{Tr}(Y_\nu^\dagger Y_\nu + Y_e^\dagger Y_e + 3(Y_u^\dagger Y_u + Y_d^\dagger Y_d)) \\
\mathfrak{b} = &\quad \mathrm{Tr}((Y_\nu^\dagger Y_\nu)^2 + (Y_e^\dagger Y_e)^2 + 3(Y_u^\dagger Y_u)^2 + 3(Y_d^\dagger Y_d)^2) \\
\mathfrak{c} = &\quad \mathrm{Tr}(MM^\dagger) \\
\mathfrak{d} = &\quad \mathrm{Tr}((MM^\dagger)^2) \\
\mathfrak{e} = &\quad \mathrm{Tr}(MM^\dagger Y_\nu^\dagger Y_\nu).
\end{aligned}$$

Gilkey's theorem using $D_A^2 = \nabla^* \nabla - E$

Differential operator $P = -\left(g^{\mu\nu} I\, \partial_\mu \partial_\nu + A^\mu \partial_\mu + B\right)$ with A, B bundle endomorphisms, $m = \dim M$

$$\mathrm{Tr}\, e^{-tP} \sim \sum_{n \geq 0} t^{\frac{n-m}{2}} \int_M a_n(x, P)\, dv(x)$$

$P = \nabla^* \nabla - E$ and $E_{;\mu}{}^{\mu} := \nabla_\mu \nabla^\mu E$

$$\nabla_\mu = \partial_\mu + \omega'_\mu, \qquad \omega'_\mu = \frac{1}{2} g_{\mu\nu} (A^\nu + \Gamma^\nu \cdot \mathrm{id})$$

$$E = B - g^{\mu\nu} (\partial_\mu \omega'_\nu + \omega'_\mu \omega'_\nu - \Gamma^\rho_{\mu\nu} \omega'_\rho)$$

$$\Omega_{\mu\nu} = \partial_\mu \omega'_\nu - \partial_\nu \omega'_\mu + [\omega'_\mu, \omega'_\nu]$$

Seeley-DeWitt coefficients

$$\begin{aligned}
a_0(x,P) &= (4\pi)^{-m/2} \mathrm{Tr}(\mathrm{id}) \\
a_2(x,P) &= (4\pi)^{-m/2} \mathrm{Tr}\left(-\tfrac{R}{6}\, \mathrm{id} + E\right) \\
a_4(x,P) &= (4\pi)^{-m/2} \tfrac{1}{360} \mathrm{Tr}(-12 R_{;\mu}{}^{\mu} + 5 R^2 - 2 R_{\mu\nu} R^{\mu\nu} \\
&+ 2 R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 60\, R\, E + 180\, E^2 + 60\, E_{;\mu}{}^{\mu} \\
&+ 30\, \Omega_{\mu\nu} \Omega^{\mu\nu})
\end{aligned}$$

Normalization and coefficients

- Rescale Higgs field $H = \frac{\sqrt{\mathfrak{a}\, f_0}}{\pi}\varphi$ to normalize kinetic term $\int \frac{1}{2}|D_\mu \mathbf{H}|^2 \sqrt{g}\, d^4x$
- Normalize Yang-Mills terms $\frac{1}{4} G^i_{\mu\nu} \overline{G}^{\mu\nu i} + \frac{1}{4} F^\alpha_{\mu\nu} \overline{F}^{\mu\nu\alpha} + \frac{1}{4} B_{\mu\nu} \overline{B}^{\mu\nu}$

Normalized form:

$$
\begin{aligned}
S = \; & \frac{1}{2\kappa_0^2} \int R \sqrt{g}\, d^4x + \gamma_0 \int \sqrt{g}\, d^4x \\
+ \; & \alpha_0 \int C_{\mu\nu\rho\sigma}\, C^{\mu\nu\rho\sigma} \sqrt{g}\, d^4x + \tau_0 \int R^* R^* \sqrt{g}\, d^4x \\
+ \; & \frac{1}{2} \int |DH|^2 \sqrt{g}\, d^4x - \mu_0^2 \int |H|^2 \sqrt{g}\, d^4x \\
- \; & \xi_0 \int R\, |H|^2 \sqrt{g}\, d^4x + \lambda_0 \int |H|^4 \sqrt{g}\, d^4x \\
+ \; & \frac{1}{4} \int (G^i_{\mu\nu}\, G^{\mu\nu i} + F^\alpha_{\mu\nu}\, F^{\mu\nu\alpha} + B_{\mu\nu}\, B^{\mu\nu}) \sqrt{g}\, d^4x
\end{aligned}
$$

where $R^* R^* = \frac{1}{4}\epsilon^{\mu\nu\rho\sigma}\epsilon_{\alpha\beta\gamma\delta} R^{\alpha\beta}_{\mu\nu} R^{\gamma\delta}_{\rho\sigma}$ integrates to the Euler characteristic $\chi(M)$ and $C^{\mu\nu\rho\sigma}$ Weyl curvature

Coefficients

$$\frac{1}{2\kappa_0^2} = \frac{96 f_2 \Lambda^2 - f_0 \mathfrak{c}}{24\pi^2} \qquad \gamma_0 = \frac{1}{\pi^2}\left(48 f_4 \Lambda^4 - f_2 \Lambda^2 \mathfrak{c} + \frac{f_0}{4}\mathfrak{d}\right)$$

$$\alpha_0 = -\frac{3 f_0}{10\pi^2} \qquad \tau_0 = \frac{11 f_0}{60\pi^2}$$

$$\mu_0^2 = 2\frac{f_2 \Lambda^2}{f_0} - \frac{\mathfrak{e}}{\mathfrak{a}} \qquad \xi_0 = \frac{1}{12}$$

$$\lambda_0 = \frac{\pi^2 \mathfrak{b}}{2 f_0 \mathfrak{a}^2}$$

Energy scale: Unification (10^{15} – 10^{17} GeV)

$$\frac{g^2 f_0}{2\pi^2} = \frac{1}{4}$$

Preferred energy scale, unification of coupling constants

Renormalization Group Equations (RGE) in the NCG Standard Model

- in QFT equations of the Renormalization Group Flow

$$\partial_t x_i(t) = \beta_{x_i}(x(t)), \quad t = \log(\Lambda/M_Z)$$

  how parameters (masses, coupling constants, etc) vary with energy scale
- the *beta function* β_{x_i} has contributions from Feynman diagrams: one-loop beta function etc.
- the NCG Standard Model described so far as a classical not a quantum field theory: as a quantum theory the quantum field should be Dirac operator

- initially RGE were "imported" in the NCG Standard Model from the usual QFT of particle physics, not computed from within the model... not satisfactory
- more recently shown that a matrix model based on the Dirac operator of the finite spectral triple of the NCG Standard Model exactly computes the RGE beta function of the Yukawa parameters of the particle physics Standard Model without the use of QFT Feynman diagrams
  - E. Gesteau, "Renormalizing Yukawa interactions in the standard model with matrices and noncommutative geometry", J. Phys A Math. Theor. 54 (2021) 035203

1-loop RGE equations for νMSN

$$\partial_t x_i(t) = \beta_{x_i}(x(t))$$

variable $t = \log(\Lambda/M_Z)$

- Coupling constants:

$$\beta_1 = \frac{41}{96\pi^2} g_1^3, \qquad \beta_2 = -\frac{19}{96\pi^2} g_2^3, \qquad \beta_3 = -\frac{7}{16\pi^2} g_3^3$$

at 1-loop decoupled from other equations (Notation: $\tilde{g}_1^2 = \frac{5}{3} g_1^2$)

- Yukawa parameters:

$$16\pi^2\ \beta_{Y_u} = Y_u\left(\frac{3}{2} Y_u^\dagger Y_u - \frac{3}{2} Y_d^\dagger Y_d + \mathfrak{a} - \frac{17}{20}\tilde{g}_1^2 - \frac{9}{4} g_2^2 - 8 g_3^2\right)$$

$$16\pi^2\ \beta_{Y_d} = Y_d\left(\frac{3}{2} Y_d^\dagger Y_d - \frac{3}{2} Y_u^\dagger Y_u + \mathfrak{a} - \frac{1}{4}\tilde{g}_1^2 - \frac{9}{4} g_2^2 - 8 g_3^2\right)$$

$$16\pi^2\ \beta_{Y_\nu} = Y_\nu\left(\frac{3}{2} Y_\nu^\dagger Y_\nu - \frac{3}{2} Y_e^\dagger Y_e + \mathfrak{a} - \frac{9}{20}\tilde{g}_1^2 - \frac{9}{4} g_2^2\right)$$

$$16\pi^2\ \beta_{Y_e} = Y_e\left(\frac{3}{2} Y_e^\dagger Y_e - \frac{3}{2} Y_\nu^\dagger Y_\nu + \mathfrak{a} - \frac{9}{4}\tilde{g}_1^2 - \frac{9}{4} g_2^2\right)$$

- Majorana mass terms:

$$16\pi^2\ \beta_M = Y_\nu Y_\nu^\dagger M + M(Y_\nu Y_\nu^\dagger)^T$$

- Higgs self coupling:

$$16\pi^2\ \beta_\lambda = 6\lambda^2 - 3\lambda(3g_2^2 + g_1^2) + 3g_2^4 + \frac{3}{2}(g_1^2 + g_2^2)^2 + 4\lambda\mathfrak{a} - 8\mathfrak{b}$$

- MSM approximation: top quark Yukawa parameter dominant term: in λ running neglect all terms except coupling constants g_i and Yukawa parameter of top quark y_t

$$\beta_\lambda = \frac{1}{16\pi^2}\left(24\lambda^2 + 12\lambda y^2 - 9\lambda(g_2^2 + \frac{1}{3}g_1^2) - 6y^4 + \frac{9}{8}g_2^4 + \frac{3}{8}g_1^4 + \frac{3}{4}g_2^2 g_1^2\right)$$

where Yukawa parameter for the top quark runs by

$$\beta_y = \frac{1}{16\pi^2}\left(\frac{9}{2}y^3 - 8g_3^2 y - \frac{9}{4}g_2^2 y - \frac{17}{12}g_1^2 y\right)$$

Renormalization group flow

- The coefficients $\mathfrak{a}, \mathfrak{b}, \mathfrak{c}, \mathfrak{d}, \mathfrak{e}$ (depend on Yukawa parameters) run with the RGE flow
- Initial conditions at unification energy: compatibility with physics at low energies

RGE in the MSM case

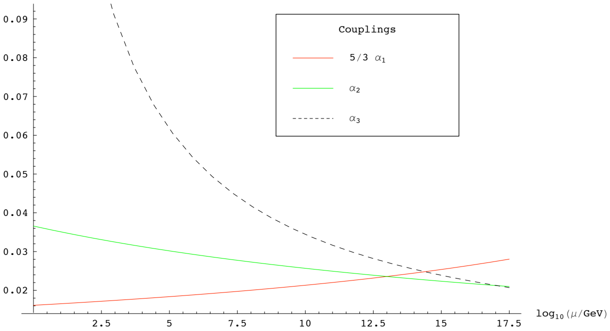
Running of coupling constants at one loop: $\alpha_i = g_i^2/(4\pi)$

$$\beta_{g_i} = (4\pi)^{-2}\, b_i\, g_i^3, \qquad \text{with} \quad b_i = (\frac{41}{6}, -\frac{19}{6}, -7),$$

$$\begin{aligned}
\alpha_1^{-1}(\Lambda) &= \alpha_1^{-1}(M_Z) - \frac{41}{12\pi} \log \frac{\Lambda}{M_Z} \\
\alpha_2^{-1}(\Lambda) &= \alpha_2^{-1}(M_Z) + \frac{19}{12\pi} \log \frac{\Lambda}{M_Z} \\
\alpha_3^{-1}(\Lambda) &= \alpha_3^{-1}(M_Z) + \frac{42}{12\pi} \log \frac{\Lambda}{M_Z}
\end{aligned}$$

$M_Z \sim 91.188$ GeV mass of Z^0 boson

At one loop RGE for coupling constants decouples from Yukawa parameters (not at 2 loops!)



Well known triangle problem: with known low energy values constants don't meet at unification $g_3^2 = g_2^2 = 5g_1^2/3$

Geometry point of view

- At one loop coupling constants decouple from Yukawa parameters
- Solving for coupling constants, RGE flow defines a vector field on moduli space $\mathcal{C}_3 \times \mathcal{C}_1$ of Dirac operators on the finite NC space F
- Subvarieties invariant under flow are relations between the SM parameters that hold at all energies
- At two loops or higher, RGE flow on a rank three vector bundle (fiber $=$ coupling constants) over the moduli space $\mathcal{C}_3 \times \mathcal{C}_1$
- Geometric problem: studying the flow and the geometry of invariant subvarieties on the moduli space

Constraints at unification

The geometry of the model imposes conditions at unification energy: specific to this NCG model

- λ parameter constraint

$$\lambda(\Lambda_{unif}) = \frac{\pi^2}{2f_0} \frac{\mathfrak{b}(\Lambda_{unif})}{\mathfrak{a}(\Lambda_{unif})^2}$$

- Higgs vacuum constraint

$$\frac{\sqrt{\mathfrak{a} f_0}}{\pi} = \frac{2 M_W}{g}$$

- See-saw mechanism and $\mathfrak{c}$ constraint

$$\frac{2 f_2 \Lambda_{unif}^2}{f_0} \leq \mathfrak{c}(\Lambda_{unif}) \leq \frac{6 f_2 \Lambda_{unif}^2}{f_0}$$

- Mass relation at unification

$$\sum_{generations} (m_\nu^2 + m_e^2 + 3m_u^2 + 3m_d^2)|_{\Lambda=\Lambda_{unif}} = 8 M_W^2|_{\Lambda=\Lambda_{unif}}$$

Need to have compatibility with low energy behavior

Mass relation at unification $Y_2(S) = 4g^2$

$$Y_2 = \sum_\sigma (y_\nu^\sigma)^2 + (y_e^\sigma)^2 + 3\,(y_u^\sigma)^2 + 3\,(y_d^\sigma)^2$$

$$\begin{array}{rcl}
(k_{(\uparrow 3)})_{\sigma\kappa} & = & \frac{g}{2M}\, m_u^\sigma\, \delta_\sigma^\kappa \\
(k_{(\downarrow 3)})_{\sigma\kappa} & = & \frac{g}{2M}\, m_d^\mu\, C_{\sigma\mu}\delta_\mu^\rho C_{\rho\kappa}^\dagger \\
(k_{(\uparrow 1)})_{\sigma\kappa} & = & \frac{g}{2M}\, m_\nu^\sigma\, \delta_\sigma^\kappa \\
(k_{(\downarrow 1)})_{\sigma\kappa} & = & \frac{g}{2M}\, m_e^\mu\, {U^{lep}}_{\sigma\mu}\delta_\mu^\rho {U^{lep}}^\dagger_{\rho\kappa}
\end{array}$$

δ_i^j = Kronecker delta, then constraint:

$$\mathrm{Tr}(k^*_{(\uparrow 1)} k_{(\uparrow 1)} + k^*_{(\downarrow 1)} k_{(\downarrow 1)} + 3(k^*_{(\uparrow 3)} k_{(\uparrow 3)} + k^*_{(\downarrow 3)} k_{(\downarrow 3)})) = 2\,g^2$$

$\Rightarrow$ mass matrices satisfy

$$\sum_\sigma (m_\nu^\sigma)^2 + (m_e^\sigma)^2 + 3\,(m_u^\sigma)^2 + 3\,(m_d^\sigma)^2 = 8\,M^2$$

See-saw mechanism: $D = D(Y)$ Dirac

$$\begin{pmatrix} 0 & M_\nu^* & M_R^* & 0 \\ M_\nu & 0 & 0 & 0 \\ M_R & 0 & 0 & \bar{M}_\nu^* \\ 0 & 0 & \bar{M}_\nu & 0 \end{pmatrix}$$

on subspace $(\nu_R, \nu_L, \bar{\nu}_R, \bar{\nu}_L)$: largest eigenvalue of $M_R \sim \Lambda$ unification scale. Take $M_R = x\, k_R$ in flat space, Higgs vacuum v small (w/resp to unif scale) $\partial_u \mathrm{Tr}(f(D_A/\Lambda)) = 0 \quad u = x^2$

$$x^2 = \frac{2\, f_2\, \Lambda^2\, \mathrm{Tr}(k_R^* k_R)}{f_0\, \mathrm{Tr}((k_R^* k_R)^2)}$$

Dirac mass $M_\nu \sim$ Fermi energy v

$$\frac{1}{2}(\pm m_R \pm \sqrt{m_R^2 + 4\, v^2})$$

two eigenvalues $\sim \pm m_R$ and two $\sim \pm \frac{v^2}{m_R}$
Compare with estimates

$$(m_R)_1 \geq 10^7\, GeV\,, \quad (m_R)_2 \geq 10^{12}\, GeV\,, \quad (m_R)_3 \geq 10^{16}\, GeV$$

Low energy limit: compatibilities and predictions

Running of top Yukawa coupling (dominant term):

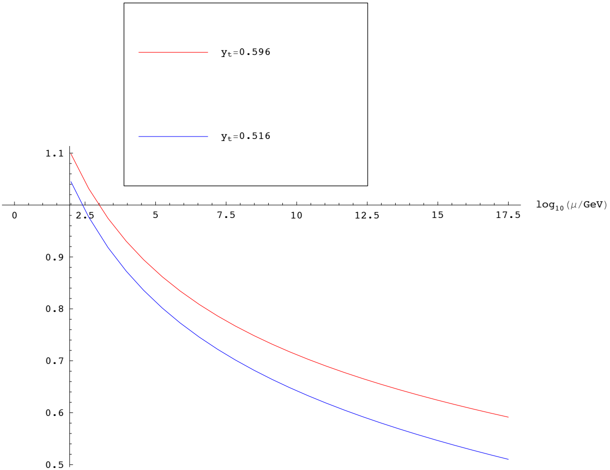
$$\frac{v}{\sqrt{2}}(y_\cdot^\sigma) = (m_\cdot^\sigma),$$

$$\frac{dy_t}{dt} = \frac{1}{16\pi^2}\left[\frac{9}{2}y_t^3 - \left(a\, g_1^2 + b\, g_2^2 + c\, g_3^2\right) y_t\right],$$

$$(a, b, c) = (\frac{17}{12}, \frac{9}{4}, 8)$$

$\Rightarrow$ value of top quark mass agrees with known (1.04 times if neglect other Yukawa couplings)

Top quark running using mass relation at unification



correction to MSM flow by y_ν^σ for τ neutrino (allowed to be comparably large by see-saw) lowers value

Higgs mass prediction using RGE for MSM

Higgs scattering parameter:

$$\frac{f_0}{2\,\pi^2}\int b\,|\varphi|^4\,\sqrt{g}\,d^4x=\frac{\pi^2}{2\,f_0}\frac{b}{a^2}\int|\mathbf{H}|^4\,\sqrt{g}\,d^4x$$

$\Rightarrow$ relation at unification ($\tilde{\lambda}$ is $|\mathbf{H}|^4$ coupling)

$$\tilde{\lambda}(\Lambda)=g_3^2\,\frac{b}{a^2}$$

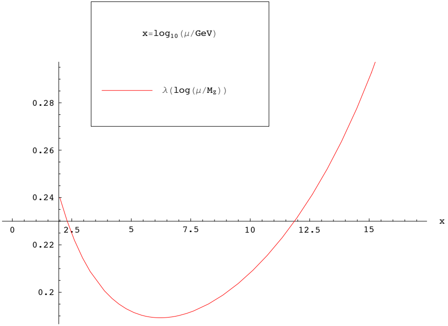
Running of Higgs scattering parameter:

$$\frac{d\lambda}{dt}=\lambda\gamma+\frac{1}{8\pi^2}(12\lambda^2+B)$$

$$\gamma=\frac{1}{16\pi^2}(12y_t^2-9g_2^2-3g_1^2)\quad B=\frac{3}{16}(3g_2^4+2g_1^2g_2^2+g_1^4)-3y_t^4$$

Higgs estimate (in MSM approximation for RGE flow)

$$m_H^2 = 8\lambda \frac{M^2}{g^2}, \quad m_H = \sqrt{2\lambda}\,\frac{2M}{g}$$



$\lambda(M_Z) \sim 0.241$ and Higgs mass ~ 170 GeV (w/correction from see-saw ~ 168 GeV) ... Problem: wrong Higgs mass! too heavy can the model be improved to accommodate a lighter Higgs? (125.5 GeV) ...yes, but with a fine-tuning problem