Mathematics as culture and knowledge

Matilde Marcolli

July 2007

Mathematics is an intellectual activity, arguably one of the most sophisticated ever produced by human civilization. Hermann Hesse sketched a portrait of the activities of mathematicians through the metaphor of the Glass Bead Game. That was perhaps the best literary attempt at catching a glimpse of some inner workings of the society of mathematicians. One does not blame a fictional work for inaccuracy, but it is indeed a difficult task to say something meaningful about what it is like to do mathematics.

There are quite a few mathematicians who entertain a Platonist view of mathematics. By this I mean essentially the belief that mathematical objects and constructs enjoy some kind of existence in a "world of ideas", independent of the human mind. As in the case of the mythical heaven, the proposers of such beliefs tend to be quite vague on the location and consistency of this extra-mental Platonic world. One reason that is often produced in support of the Platonic standpoint is the effectiveness of mathematics in modeling the physical world. No doubt that Kepler's laws would eventually be observed and understood by any technological intelligence living on a planet bound by gravity to revolve around a star (but would such a discovery follow the course we know had the planet been revolving around two stars?) but one can hardly make an equally strong case in support of other very beautiful but far more abstract branches of mathematics.

If no one would perhaps doubt that any sufficiently evolved extraterrestrial intelligence would understand the notion of prime number, there is far less compelling evidence that they would have our same notions of derived categories or shtukas. Recent years have gotten us used to more and more sophisticated mathematics being recruited to account for increasingly complicated models of high-energy physics. This sort of evidence notwithstanding, I personally remain extremely skeptical about the Platonist hypothesis.

Our brains developed over millions of years of evolution by natural selection. The capacity to produce mathematics has an obvious evolutionary advantage in as it is the key to the development of a scientific and technological civilization. The prominent position that this species of apes has acquired, compared to the other animal species of the planet, is a clear demonstration of the evolutionary advantage of the brain's capacity for scientific thought.

Other brains that are the product of a completely different evolutionary process in an entirely different environment might as well achieve the same goal of a technological civilization while producing a substantially different kind of mathematics from the one we know. Not entirely disjoint surely (prime numbers), but with possibly a vast symmetric difference. The existence of extraterrestrial intelligence is purely hypothetical. Sagan and Shklovskii speculated beautifully about it in the seventies and I will leave it at that, Platonism and all.

If mathematics (a large part of mathematics at least) is not a glimpse of the Platonic paradise but merely a byproduct of our brains and evolutionary processes, it loses none of its beauty because of that. It is all the more interesting *because* it is a part of human culture and it moves along with and is influenced by the development of the rest of our civilization.

The mathematics that we know today is the result of a long and tortuous itinerary of cultural development. It is far from being a static edifice, however. Its continuous rapid pace evolution can be seen easily by mentioning a couple of significant statistics. Math Sci Net, the main source of reviews of mathematical publications lists a total of 2,245,194 items, growing at the speed of 60,000 per year (and those listed by MathSciNet are just a selection of the total number of mathematical publications).

The first important step for anyone interested in working in mathematics is an awareness of the vastness of the landscape. One of the major risks, in my opinion, in mathematics and in many other fields of human knowledge, is that of being naive. One does not improvise oneself a mathematician. Becoming one takes about ten years of intensive training and careful study. That's just in order to accumulate the minimal amount of knowledge and skills that are needed to understand what doing mathematics is about. To start to actually *do* something still requires a few further steps.

One, which is extremely difficult to acquire, and is a good sign of having achieved maturity as a professional mathematician, is the capacity to sniff out what is *interesting*. There are many things in mathematics one can do for the sake of doing them. Marcel Duchamp entitled one of his provocative sculptures "Classify the combs by the number of their teeth".

Truly interesting mathematics is not a technical exercise in classifying combs. Often what makes a mathematical result surprising and interesting lies in discovering unexpected connections: a way of relating results and constructions that were seemingly unrelated, recognizing a similarity of structures across apparently different phenomena. This requires knowledge. One has to be able to comfortably navigate the existent in order to be able to envision the non-existent.

Being naive in mathematics has (with rare exceptions) the sole effect of digging oneself into an obscure corner of useless game-playing. Knowledge is what provides the crucial lighthouses and nautical charts that allow the working mathematicians to navigate their way safely across rough waters.

There are widespread romantic mythologies about lonely geniuses who don't read but still generate beautiful theorems. These are based on largely fabricated anecdotal accounts. In truth, a long time spent reading and acquiring knowledge of present and past mathematics is essential to creating interesting future mathematics. Isolation only means the drying up of creative abilities.

Besides its effectiveness as a catalyst of invention, the transmission of knowledge through the written word is what makes us human. It is the key to the advancement of civilization. We read and learn because it is a great pleasure to do so, because we are human beings who care for being not an isolated fragment but a part of humanity as a whole. As in John Donne's famous poem "No man is an island, entire of itself every man is a piece of the continent, a part of the main."

Mathematics is interesting to an especially high degree among the achievements of humanity, because it has a universality that can provide us with a way to bridge across and transcend the insignificant geographic and historic differences that split humankind. It is the common language our brains are hardwired to produce, the one that drives our scientific and technological development and is at the same time a deeply philosophical and artistic kind of endeavor.

Indeed, this is a peculiar aspect of mathematics, that singles it out among the various fields of human knowledge. It functions simultaneously with the modes of the hard sciences and with those of the fine arts. Flights of the imagination, visual and poetic imageries and esthetic considerations drive the development of the field and live side by side with the most stringent rules of scientific rigor.

It is a pity that neuroscientists trying to understand how the brain develops mathematics generally tend to confuse mathematics with "number sense". The latter is a completely different faculty of the intellect, which is often completely disjoint from mathematics itself (there are plenty of examples of famous mathematicians who have no number sense at all). Mathematics is about the creation of structures and, in particular, numbers also exhibit interesting structures, but that is as far as the connection goes.

Understanding how mathematics is created in our brains will be a wonderful way to discover more of the functioning of the brain itself, since it provides a full spectrum of modes of operation of creativity and imagination as well as of manipulation of images and symbols, with a very precise and well defined focus.

The final answer, if one is needed, to the question of why we do mathematics, is that we derive pleasure in doing so. It is an obvious byproduct of evolution by natural selection that we derive pleasure in doing things that are also beneficial to the survival of our genes. Mathematics is beneficial to our species for the applications it has to our science and technology, but that's not the reason why we enjoy doing mathematics. We do not think of the importance of practical applications when we enjoy creating new mathematics any more than we think of the importance of mixing our DNAs when we enjoy having sex.