Noncommutative Geometry and Cosmology
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Joint work with Elena Pierpaoli (Astronomy, USC/Caltech),
2009
The NCG standard model


The noncommutative space $X \times F$
product of 4-dim spacetime and finite NC space
The spectral action functional

$$\text{Tr}(f(D_A/\Lambda)) + \frac{1}{2} \langle J \tilde{\xi}, D_A \tilde{\xi} \rangle$$

$D_A = D + A + \epsilon' J A J^{-1}$ Dirac operator with inner fluctuations
$A = A^* = \sum_k a_k[D, b_k]$
Spectral triples \((\mathcal{A}, \mathcal{H}, D)\):
- involutive algebra \(\mathcal{A}\)
- representation \(\pi : \mathcal{A} \rightarrow \mathcal{L}(\mathcal{H})\)
- self adjoint operator \(D\) on \(\mathcal{H}\)
- compact resolvent \((1 + D^2)^{-1/2} \in \mathcal{K}\)
- \([a, D]\) bounded \(\forall a \in \mathcal{A}\)
- even \(\mathbb{Z}/2\)-grading \([\gamma, a] = 0\) and \(D\gamma = -\gamma D\)
- real structure: antilinear isom \(J : \mathcal{H} \rightarrow \mathcal{H}\) with \(J^2 = \varepsilon\), \(JD = \varepsilon' DJ\), and \(J\gamma = \varepsilon'' \gamma J\)

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- bimodule: \([a, b^0] = 0\) for \(b^0 = Jb^*J^{-1}\)
- order one condition: \([[D, a], b^0] = 0\)
Ansatz for the NC space $F$

$$\mathcal{A}_{LR} = \mathbb{C} \oplus \mathbb{H}_L \oplus \mathbb{H}_R \oplus M_3(\mathbb{C})$$

(or more general) $\Rightarrow$ everything else follows by computation

- Representation: $\mathcal{M}_F$ sum of all inequiv irred odd $\mathcal{A}_{LR}$-bimodules (fix $N$ generations) $\mathcal{H}_F = \oplus^N \mathcal{M}_F$ fermions
- Algebra $\mathcal{A}_F = \mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$: order one condition
- $F$ zero dimensional but KO-dim 6
- $J_F =$ matter/antimatter, $\gamma_F =$ L/R chirality
- Classification of Dirac operators (moduli spaces)
Dirac operators and Majorana mass terms

\[ D(Y) = \begin{pmatrix} S & T^* \\ T & \bar{S} \end{pmatrix}, \quad S = S_1 \oplus (S_3 \otimes 1_3), \quad T = Y_R : |\nu_R\rangle \rightarrow J_F |\nu_R\rangle \]

\[
S_1 = \begin{pmatrix}
0 & 0 & Y_{(\uparrow 1)} & 0 \\
0 & 0 & 0 & Y_{(\downarrow 1)} \\
Y_{(\uparrow 1)} & 0 & 0 & 0 \\
0 & Y_{(\downarrow 1)} & 0 & 0
\end{pmatrix}
\]

\[
S_3 = \begin{pmatrix}
0 & 0 & Y_{(\uparrow 3)} & 0 \\
0 & 0 & 0 & Y_{(\downarrow 3)} \\
Y_{(\uparrow 3)} & 0 & 0 & 0 \\
0 & Y_{(\downarrow 3)} & 0 & 0
\end{pmatrix}
\]

Yukawa matrices: Dirac masses and mixing angles in $GL_{N=3}(\mathbb{C})$

\[ Y_e = Y_{(\downarrow 1)} \text{ (charged leptons)} \]
\[ Y_\nu = Y_{(\uparrow 1)} \text{ (neutrinos)} \]
\[ Y_d = Y_{(\downarrow 3)} \text{ (d/s/b quarks)} \]
\[ Y_u = Y_{(\uparrow 3)} \text{ (u/c/t quarks)} \]
\[ M = Y^t_R \text{ Majorana mass terms symm matrix} \]
Product geometry \((C^\infty(X), L^2(X, S), D_X) \cup (\mathcal{A}_F, \mathcal{H}_F, D_F)\)

- \(\mathcal{A} = C^\infty(X) \otimes \mathcal{A}_F = C^\infty(X, \mathcal{A}_F)\)
- \(\mathcal{H} = L^2(X, S) \otimes \mathcal{H}_F = L^2(X, S \otimes \mathcal{H}_F)\)
- \(D = D_X \otimes 1 + \gamma_5 \otimes D_F\)

Asymptotic formula for the spectral action \((\text{Chamseddine–Connes})\)

\[
\text{Tr}(f(D/\Lambda)) \sim \sum_{k \in \text{DimSp}} f_k \Lambda^k \int |D|^{-k} + f(0) \zeta_D(0) + o(1)
\]

for large \(\Lambda\) with \(f_k = \int_0^\infty f(\nu) \nu^{k-1} d\nu\) and integration given by residues of zeta function \(\zeta_D(s) = \text{Tr}(|D|^{-s})\); \(\text{DimSp}\) poles of zeta functions
The asymptotic expansion of the spectral action from [CCM]

\[ S = \frac{1}{\pi^2} (48 f_4 \Lambda^4 - f_2 \Lambda^2 c + \frac{f_0}{4} d) \int \sqrt{g} \, d^4 x 
+ \frac{96 f_2 \Lambda^2 - f_0 c}{24\pi^2} \int R \sqrt{g} \, d^4 x 
+ \frac{f_0}{10 \pi^2} \int \left( \frac{11}{6} R^* R^* - 3 C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} \right) \sqrt{g} \, d^4 x
+ \left( -2 a f_2 \Lambda^2 + e f_0 \right) \int |\varphi|^2 \sqrt{g} \, d^4 x 
+ \frac{f_0 a}{2 \pi^2} \int |D_\mu \varphi|^2 \sqrt{g} \, d^4 x 
- \frac{f_0 a}{12 \pi^2} \int R |\varphi|^2 \sqrt{g} \, d^4 x 
+ \frac{f_0 b}{2 \pi^2} \int |\varphi|^4 \sqrt{g} \, d^4 x 
+ \frac{f_0}{2 \pi^2} \int \left( g_3^2 G^i_{\mu\nu} G^{\mu\nu i} + g_2^2 F^\alpha_{\mu\nu} F^{\mu\nu\alpha} + \frac{5}{3} g_1^2 B_{\mu\nu} B^{\mu\nu} \right) \sqrt{g} \, d^4 x, \]
Parameters:

- $f_0$, $f_2$, $f_4$ free parameters, $f_0 = f(0)$ and, for $k > 0$,

  $f_k = \int_0^\infty f(\nu)\nu^{k-1}d\nu.$

- $a, b, c, d, e$ functions of Yukawa parameters of SM+r.h.$\nu$

\[
\begin{align*}
  a &= \text{Tr}(Y_\nu^\dagger Y_\nu + Y_e^\dagger Y_e + 3(Y_u^\dagger Y_u + Y_d^\dagger Y_d)) \\
  b &= \text{Tr}((Y_\nu^\dagger Y_\nu)^2 + (Y_e^\dagger Y_e)^2 + 3(Y_u^\dagger Y_u)^2 + 3(Y_d^\dagger Y_d)^2) \\
  c &= \text{Tr}(MM^\dagger) \\
  d &= \text{Tr}((MM^\dagger)^2) \\
  e &= \text{Tr}(MM^\dagger Y_\nu^\dagger Y_\nu).
\end{align*}
\]
Normalization and coefficients

\[
S = \frac{1}{2\kappa_0^2} \int R \sqrt{g} \, d^4x + \gamma_0 \int \sqrt{g} \, d^4x + \alpha_0 \int C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} \sqrt{g} \, d^4x + \tau_0 \int R^* R^* \sqrt{g} \, d^4x + \frac{1}{2} \int |DH|^2 \sqrt{g} \, d^4x - \mu_0^2 \int |H|^2 \sqrt{g} \, d^4x - \xi_0 \int R |H|^2 \sqrt{g} \, d^4x + \lambda_0 \int |H|^4 \sqrt{g} \, d^4x + \frac{1}{4} \int (G_{\mu\nu}^i G^{\mu\nu i} + F_{\mu\nu}^\alpha F^{\mu\nu\alpha} + B_{\mu\nu} B^{\mu\nu}) \sqrt{g} \, d^4x,
\]

Energy scale: Unification (10^{15} – 10^{17} GeV)

\[
g^2 f_0 \over 2\pi^2 = \frac{1}{4}
\]

Preferred energy scale, unification of coupling constants
Coefficients

\[
\frac{1}{2\kappa_0^2} = \frac{96f_2\Lambda^2 - f_0c}{24\pi^2}
\]

\[
\gamma_0 = \frac{1}{\pi^2} \left(48f_4\Lambda^4 - f_2\Lambda^2c + \frac{f_0d}{4}\right)
\]

\[
\alpha_0 = -\frac{3f_0}{10\pi^2}
\]

\[
\tau_0 = \frac{11f_0}{60\pi^2}
\]

\[
\mu_0^2 = 2\frac{f_2\Lambda^2}{f_0} - \frac{e}{a}
\]

\[
\xi_0 = \frac{1}{12}
\]

\[
\lambda_0 = \frac{\pi^2b}{2f_0a^2}
\]
Renormalization group equations for SM with right handed neutrinos and Majorana mass terms, from unification energy ($2 \times 10^{16}$ GeV) down to the electroweak scale ($10^2$ GeV)

AKLRS S. Antusch, J. Kersten, M. Lindner, M. Ratz, M.A. Schmidt

*Running neutrino mass parameters in see-saw scenarios*, JHEP 03 (2005) 024.

Remark: RGE analysis in [CCM] only done using minimal SM
1-loop RGE equations: \( \Lambda \frac{df}{d\Lambda} = \beta_f(\Lambda) \)

\[
16\pi^2 \beta_{g_i} = b_i g_i^3 \quad \text{with} \quad (b_{SU(3)}, b_{SU(2)}, b_{U(1)}) = (-7, -\frac{19}{6}, \frac{41}{10})
\]

\[
16\pi^2 \beta_{Y_u} = Y_u(\frac{3}{2} Y_u^\dagger Y_u - \frac{3}{2} Y_d^\dagger Y_d + a - \frac{17}{20} g_1^2 - \frac{9}{4} g_2^2 - 8g_3^2)
\]

\[
16\pi^2 \beta_{Y_d} = Y_d(\frac{3}{2} Y_d^\dagger Y_d - \frac{3}{2} Y_u^\dagger Y_u + a - \frac{1}{4} g_1^2 - \frac{9}{4} g_2^2 - 8g_3^2)
\]

\[
16\pi^2 \beta_{Y_\nu} = Y_\nu(\frac{3}{2} Y_\nu^\dagger Y_\nu - \frac{3}{2} Y_e^\dagger Y_e + a - \frac{9}{20} g_1^2 - \frac{9}{4} g_2^2)
\]

\[
16\pi^2 \beta_{Y_e} = Y_e(\frac{3}{2} Y_e^\dagger Y_e - \frac{3}{2} Y_\nu^\dagger Y_\nu + a - \frac{9}{4} g_1^2 - \frac{9}{4} g_2^2)
\]

\[
16\pi^2 \beta_{M} = Y_\nu Y_\nu^\dagger M + M(Y_\nu Y_\nu^\dagger)^T
\]

\[
16\pi^2 \beta_{\lambda} = 6\lambda^2 - 3\lambda(3g_2^2 + \frac{3}{5} g_1^2) + 3g_4^2 + \frac{3}{2}(\frac{3}{5} g_1^2 + g_2^2)^2 + 4\lambda a - 8b
\]

Note: different normalization from [CCM] and 5/3 factor included in \( g_1^2 \)
Method of AKLRS: non-degenerate spectrum of Majorana masses, different effective field theories in between the three see-saw scales:

- RGE from unification $\Lambda_{\text{unif}}$ down to first see-saw scale (largest eigenvalue of $M$)
- Introduce $Y_{\nu}^{(3)}$ removing last row of $Y_{\nu}$ in basis where $M$ diagonal and $M^{(3)}$ removing last row and column.
- Induced RGE down to second see-saw scale
- Introduce $Y_{\nu}^{(2)}$ and $M^{(2)}$, matching boundary conditions
- Induced RGE down to first see-saw scale
- Introduce $Y_{\nu}^{(1)}$ and $M^{(1)}$, matching boundary conditions
- Induced RGE down to electroweak energy $\Lambda_{\text{ew}}$

Use effective field theories $Y_{\nu}^{(N)}$ and $M^{(N)}$ between see-saw scales
Running of coefficients $a, b$ with RGE

Coefficients $a$ and $b$ near the top see-saw scale
Running of coefficient $c$ with RGE

Effect of the three see-saw scales
Running of coefficient $d$ with RGE

Effect of the three see-saw scales
Running of coefficient $\epsilon$ with RGE

Effect of the three see-saw scales
Cosmology timeline

- Planck epoch: $t \leq 10^{-43}$ s after the Big Bang (unification of forces with gravity, quantum gravity)
- Grand Unification epoch: $10^{-43} s \leq t \leq 10^{-36}$ s (electroweak and strong forces unified; Higgs)
- Electroweak epoch: $10^{-36} s \leq t \leq 10^{-12}$ s (strong and electroweak forces separated)
- Inflationary epoch: possibly $10^{-36} s \leq t \leq 10^{-32}$ s

- NCG SM preferred scale at unification; RGE running between unification and electroweak $\Rightarrow$ info on inflationary epoch.
- Remark: Cannot extrapolate to modern universe, nonperturbative effects in the spectral action
Cosmological implications of the NCG SM

- Linde’s hypothesis (antigravity in the early universe)
- Primordial black holes and gravitational memory
- Gravitational waves in modified gravity
- Gravity balls
- Varying effective cosmological constant
- Higgs based slow-roll inflation
- Spontaneously arising Hoyle-Narlikar in EH backgrounds
Effective gravitational constant

\[ G_{\text{eff}} = \frac{\kappa_0^2}{8\pi} = \frac{3\pi}{192 f_2 \Lambda^2 - 2 f_0 c(\Lambda)} \]

Effective cosmological constant

\[ \gamma_0 = \frac{1}{4\pi^2} (192 f_4 \Lambda^4 - 4 f_2 \Lambda^2 c(\Lambda) + f_0 d(\Lambda)) \]
Conformal non-minimal coupling of Higgs and gravity

\[ \frac{1}{16\pi G_{\text{eff}}} \int R \sqrt{g} d^4x - \frac{1}{12} \int R |H|^2 \sqrt{g} d^4x \]

Conformal gravity

\[ -\frac{3f_0}{10\pi^2} \int C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} \sqrt{g} d^4x \]

\( C^{\mu\nu\rho\sigma} = \) Weyl curvature tensor (trace free part of Riemann tensor)
An example: \( G_{\text{eff}}(\Lambda_{\text{ew}}) = G \) (at electroweak phase transition \( G_{\text{eff}} \) is already modern universe Newton constant)

\[
1/\sqrt{G} = 1.22086 \times 10^{19} \text{ GeV} \Rightarrow f_2 = 7.31647 \times 10^{32}
\]

\[
G_{\text{eff}}^{-1}(\Lambda) \sim \frac{96 f_2 \Lambda^2}{24 \pi^2}
\]

Term \( \epsilon/\alpha \) lower order

Dominant terms in the spectral action:

\[
\Lambda^2 \left( \frac{1}{2\tilde{\kappa}_0^2} \int R \sqrt{g} d^4x - \tilde{\mu}_0^2 \int |H|^2 \sqrt{g} d^4x \right)
\]

\( \tilde{\kappa}_0 = \Lambda \kappa_0 \) and \( \tilde{\mu}_0 = \mu_0/\Lambda \), where \( \mu_0^2 \sim \frac{2 f_2 \Lambda^2}{f_0} \)
Detectable by gravitational waves:

Einstein equations $R^\mu_\nu - \frac{1}{2} g^\mu_\nu R = \kappa_0^2 T^\mu_\nu$

$$g^\mu_\nu = a(t)^2 \begin{pmatrix} -1 & 0 \\ 0 & \delta_{ij} + h_{ij}(x) \end{pmatrix}$$

trace and traceless part of $h_{ij} \Rightarrow$ Friedmann equation

$$-3 \left( \frac{\dot{a}}{a} \right)^2 + \frac{1}{2} \left( 4 \left( \frac{\dot{a}}{a} \right) \dot{h} + 2\ddot{h} \right) = \frac{\tilde{\kappa}_0^2}{\Lambda^2} T_{00}$$
\( \Lambda(t) = 1/a(t) \) (\( f_2 \) large) **Inflationary epoch:** \( a(t) \sim e^{\alpha t} \)

**NCG model solutions:**

\[
h(t) = \frac{3\pi^2 T_{00}}{192 f_2 \alpha^2} e^{2\alpha t} + \frac{3\alpha}{2} t + \frac{A}{2\alpha} e^{-2\alpha t} + B
\]

**Ordinary cosmology:**

\[
\left( \frac{4\pi G T_{00}}{\alpha} + \frac{3\alpha}{2} \right) t + \frac{A}{2\alpha} e^{-2\alpha t} + B
\]

**Radiation dominated epoch:** \( a(t) \sim t^{1/2} \)

**NCG model solutions:**

\[
h(t) = \frac{4\pi^2 T_{00}}{288 f_2} t^3 + B + A \log(t) + \frac{3}{8} \log(t)^2
\]

**Ordinary cosmology:**

\[
h(t) = 2\pi G T_{00} t^2 + B + A \log(t) + \frac{3}{8} \log(t)^2
\]
Same example, special case:

\[ R \sim \frac{2\tilde{\kappa}^2_0 \tilde{\mu}^2_0 a f_0}{\pi^2} \sim 1 \quad \text{and} \quad H \sim \sqrt{a f_0 / \pi} \]

Leaves conformally coupled matter and gravity

\[
S_c = \alpha_0 \int C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} \sqrt{g} \, d^4 x + \frac{1}{2} \int |DH|^2 \sqrt{g} \, d^4 x \\
- \xi_0 \int R |H|^2 \sqrt{g} \, d^4 x + \lambda_0 \int |H|^4 \sqrt{g} \, d^4 x \\
+ \frac{1}{4} \int (G^i_{\mu\nu} G^{\mu\nu i} + F^\alpha_{\mu\nu} F^{\mu\nu\alpha} + B_{\mu\nu} B^{\mu\nu}) \sqrt{g} \, d^4 x.
\]

A Hoyle-Narlikar type cosmology, normally suppressed by dominant Einstein–Hilbert term, arises when \( R \sim 1 \) and \( H \sim v \).
Cosmological term controlled by additional parameter $f_4$, vanishing condition:

$$f_4 = \frac{4f_2\Lambda^2 - f_0 d}{192\Lambda^4}$$

Example: vanishing at unification $\gamma_0(\Lambda_{unif}) = 0$

Running of $\gamma_0(\Lambda)$: possible inflationary mechanism
The $\lambda_0$-ansatz

$$
\lambda_0|_{\Lambda=\Lambda_{\text{unif}}} = \lambda(\Lambda_{\text{unif}}) \frac{\pi^2 b(\Lambda_{\text{unif}})}{f_0 a^2(\Lambda_{\text{unif}})},
$$

- Run like $\lambda(\Lambda)$ but change boundary condition to $\lambda_0|_{\Lambda=\Lambda_{\text{unif}}}$
- Run like

$$
\lambda_0(\Lambda) = \lambda(\Lambda) \frac{\pi^2 b(\Lambda)}{f_0 a^2(\Lambda)}
$$

For most of our cosmological estimates no serious difference
The running of $\lambda_0(\Lambda)$ near the top see-saw scale.
Tongue-in-cheek remark:

- Higgs mass estimate in [CCM] from low energy limit of $\lambda$ (running with RGE of minimal SM)

$$\sqrt{2\lambda} \frac{2M}{g} \sim 170 \text{ GeV}$$

Higgs vacuum $2M/g \sim 246$ GeV

- Estimate using the ansatz for $\lambda_0(\Lambda)$:

$$\sqrt{2\lambda_0} \frac{2M}{g} < 158 \text{ GeV}$$

Tevatron collaboration: projected window of exclusion for the Higgs starts at 158 GeV
Linde’s hypothesis antigravity in the early universe


Based on a conformal coupling

\[
\frac{1}{16\pi G} \int R \sqrt{g} d^4x - \frac{1}{12} \int R \phi^2 \sqrt{g} d^4x
\]

giving an effective

\[
G_{\text{eff}}^{-1} = G^{-1} - \frac{4}{3} \pi \phi^2
\]

In the NCG SM model two sources of negative gravity

- Running of \(G_{\text{eff}}(\Lambda)\)
- Conformal coupling to the Higgs field
The effective gravitational constant surface

The surface $G_{\text{eff}}(\Lambda, f_2)$
Effective $G_{\text{eff}}^{-1}(\Lambda, f_2)$ near the top see-saw scale
Example: fixing $G_{\text{eff}}(\Lambda_{\text{unif}}) = G$ (as in [CCM] but different RGE)

\begin{align*}
1.1 \times 10^{12} & \\
1.2 \times 10^{12} & \\
1.3 \times 10^{12} & \\
1.4 \times 10^{12} & \\
1.5 \times 10^{12} & \\
-2 \times 10^{29} & \\
-1 \times 10^{29} & \\
1 \times 10^{29} & \\
2 \times 10^{29} & \\
\end{align*}

gives an example of negative gravity regime with conformal gravity becoming dominant near sign change of $G_{\text{eff}}(\Lambda)^{-1}$ at $\sim 10^{12}$ GeV
Gravity balls (or “Space Balls”)

\[ G_{\text{eff}, H} = \frac{G_{\text{eff}}}{1 - \frac{4\pi}{3} G_{\text{eff}} |H|^2} \]

combines running of \( G_{\text{eff}} \) with Linde mechanism

Suppose \( f_2 \) such that \( G_{\text{eff}}(\Lambda) > 0 \)

\[
\begin{cases}
G_{\text{eff}, H} < 0 & \text{for } |H|^2 > \frac{3}{4\pi G_{\text{eff}}(\Lambda)}, \\
G_{\text{eff}, H} > 0 & \text{for } |H|^2 < \frac{3}{4\pi G_{\text{eff}}(\Lambda)}.
\end{cases}
\]

Unstable and stable equilibrium for \( H \):

\[ \ell_H(\Lambda, f_2) := \frac{\mu_0^2}{2\lambda_0}(\Lambda) = \frac{2 f_2 \Lambda^2}{f_0} - \frac{\varepsilon(\Lambda)}{a(\Lambda)} = \frac{(2 f_2 \Lambda^2 a(\Lambda) - f_0 \varepsilon(\Lambda))a(\Lambda)}{\pi^2 \lambda(\Lambda)b(\Lambda)} \]

(with \( \lambda_0 \)-ansatz)
Surface $\ell_H(\Lambda, f_2)$ near the top see-saw scale

Negative gravity regime where

$$\ell_H(\Lambda, f_2) > \frac{3}{4\pi G_{\text{eff}}(\Lambda, f_2)}$$
An example of transition to negative gravity

Gravity balls: regions where $|H|^2 \sim 0$ unstable equilibrium (positive gravity) surrounded by region with $|H|^2 \sim \ell_H(\Lambda, f_2)$ stable (negative gravity): possible model of dark energy
Primordial black holes (Zeldovich–Novikov, 1967)


Caused by: collapse of overdense regions, phase transitions in the early universe, cosmic loops and strings, inflationary reheating, etc

**Gravitational memory**: if gravity balls with different $G_{\text{eff},H}$ primordial black holes can evolve with different $G_{\text{eff},H}$ from surrounding space
Evaporation of PBHs by Hawking radiation

\[
\frac{dM(t)}{dt} \sim -(G_{\text{eff}}(t)M(t))^{-2}
\]

with Hawking temperature \( T = (8\pi G_{\text{eff}}(t)M(t))^{-1} \). In terms of energy:

\[
M^{2} dM = \frac{1}{\Lambda^{2} G_{\text{eff}}^{2}(\Lambda, f_{2})} d\Lambda
\]

Without gravitational memory:

\[
M(\Lambda, f_{2}) = 3 \sqrt{M^{3}(\Lambda_{in}) - \frac{2}{3\pi^{2}} \int_{\Lambda}^{\Lambda_{in}} \frac{(192 f_{2} x^{2} - 2 f_{0} c(x))^{2}}{x^{3}} dx}
\]

With gravitational memory:

\[
M(\Lambda, f_{2}) = 3 \sqrt{M^{3}(\Lambda_{in}) - \frac{2}{3\pi^{2}} \int_{\Lambda}^{\Lambda_{in}} \frac{(1 - \frac{4\pi}{3} G_{\text{eff}}(x)|H|^{2})^{2}}{x^{3} G_{\text{eff}}(x)^{2}} dx}
\]

Evaporation of PBHs linked to \( \gamma \)-ray bursts
Higgs based slow-roll inflation


Minimal SM and non-minimal coupling of Higgs and gravity. Non-conformal coupling $\xi_0 \neq 1/12$, running of $\xi_0$
Effective Higgs potential: inflation parameter $\psi = \sqrt{\xi_0 \kappa_0} |H|$

inflationary period $\psi \gg 1$, end of inflation $\psi \sim 1$, low energy regime $\psi \ll 1$
In the NCG SM have $\xi_0 = 1/12$ but same Higgs based slow-roll inflation due to $\kappa_0$ running (say $\kappa_0 > 0$)

$$\psi(\Lambda) = \sqrt{\xi_0(\Lambda)\kappa_0(\Lambda)}|H| = \sqrt{\frac{\pi^2}{96f_2\Lambda^2 - f_0c(\Lambda)}}|H|$$

Einstein metric $g^{E}_{\mu\nu} = f(H)g_{\mu\nu}$, for $f(H) = 1 + \xi_0\kappa_0|H|^2$

Higgs potential

$$V_E(H) = \frac{\lambda_0|H|^4}{(1 + \xi_0\kappa_0^2|H|^2)^2}$$

For $\psi >> 1$ approaches constant

$$V_E = \frac{\lambda_0(\Lambda)}{4\xi_0^2(\Lambda)\kappa_0^4(\Lambda)} = \frac{\lambda(\Lambda)b(\Lambda)(96f_2\Lambda^2 - f_0c(\Lambda))^2}{4f_0a^2(\Lambda)}$$

usual quartic potential for $\psi << 1$
Running of $V_E$ (for different values of $f_2$)

near the top see-saw scale
Slow roll parameters \( V_E(s) = \frac{\lambda_0 s^4}{(1+\xi_0 \kappa_0^2 s^2)^2} \)

\[
C(s) := \frac{1}{2(1 + \xi_0 \kappa_0^2 s^2)} + \frac{3}{2\kappa_0^2} \frac{(2\xi_0 \kappa_0^2 s)^2}{(1 + \xi_0 \kappa_0^2 s^2)^2}
\]

\[
\varepsilon(s) = \frac{1}{2\kappa_0^2} \left( \frac{V'_E(s)}{V_E(s)} \right)^2 C(s)^{-1} = \frac{16\kappa_0^2}{s^2 + \xi_0 \kappa_0^2(1 + (\kappa_0^2)^2)s^4}
\]

\[
\eta(s) = \frac{1}{\kappa_0^2} \left( \frac{V''_E(s)}{V_E(s)} C(s)^{-1} - \frac{V'_E(s)}{V_E(s)} C(s)^{-3/2} \frac{d}{ds} C(s)^{1/2} \right)
\]

\[
= \frac{8(3 + \xi_0 \kappa_0^2 s^2(1 - 2\xi_0 \kappa_0^2(6s^2 + \kappa_0^2(2 + 3(\kappa_0^2)^2)s^2 + (1 + (\kappa_0^2)^2)s^4))))}{\kappa_0^2(s + \xi_0 \kappa_0^2(1 + (\kappa_0^2)^2)s^3)^2}
\]

Spectral index and tensor to scalar ratio

\[
n_s = 1 + \frac{32(216 + \kappa_0^2(6s^2 - \kappa_0^2(432 + 12\kappa_0^2(2 + 3(\kappa_0^2)^2)s^2 + (1 + (\kappa_0^2)^2)s^4))))}{\kappa_0^2(12s + \kappa_0^2(1 + (\kappa_0^2)^2)s^3)^2}
\]

\[
r = \frac{256\kappa_0^2}{s^2 + \frac{\kappa_0^2}{12}(1 + (\kappa_0^2)^2)s^4}
\]
Currently under investigation

- Varying boundary conditions for the RGE flow
- Dark matter models with Majorana see-saw (Kusenko, Shaposhnikov–Tkachev)
- Other models of $G_{\text{eff}}$ runnings (Buchbinder–Odintsov–Shapiro, Dou–Percacci, Reuter, Hamber–Williams)
- NCG SM with dilaton field
- Nonperturbative effects in the spectral action
- Quantum corrections from field vacua