Early Universe Models

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Ma148b Spring 2016
Topics in Mathematical Physics

This lecture based on

- Matilde Marcolli, Elena Pierpaoli, *Early universe models from Noncommutative Geometry*, Advances in Theoretical and Mathematical Physics, Vol.14 (2010) N.5, 1373–1432.
- Daniel Kolodrubetz, Matilde Marcolli, *Boundary conditions of the RGE flow in the noncommutative geometry approach to particle physics and cosmology*, Phys. Lett. B, Vol.693 (2010) 166–174

Re-examine RGE flow; gravitational terms in the asymptotic form of the spectral action; cosmological timeline; running in the very early universe; inflation and other gravitational phenomena

Asymptotic expansion of spectral action for large energies

$$
\begin{aligned}
S = \;& \frac{1}{2\kappa_0^2} \int R \sqrt{g}\, d^4x + \gamma_0 \int \sqrt{g}\, d^4x \\
+ \;& \alpha_0 \int C_{\mu\nu\rho\sigma}\, C^{\mu\nu\rho\sigma} \sqrt{g}\, d^4x + \tau_0 \int R^* R^* \sqrt{g}\, d^4x \\
+ \;& \frac{1}{2} \int |DH|^2 \sqrt{g}\, d^4x - \mu_0^2 \int |H|^2 \sqrt{g}\, d^4x \\
- \;& \xi_0 \int R\, |H|^2 \sqrt{g}\, d^4x + \lambda_0 \int |H|^4 \sqrt{g}\, d^4x \\
+ \;& \frac{1}{4} \int (G^i_{\mu\nu}\, G^{\mu\nu i} + F^\alpha_{\mu\nu}\, F^{\mu\nu\alpha} + B_{\mu\nu}\, B^{\mu\nu}) \sqrt{g}\, d^4x
\end{aligned}
$$

A modified gravity model with non minimal coupling to Higgs and minimal coupling to gauge theories

Coefficients and parameters:

$$\tfrac{1}{2\kappa_0^2} = \frac{96 f_2 \Lambda^2 - f_0 \mathfrak{c}}{24\pi^2} \qquad \gamma_0 = \frac{1}{\pi^2}(48 f_4 \Lambda^4 - f_2 \Lambda^2 \mathfrak{c} + \frac{f_0}{4}\mathfrak{d})$$

$$\alpha_0 = -\frac{3 f_0}{10\pi^2} \qquad \tau_0 = \frac{11 f_0}{60\pi^2}$$

$$\mu_0^2 = 2\frac{f_2 \Lambda^2}{f_0} - \frac{\mathfrak{e}}{\mathfrak{a}} \qquad \xi_0 = \tfrac{1}{12} \qquad \lambda_0 = \frac{\pi^2 \mathfrak{b}}{2 f_0 \mathfrak{a}^2}$$

- f_0, f_2, f_4 free parameters, $f_0 = f(0)$ and, for $k > 0$,

$$f_k = \int_0^\infty f(v) v^{k-1} dv.$$

- $\mathfrak{a}, \mathfrak{b}, \mathfrak{c}, \mathfrak{d}, \mathfrak{e}$ functions of Yukawa parameters of νMSM

$$\begin{aligned}
\mathfrak{a} &= \mathrm{Tr}(Y_\nu^\dagger Y_\nu + Y_e^\dagger Y_e + 3(Y_u^\dagger Y_u + Y_d^\dagger Y_d)) \\
\mathfrak{b} &= \mathrm{Tr}((Y_\nu^\dagger Y_\nu)^2 + (Y_e^\dagger Y_e)^2 + 3(Y_u^\dagger Y_u)^2 + 3(Y_d^\dagger Y_d)^2) \\
\mathfrak{c} &= \mathrm{Tr}(MM^\dagger) \qquad \mathfrak{d} = \mathrm{Tr}((MM^\dagger)^2) \\
\mathfrak{e} &= \mathrm{Tr}(MM^\dagger Y_\nu^\dagger Y_\nu).
\end{aligned}$$

Two different perspectives on running: parameters $\mathfrak{a}, \mathfrak{b}, \mathfrak{c}, \mathfrak{d}, \mathfrak{e}$ run with RGE

- The relation between coefficients κ_0, γ_0, α_0, τ_0, μ_0, ξ_0, λ_0 and parameters $\mathfrak{a}, \mathfrak{b}, \mathfrak{c}, \mathfrak{d}, \mathfrak{e}$ hold only at Λ_{unif}: constraint on initial conditions; independent running
- There is a range of energies $\Lambda_{min} \leq \Lambda \leq \Lambda_{unif}$ where the running of coefficients κ_0, γ_0, α_0, τ_0, μ_0, ξ_0, λ_0 determined by running of $\mathfrak{a}, \mathfrak{b}, \mathfrak{c}, \mathfrak{d}, \mathfrak{e}$ and relation continues to hold (very early universe only)

The first perspective requires independent running of gravitational parameters with given condition at unification; the second perspective allows for interesting gravitational phenomena in the very early universe specific to NCG model only

The first approach followed in

- Ali Chamseddine, Alain Connes, Matilde Marcolli, *Gravity and the Standard Model with neutrino mixing*, ATMP 11 (2007) 991–1090, arXiv:hep-th/0610241

For modified gravity models

$$\int \left(\frac{1}{2\eta} C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} - \frac{\omega}{3\eta} R^2 + \frac{\theta}{\eta} R^* R^* \right) \sqrt{g}\, d^4 x$$

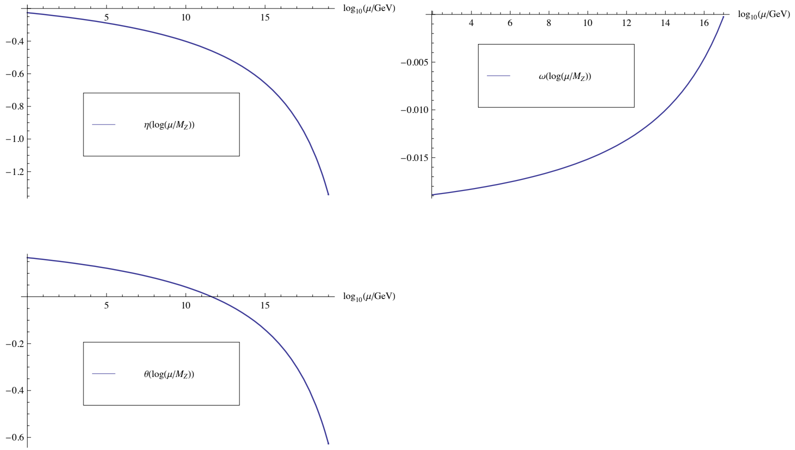
Running of gravitational parameters (Avramidi, Codello–Percacci, Donoghue) by

$$\beta_\eta = -\frac{1}{(4\pi)^2} \frac{133}{10} \eta^2$$

$$\beta_\omega = -\frac{1}{(4\pi)^2} \frac{25 + 1098\,\omega + 200\,\omega^2}{60} \eta$$

$$\beta_\theta = \frac{1}{(4\pi)^2} \frac{7(56 - 171\,\theta)}{90} \eta.$$

With relations at unification get running like



Plausible values in low energy limit (near fixed points)

Look then at second point of view: M.M.-E.Pierpaoli (2009)

Renormalization group equations for SM with right handed neutrinos and Majorana mass terms, from unification energy (2×10^{16} GeV) down to the electroweak scale (10^2 GeV)

AKLRS S. Antusch, J. Kersten, M. Lindner, M. Ratz, M.A. Schmidt *Running neutrino mass parameters in see-saw scenarios*, JHEP 03 (2005) 024.

Remark: RGE analysis in [CCM] only done using minimal SM

1-loop RGE equations: $\Lambda \frac{df}{d\Lambda} = \beta_f(\Lambda)$

$$16\pi^2 \ \beta_{g_i} = b_i \, g_i^3 \quad \text{with } (b_{SU(3)}, b_{SU(2)}, b_{U(1)}) = (-7, -\frac{19}{6}, \frac{41}{10})$$

$$16\pi^2 \ \beta_{Y_u} = Y_u(\frac{3}{2} Y_u^\dagger Y_u - \frac{3}{2} Y_d^\dagger Y_d + \mathfrak{a} - \frac{17}{20} g_1^2 - \frac{9}{4} g_2^2 - 8 g_3^2)$$

$$16\pi^2 \ \beta_{Y_d} = Y_d(\frac{3}{2} Y_d^\dagger Y_d - \frac{3}{2} Y_u^\dagger Y_u + \mathfrak{a} - \frac{1}{4} g_1^2 - \frac{9}{4} g_2^2 - 8 g_3^2)$$

$$16\pi^2 \ \beta_{Y_\nu} = Y_\nu(\frac{3}{2} Y_\nu^\dagger Y_\nu - \frac{3}{2} Y_e^\dagger Y_e + \mathfrak{a} - \frac{9}{20} g_1^2 - \frac{9}{4} g_2^2)$$

$$16\pi^2 \ \beta_{Y_e} = Y_e(\frac{3}{2} Y_e^\dagger Y_e - \frac{3}{2} Y_\nu^\dagger Y_\nu + \mathfrak{a} - \frac{9}{4} g_1^2 - \frac{9}{4} g_2^2)$$

$$16\pi^2 \ \beta_M = Y_\nu Y_\nu^\dagger M + M (Y_\nu Y_\nu^\dagger)^T$$

$$16\pi^2 \ \beta_\lambda = 6\lambda^2 - 3\lambda(3 g_2^2 + \frac{3}{5} g_1^2) + 3 g_2^4 + \frac{3}{2}(\frac{3}{5} g_1^2 + g_2^2)^2 + 4\lambda\mathfrak{a} - 8\mathfrak{b}$$

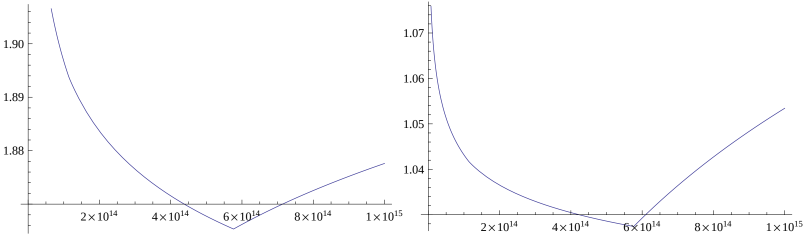
Note: different normalization from [CCM] and 5/3 factor included in g_1^2

Method of AKLRS: non-degenerate spectrum of Majorana masses, different effective field theories in between the three see-saw scales:

- RGE from unification Λ_{unif} down to first see-saw scale (largest eigenvalue of M)
- Introduce $Y_\nu^{(3)}$ removing last row of Y_ν in basis where M diagonal and $M^{(3)}$ removing last row and column.
- Induced RGE down to second see-saw scale
- Introduce $Y_\nu^{(2)}$ and $M^{(2)}$, matching boundary conditions
- Induced RGE down to first see-saw scale
- Introduce $Y_\nu^{(1)}$ and $M^{(1)}$, matching boundary conditions
- Induced RGE down to electoweak energy Λ_{ew}

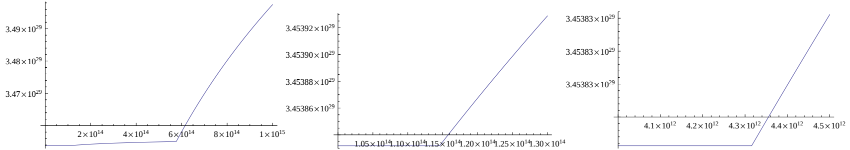
Use effective field theories $Y_\nu^{(N)}$ and $M^{(N)}$ between see-saw scales

Running of coefficients $\mathfrak{a}, \mathfrak{b}$ with RGE

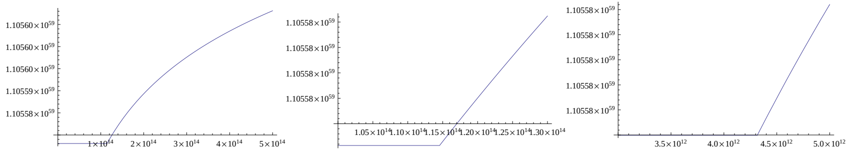


Coefficients $\mathfrak{a}$ and $\mathfrak{b}$ near the top see-saw scale

Running of coefficient $\mathfrak{c}$ with RGE

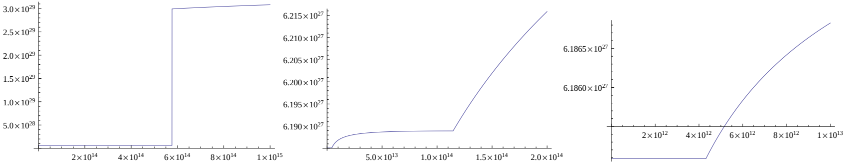


Running of coefficient $\mathfrak{d}$ with RGE



Effect of the three see-saw scales

Running of coefficient $\mathfrak{c}$ with RGE



Effect of the three see-saw scales

With default boundary conditions at unification of AKLRS

...but sensitive dependence on the initial conditions $\Rightarrow$ fine tuning

Changing initial conditions: maximal mixing conditions at unification

$$\zeta = exp(2\pi i/3)$$

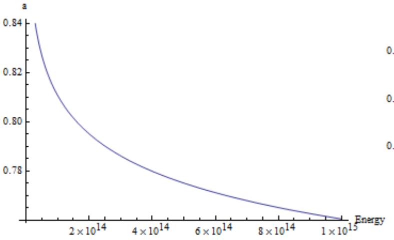
$$U_{PMNS}(\Lambda_{unif}) = \frac{1}{3} \begin{pmatrix} 1 & \zeta & \zeta^2 \\ \zeta & 1 & \zeta \\ \zeta^2 & \zeta & 1 \end{pmatrix}$$

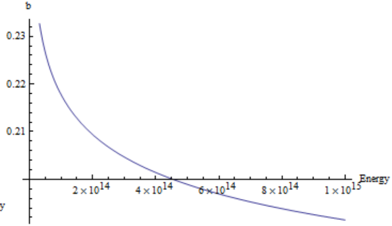
$$\delta_{(\uparrow 1)} = \frac{1}{246} \begin{pmatrix} 12.2 \times 10^{-9} & 0 & 0 \\ 0 & 170 \times 10^{-6} & 0 \\ 0 & 0 & 15.5 \times 10^{-3} \end{pmatrix}$$

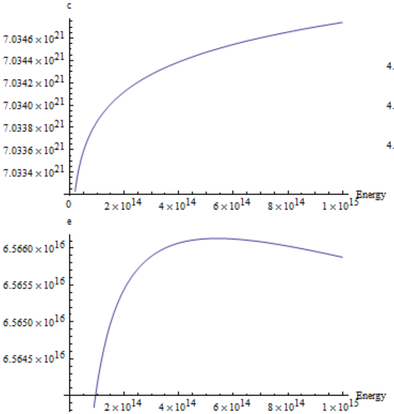
$$Y_\nu = U^\dagger_{PMNS} \delta_{(\uparrow 1)} U_{PMNS}$$

- Daniel Kolodrubetz, Matilde Marcolli, *Boundary conditions of the RGE flow in the noncommutative geometry approach to particle physics and cosmology*, arXiv:1006.4000

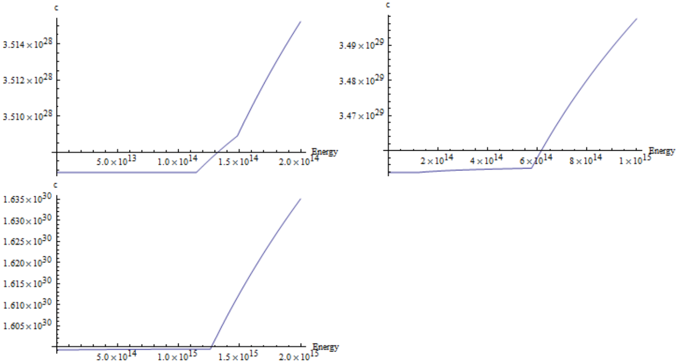
Effect on coefficients running






c
7.0346 × 10^21
7.0344 × 10^21
7.0342 × 10^21
7.0340 × 10^21
7.0338 × 10^21
7.0336 × 10^21
7.0334 × 10^21
0
2 × 10^14
4 × 10^14
6 × 10^14
8 × 10^14
1 × 10^15
Energy
d
4.9475 × 10^43
4.9470 × 10^43
4.9465 × 10^43
1 × 10^14
2 × 10^14
3 × 10^14
4 × 10^14
5 × 10^14
Energy
e
6.5660 × 10^16
6.5655 × 10^16
6.5650 × 10^16
6.5645 × 10^16
2 × 10^14
4 × 10^14
6 × 10^14
8 × 10^14
1 × 10^15
Energy

Further evidence of sensitive dependence: changing only one parameter in diagonal matrix Y_ν get running of top term:



Geometric constraints at unification energy

- λ parameter constraint

$$\lambda(\Lambda_{unif}) = \frac{\pi^2}{2f_0} \frac{\mathfrak{b}(\Lambda_{unif})}{\mathfrak{a}(\Lambda_{unif})^2}$$

- Higgs vacuum constraint

$$\frac{\sqrt{\mathfrak{a} f_0}}{\pi} = \frac{2M_W}{g}$$

- See-saw mechanism and $\mathfrak{c}$ constraint

$$\frac{2f_2\Lambda_{unif}^2}{f_0} \leq \mathfrak{c}(\Lambda_{unif}) \leq \frac{6f_2\Lambda_{unif}^2}{f_0}$$

- Mass relation at unification

$$\sum_{generations} (m_\nu^2 + m_e^2 + 3m_u^2 + 3m_d^2)|_{\Lambda=\Lambda_{unif}} = 8M_W^2|_{\Lambda=\Lambda_{unif}}$$

Choice of initial conditions at unification:

- Compatibility with low energy values: experimental constraints
- Compatibility with geometric constraints at unification

It is possible to modify boundary conditions to achieve both compatibilities

Example: using maximal mixing conditions but modify parameters in the Majorana mass matrix and initial condition of Higgs parameter to satisfy geometric constraints

Cosmology timeline

- Planck epoch: $t \leq 10^{-43}\,s$ after the Big Bang (unification of forces with gravity, quantum gravity)
- Grand Unification epoch: $10^{-43}\,s \leq t \leq 10^{-36}\,s$ (electroweak and strong forces unified; Higgs)
- Electroweak epoch: $10^{-36}\,s \leq t \leq 10^{-12}\,s$ (strong and electroweak forces separated)
- Inflationary epoch: possibly $10^{-36}\,s \leq t \leq 10^{-32}\,s$

- NCG SM preferred scale at unification; RGE running between unification and electroweak $\Rightarrow$ info on inflationary epoch.
- Remark: Cannot extrapolate to modern universe, nonperturbative effects in the spectral action and phase transitions in the RGE flow

Cosmological implications of the NCG SM

- Linde's hypothesis (antigravity in the early universe)
- Primordial black holes and gravitational memory
- Running effective gravitational constant affecting gravitational waves
- Gravity balls
- Varying effective cosmological constant
- Higgs based slow-roll inflation
- Spontaneously arising Hoyle-Narlikar in EH backgrounds

Effective gravitational constant

$$G_{\text{eff}} = \frac{\kappa_0^2}{8\pi} = \frac{3\pi}{192 f_2 \Lambda^2 - 2 f_0 \mathfrak{c}(\Lambda)}$$

Effective cosmological constant

$$\gamma_0 = \frac{1}{4\pi^2}(192 f_4 \Lambda^4 - 4 f_2 \Lambda^2 \mathfrak{c}(\Lambda) + f_0 \mathfrak{d}(\Lambda))$$

Conformal non-minimal coupling of Higgs and gravity

$$\frac{1}{16\pi G_{\rm eff}}\int R\sqrt{g}d^4x - \frac{1}{12}\int R\,|H|^2\sqrt{g}d^4x$$

Conformal gravity

$$\frac{-3f_0}{10\pi^2}\int C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}\sqrt{g}d^4x$$

$C^{\mu\nu\rho\sigma}$ = Weyl curvature tensor (trace free part of Riemann tensor)

$$C_{\lambda\mu\nu\kappa} = R_{\lambda\mu\nu\kappa} - \frac{1}{2}(g_{\lambda\nu}R_{\mu\kappa} - g_{\mu\nu}R_{\lambda\kappa} + g_{\mu\kappa}R_{\lambda\nu}) + \frac{1}{6}(g_{\lambda\nu}g_{\mu\kappa} - g_{\lambda\kappa}g_{\mu\nu})$$

An example: $G_{\rm eff}(\Lambda_{ew}) = G$ (at electroweak phase transition $G_{\rm eff}$ is already modern universe Newton constant)
$1/\sqrt{G} = 1.22086 \times 10^{19}$ GeV $\Rightarrow f_2 = 7.31647 \times 10^{32}$

$$G_{\rm eff}^{-1}(\Lambda) \sim \frac{96 f_2 \Lambda^2}{24\pi^2}$$

Term $\mathfrak{e}/\mathfrak{a}$ lower order
Dominant terms in the spectral action:

$$\Lambda^2 \left(\frac{1}{2\tilde{\kappa}_0^2} \int R\sqrt{g} d^4x - \tilde{\mu}_0^2 \int |H|^2 \sqrt{g} d^4x \right)$$

$\tilde{\kappa}_0 = \Lambda\kappa_0$ and $\tilde{\mu}_0 = \mu_0/\Lambda$, where $\mu_0^2 \sim \frac{2f_2\Lambda^2}{f_0}$

Detectable by gravitational waves:
Einstein equations $R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = \kappa_0^2 T^{\mu\nu}$

$$g_{\mu\nu} = a(t)^2 \begin{pmatrix} -1 & 0 \\ 0 & \delta_{ij} + h_{ij}(x) \end{pmatrix}$$

trace and traceless part of h_{ij} $\Rightarrow$ Friedmann equation

$$-3 \left(\frac{\dot{a}}{a}\right)^2 + \frac{1}{2} \left(4 \left(\frac{\dot{a}}{a}\right) \dot{h} + 2\ddot{h} \right) = \frac{\tilde{\kappa}_0^2}{\Lambda^2} \, T_{00}$$

$\Lambda(t) = 1/a(t)$ (f_2 large) Inflationary epoch: $a(t) \sim e^{\alpha t}$
NCG model solutions:

$$h(t) = \frac{3\pi^2 T_{00}}{192 f_2 \alpha^2} e^{2\alpha t} + \frac{3\alpha}{2} t + \frac{A}{2\alpha} e^{-2\alpha t} + B$$

Ordinary cosmology:

$$(\frac{4\pi G T_{00}}{\alpha} + \frac{3\alpha}{2}) t + \frac{A}{2\alpha} e^{-2\alpha t} + B$$

Radiation dominated epoch: $a(t) \sim t^{1/2}$
NCG model solutions:

$$h(t) = \frac{4\pi^2 T_{00}}{288 f_2} t^3 + B + A \log(t) + \frac{3}{8} \log(t)^2$$

Ordinary cosmology:

$$h(t) = 2\pi G T_{00} t^2 + B + A \log(t) + \frac{3}{8} \log(t)^2$$

Same example, special case:

$$R \sim \frac{2\tilde{\kappa}_0^2 \tilde{\mu}_0^2 \mathfrak{a} f_0}{\pi^2} \sim 1 \quad \text{and} \quad H \sim \sqrt{\mathfrak{a} f_0}/\pi$$

Leaves conformally coupled matter and gravity

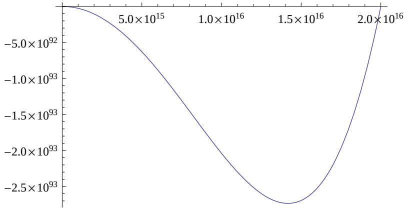
$$\begin{aligned} S_c = \; & \alpha_0 \int C_{\mu\nu\rho\sigma}\, C^{\mu\nu\rho\sigma} \sqrt{g}\, d^4x + \frac{1}{2} \int |DH|^2 \sqrt{g}\, d^4x \\ & -\xi_0 \int R\, |H|^2 \sqrt{g}\, d^4x + \lambda_0 \int |H|^4 \sqrt{g}\, d^4x \\ & +\frac{1}{4} \int (G^i_{\mu\nu}\, G^{\mu\nu i} + F^\alpha_{\mu\nu}\, F^{\mu\nu\alpha} + B_{\mu\nu}\, B^{\mu\nu}) \sqrt{g}\, d^4x. \end{aligned}$$

A Hoyle-Narlikar type cosmology, normally suppressed by dominant Einstein–Hilbert term, arises when $R \sim 1$ and $H \sim v$, near see-saw scale.

Cosmological term controlled by additional parameter f_4, vanishing condition:

$$f_4 = \frac{(4 f_2 \Lambda^2 \mathfrak{c} - f_0 \mathfrak{d})}{192 \Lambda^4}$$

Example: vanishing at unification $\gamma_0(\Lambda_{unif}) = 0$



Running of $\gamma_0(\Lambda)$: possible inflationary mechanism

The λ_0-ansatz

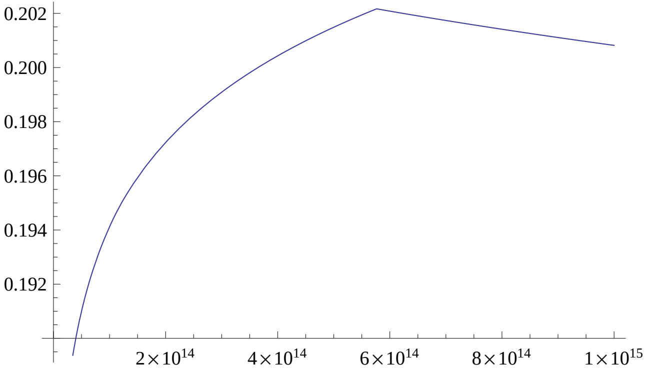
$$\lambda_0|_{\Lambda=\Lambda_{unif}} = \lambda(\Lambda_{unif}) \frac{\pi^2 \mathfrak{b}(\Lambda_{unif})}{f_0 \mathfrak{a}^2(\Lambda_{unif})},$$

- Run like $\lambda(\Lambda)$ but change boundary condition to $\lambda_0|_{\Lambda=\Lambda_{unif}}$
- Run like

$$\lambda_0(\Lambda) = \lambda(\Lambda) \frac{\pi^2 \mathfrak{b}(\Lambda)}{f_0 \mathfrak{a}^2(\Lambda)}$$

For most of our cosmological estimates no serious difference

The running of $\lambda_0(\Lambda)$ near the top see-saw scale



Linde's hypothesis antigravity in the early universe

- A.D. Linde, *Gauge theories, time-dependence of the gravitational constant and antigravity in the early universe*, Phys. Letters B, Vol.93 (1980) N.4, 394–396

Based on a conformal coupling

$$\frac{1}{16\pi G}\int R\,\sqrt{g}\,d^4x - \frac{1}{12}\int R\,\phi^2\sqrt{g}\,d^4x$$

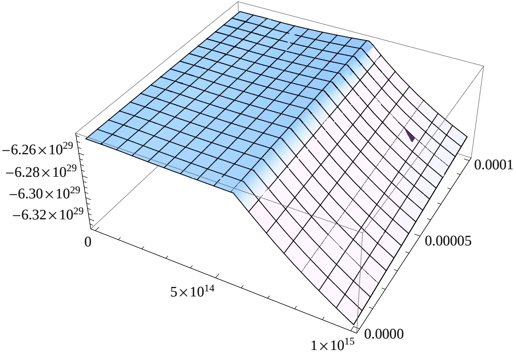
giving an effective

$$G_{\rm eff}^{-1} = G^{-1} - \frac{4}{3}\pi\phi^2$$

In the NCG SM model two sources of negative gravity

- Running of $G_{\rm eff}(\Lambda)$
- Conformal coupling to the Higgs field

Example of effective $G_{\text{eff}}^{-1}(\Lambda, f_2)$ near the top see-saw scale



Example: fixing $G_{\text{eff}}(\Lambda_{unif}) = G$ gives a phase of negative gravity with conformal gravity becoming dominant near sign change of $G_{\text{eff}}(\Lambda)^{-1}$ at $\sim 10^{12}$ GeV

Gravity balls

$$G_{\mathrm{eff},H} = \frac{G_{\mathrm{eff}}}{1 - \frac{4\pi}{3} G_{\mathrm{eff}} |H|^2}$$

combines running of G_{eff} with Linde mechanism

Suppose f_2 such that $G_{\mathrm{eff}}(\Lambda) > 0$

$$\begin{cases} G_{\mathrm{eff},H} < 0 & \text{for } |H|^2 > \frac{3}{4\pi G_{\mathrm{eff}}(\Lambda)}, \\ G_{\mathrm{eff},H} > 0 & \text{for } |H|^2 < \frac{3}{4\pi G_{\mathrm{eff}}(\Lambda)}. \end{cases}$$

Unstable and stable equilibrium for H:

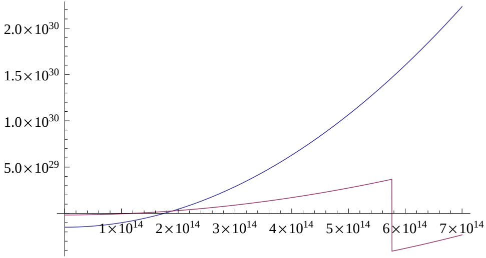
$$\ell_H(\Lambda, f_2) := \frac{\mu_0^2}{2\lambda_0}(\Lambda) = \frac{2\frac{f_2\Lambda^2}{f_0} - \frac{\mathfrak{e}(\Lambda)}{\mathfrak{a}(\Lambda)}}{\lambda(\Lambda)\frac{\pi^2\mathfrak{b}(\Lambda)}{f_0\mathfrak{a}^2(\Lambda)}} = \frac{(2f_2\Lambda^2\mathfrak{a}(\Lambda) - f_0\mathfrak{e}(\Lambda))\mathfrak{a}(\Lambda)}{\pi^2\lambda(\Lambda)\mathfrak{b}(\Lambda)}$$

(with λ_0-ansatz)

Negative gravity regime where

$$\ell_H(\Lambda, f_2) > \frac{3}{4\pi G_{\mathrm{eff}}(\Lambda, f_2)}$$

An example of transition to negative gravity



Gravity balls: regions where $|H|^2 \sim 0$ unstable equilibrium (positive gravity) surrounded by region with $|H|^2 \sim \ell_H(\Lambda, f_2)$ stable (negative gravity): possible model of dark energy

Primordial black holes (Zeldovich–Novikov, 1967)

- I.D. Novikov, A.G. Polnarev, A.A. Starobinsky, Ya.B. Zeldovich, *Primordial black holes*, Astron. Astrophys. 80 (1979) 104–109
- J.D. Barrow, *Gravitational memory?* Phys. Rev. D Vol.46 (1992) N.8 R3227, 4pp.

Caused by: collapse of overdense regions, phase transitions in the early universe, cosmic loops and strings, inflationary reheating, etc

Gravitational memory: if gravity balls with different $G_{\mathrm{eff},H}$ primordial black holes can evolve with different $G_{\mathrm{eff},H}$ from surrounding space

Evaporation of PBHs by Hawking radiation

$$\frac{d\mathcal{M}(t)}{dt} \sim -(G_{\rm eff}(t)\mathcal{M}(t))^{-2}$$

with Hawking temperature $T = (8\pi G_{\rm eff}(t)\mathcal{M}(t))^{-1}$.
In terms of energy:

$$\mathcal{M}^2\, d\mathcal{M} = \frac{1}{\Lambda^2 G_{\rm eff}^2(\Lambda, f_2)} d\Lambda$$

With or without gravitational memory depending on $G_{\rm eff}$ behavior

Evaporation of PBHs linked to γ-ray bursts

Higgs based slow-roll inflation

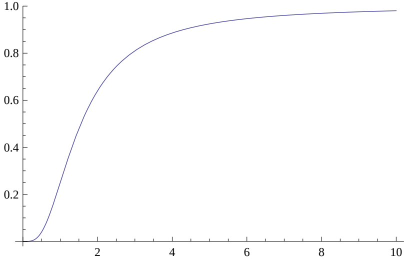
dSHW A. De Simone, M.P. Hertzberg, F. Wilczek, *Running inflation in the Standard Model*, hep-ph/0812.4946v2

Minimal SM and non-minimal coupling of Higgs and gravity.

$$\xi_0 \int R\,|H|^2 \sqrt{g}\, d^4x$$

Non-conformal coupling $\xi_0 \neq 1/12$, running of ξ_0

Effective Higgs potential: inflation parameter $\psi = \sqrt{\xi_0}\kappa_0 |H|$



inflationary period $\psi >> 1$, end of inflation $\psi \sim 1$, low energy regime $\psi << 1$

In the NCG SM have $\xi_0 = 1/12$ but same Higgs based slow-roll inflation due to κ_0 running (say $\kappa_0 > 0$)

$$\psi(\Lambda) = \sqrt{\xi_0(\Lambda)}\kappa_0(\Lambda)|H| = \sqrt{\frac{\pi^2}{96 f_2 \Lambda^2 - f_0 \mathfrak{c}(\Lambda)}}\,|H|$$

Einstein metric $g^E_{\mu\nu} = f(H) g_{\mu\nu}$, for $f(H) = 1 + \xi_0 \kappa_0 |H|^2$
Higgs potential

$$V_E(H) = \frac{\lambda_0 |H|^4}{(1 + \xi_0 \kappa_0^2 |H|^2)^2}$$

For $\psi >> 1$ approaches constant; usual quartic potential for $\psi << 1$

Conclusion:
Various possible inflation scenarios in the very early universe from running of coefficients of the spectral action according to the relation to Yukawa parameters: phases and regions of negative gravity, variable gravitational and cosmological constants, inflation potential from nonmininal coupling of Higgs to gravity

Main problem: these effects depend on choice of initial conditions at unification (sensitive dependence) and several of these scenarios are ruled out when moving boundary conditions

Other cosmological aspects of the Spectral Action

- The problem of cosmic topology
- The Poisson summation formula
- Nonperturbative computation of the spectral action on 3-dimensional space forms
- Slow-roll inflation: potential, slow-roll coefficients, power spectra
- Slow-roll inflation from the nonperturbative spectral action and cosmic topology