Noncommutative Geometry and Cosmology arXiv:0908.3683

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Joint work with Elena Pierpaoli (Astronomy, USC/Caltech), 2009

The NCG standard model

CCM A. Chamseddine, A. Connes, M. Marcolli, *Gravity and the standard model with neutrino mixing*, Adv. Theor. Math. Phys. 11 (2007), no. 6, 991–1089.

The noncommutative space $X \times F$

product of 4-dim spacetime and finite NC space

The spectral action functional

$$\mathrm{Tr}(f(D_A/\Lambda)) + \frac{1}{2}\langle J\tilde{\xi}, D_A\tilde{\xi}\rangle$$

$D_A = D + A + \varepsilon' J A J^{-1}$ Dirac operator with inner fluctuations

$A = A^* = \sum_k a_k[D, b_k]$

Spectral triples $(\mathcal{A}, \mathcal{H}, D)$:

- involutive algebra $\mathcal{A}$
- representation $\pi : \mathcal{A} \to \mathcal{L}(\mathcal{H})$
- self adjoint operator D on $\mathcal{H}$
- compact resolvent $(1 + D^2)^{-1/2} \in \mathcal{K}$
- $[a, D]$ bounded $\forall a \in \mathcal{A}$
- even $\mathbb{Z}/2$-grading $[\gamma, a] = 0$ and $D\gamma = -\gamma D$
- real structure: antilinear isom $J : \mathcal{H} \to \mathcal{H}$ with $J^2 = \varepsilon$, $JD = \varepsilon' DJ$, and $J\gamma = \varepsilon'' \gamma J$

n	0	1	2	3	4	5	6	7
ε	1	1	-1	-1	-1	-1	1	1
ε'	1	-1	1	1	1	-1	1	1
ε''	1		-1		1		-1	

- bimodule: $[a, b^0] = 0$ for $b^0 = Jb^* J^{-1}$
- order one condition: $[[D, a], b^0] = 0$

Ansatz for the NC space F

$$\mathcal{A}_{LR} = \mathbb{C} \oplus \mathbb{H}_L \oplus \mathbb{H}_R \oplus M_3(\mathbb{C})$$

(or more general) $\Rightarrow$ everything else follows by *computation*

- Representation: $\mathcal{M}_F$ sum of all inequiv irred odd $\mathcal{A}_{LR}$-bimodules (fix N generations) $\mathcal{H}_F = \oplus^N \mathcal{M}_F$ fermions
- Algebra $\mathcal{A}_F = \mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$: order one condition
- F zero dimensional but KO-dim 6
- $J_F =$ matter/antimatter, $\gamma_F =$ L/R chirality
- Classification of Dirac operators (moduli spaces)

Dirac operators and Majorana mass terms

$$D(Y) = \begin{pmatrix} S & T^* \\ T & \bar{S} \end{pmatrix}, \quad S = S_1 \oplus (S_3 \otimes 1_3), \quad T = Y_R : |\nu_R\rangle \to J_F |\nu_R\rangle$$

$$S_1 = \begin{pmatrix} 0 & 0 & Y^*_{(\uparrow 1)} & 0 \\ 0 & 0 & 0 & Y^*_{(\downarrow 1)} \\ Y_{(\uparrow 1)} & 0 & 0 & 0 \\ 0 & Y_{(\downarrow 1)} & 0 & 0 \end{pmatrix}$$

$$S_3 = \begin{pmatrix} 0 & 0 & Y^*_{(\uparrow 3)} & 0 \\ 0 & 0 & 0 & Y^*_{(\downarrow 3)} \\ Y_{(\uparrow 3)} & 0 & 0 & 0 \\ 0 & Y_{(\downarrow 3)} & 0 & 0 \end{pmatrix}$$

Yukawa matrices: Dirac masses and mixing angles in $\mathrm{GL}_{N=3}(\mathbb{C})$

$Y_e = Y_{(\downarrow 1)}$ (charged leptons)

$Y_\nu = Y_{(\uparrow 1)}$ (neutrinos)

$Y_d = Y_{(\downarrow 3)}$ (d/s/b quarks)

$Y_u = Y_{(\uparrow 3)}$ (u/c/t quarks)

$M = Y_R^t$ Majorana mass terms symm matrix

Product geometry $(C^\infty(X), L^2(X,S), D_X) \cup (\mathcal{A}_F, \mathcal{H}_F, D_F)$

- $\mathcal{A} = C^\infty(X) \otimes \mathcal{A}_F = C^\infty(X, \mathcal{A}_F)$
- $\mathcal{H} = L^2(X,S) \otimes \mathcal{H}_F = L^2(X, S \otimes \mathcal{H}_F)$
- $D = D_X \otimes 1 + \gamma_5 \otimes D_F$

Asymptotic formula for the spectral action (Chamseddine–Connes)

$$\mathrm{Tr}(f(D/\Lambda)) \sim \sum_{k \in \mathrm{DimSp}} f_k \Lambda^k \fint |D|^{-k} + f(0)\zeta_D(0) + o(1)$$

for large Λ with $f_k = \int_0^\infty f(v) v^{k-1} dv$ and integration given by residues of zeta function $\zeta_D(s) = \mathrm{Tr}(|D|^{-s})$; DimSp poles of zeta functions

The asymptotic expansion of the spectral action from [CCM]

$$
\begin{aligned}
S = \; & \frac{1}{\pi^2}\left(48\, f_4\, \Lambda^4 - f_2\, \Lambda^2\, \mathfrak{c} + \frac{f_0}{4}\, \mathfrak{d}\right) \int \sqrt{g}\, d^4x \\
+ \; & \frac{96\, f_2\, \Lambda^2 - f_0\, \mathfrak{c}}{24\pi^2} \int R\, \sqrt{g}\, d^4x \\
+ \; & \frac{f_0}{10\, \pi^2} \int \left(\frac{11}{6}\, R^* R^* - 3\, C_{\mu\nu\rho\sigma}\, C^{\mu\nu\rho\sigma}\right) \sqrt{g}\, d^4x \\
+ \; & \frac{(-2\, \mathfrak{a}\, f_2\, \Lambda^2 \, + \, \mathfrak{e}\, f_0)}{\pi^2} \int |\varphi|^2\, \sqrt{g}\, d^4x \\
+ \; & \frac{f_0 \mathfrak{a}}{2\, \pi^2} \int |D_\mu \varphi|^2\, \sqrt{g}\, d^4x \\
- \; & \frac{f_0 \mathfrak{a}}{12\, \pi^2} \int R\, |\varphi|^2\, \sqrt{g}\, d^4x \\
+ \; & \frac{f_0 \mathfrak{b}}{2\, \pi^2} \int |\varphi|^4\, \sqrt{g}\, d^4x \\
+ \; & \frac{f_0}{2\, \pi^2} \int \left(g_3^2\, G^i_{\mu\nu}\, G^{\mu\nu i} + g_2^2\, F^\alpha_{\mu\nu}\, F^{\mu\nu\alpha} + \frac{5}{3}\, g_1^2\, B_{\mu\nu}\, B^{\mu\nu}\right) \sqrt{g}\, d^4x,
\end{aligned}
$$

Parameters:

- f_0, f_2, f_4 free parameters, $f_0 = f(0)$ and, for $k > 0$,

$$f_k = \int_0^\infty f(v) v^{k-1} dv.$$

- $\mathfrak{a}, \mathfrak{b}, \mathfrak{c}, \mathfrak{d}, \mathfrak{e}$ functions of Yukawa parameters of SM+r.h.ν

$$\begin{aligned}
\mathfrak{a} = & \quad \mathrm{Tr}(Y_\nu^\dagger Y_\nu + Y_e^\dagger Y_e + 3(Y_u^\dagger Y_u + Y_d^\dagger Y_d)) \\
\mathfrak{b} = & \quad \mathrm{Tr}((Y_\nu^\dagger Y_\nu)^2 + (Y_e^\dagger Y_e)^2 + 3(Y_u^\dagger Y_u)^2 + 3(Y_d^\dagger Y_d)^2) \\
\mathfrak{c} = & \quad \mathrm{Tr}(MM^\dagger) \\
\mathfrak{d} = & \quad \mathrm{Tr}((MM^\dagger)^2) \\
\mathfrak{e} = & \quad \mathrm{Tr}(MM^\dagger Y_\nu^\dagger Y_\nu).
\end{aligned}$$

Normalization and coefficients

$$
\begin{aligned}
S = & \frac{1}{2\kappa_0^2}\int R\,\sqrt{g}\,d^4x + \gamma_0\int \sqrt{g}\,d^4x \\
+ & \alpha_0\int C_{\mu\nu\rho\sigma}\,C^{\mu\nu\rho\sigma}\sqrt{g}\,d^4x + \tau_0\int R^*R^*\sqrt{g}\,d^4x \\
+ & \frac{1}{2}\int |DH|^2\sqrt{g}\,d^4x - \mu_0^2\int |H|^2\sqrt{g}\,d^4x \\
- & \xi_0\int R\,|H|^2\sqrt{g}\,d^4x + \lambda_0\int |H|^4\sqrt{g}\,d^4x \\
+ & \frac{1}{4}\int (G^i_{\mu\nu}\,G^{\mu\nu i} + F^\alpha_{\mu\nu}\,F^{\mu\nu\alpha} + B_{\mu\nu}\,B^{\mu\nu})\sqrt{g}\,d^4x,
\end{aligned}
$$

Energy scale: Unification ($10^{15} - 10^{17}$ GeV)

$$\frac{g^2 f_0}{2\pi^2} = \frac{1}{4}$$

Preferred energy scale, unification of coupling constants

Coefficients

$$\frac{1}{2\kappa_0^2} = \frac{96 f_2 \Lambda^2 - f_0 \mathfrak{c}}{24\pi^2} \qquad \gamma_0 = \frac{1}{\pi^2}(48 f_4 \Lambda^4 - f_2 \Lambda^2 \mathfrak{c} + \frac{f_0}{4}\mathfrak{d})$$

$$\alpha_0 = -\frac{3 f_0}{10\pi^2} \qquad \tau_0 = \frac{11 f_0}{60\pi^2}$$

$$\mu_0^2 = 2\frac{f_2 \Lambda^2}{f_0} - \frac{\mathfrak{e}}{\mathfrak{a}} \qquad \xi_0 = \frac{1}{12}$$

$$\lambda_0 = \frac{\pi^2 \mathfrak{b}}{2 f_0 \mathfrak{a}^2}$$

Renormalization group equations for SM with right handed neutrinos and Majorana mass terms, from unification energy (2×10^{16} GeV) down to the electroweak scale (10^2 GeV)

AKLRS S. Antusch, J. Kersten, M. Lindner, M. Ratz, M.A. Schmidt *Running neutrino mass parameters in see-saw scenarios*, JHEP 03 (2005) 024.

Remark: RGE analysis in [CCM] only done using minimal SM

1-loop RGE equations: $\Lambda \frac{df}{d\Lambda} = \beta_f(\Lambda)$

$$16\pi^2\ \beta_{g_i} = b_i\, g_i^3 \quad \text{with } (b_{SU(3)}, b_{SU(2)}, b_{U(1)}) = (-7, -\frac{19}{6}, \frac{41}{10})$$

$$16\pi^2\ \beta_{Y_u} = Y_u(\frac{3}{2} Y_u^\dagger Y_u - \frac{3}{2} Y_d^\dagger Y_d + \mathfrak{a} - \frac{17}{20} g_1^2 - \frac{9}{4} g_2^2 - 8 g_3^2)$$

$$16\pi^2\ \beta_{Y_d} = Y_d(\frac{3}{2} Y_d^\dagger Y_d - \frac{3}{2} Y_u^\dagger Y_u + \mathfrak{a} - \frac{1}{4} g_1^2 - \frac{9}{4} g_2^2 - 8 g_3^2)$$

$$16\pi^2\ \beta_{Y_\nu} = Y_\nu(\frac{3}{2} Y_\nu^\dagger Y_\nu - \frac{3}{2} Y_e^\dagger Y_e + \mathfrak{a} - \frac{9}{20} g_1^2 - \frac{9}{4} g_2^2)$$

$$16\pi^2\ \beta_{Y_e} = Y_e(\frac{3}{2} Y_e^\dagger Y_e - \frac{3}{2} Y_\nu^\dagger Y_\nu + \mathfrak{a} - \frac{9}{4} g_1^2 - \frac{9}{4} g_2^2)$$

$$16\pi^2\ \beta_M = Y_\nu Y_\nu^\dagger M + M(Y_\nu Y_\nu^\dagger)^T$$

$$16\pi^2\ \beta_\lambda = 6\lambda^2 - 3\lambda(3 g_2^2 + \frac{3}{5} g_1^2) + 3 g_2^4 + \frac{3}{2}(\frac{3}{5} g_1^2 + g_2^2)^2 + 4\lambda\mathfrak{a} - 8\mathfrak{b}$$

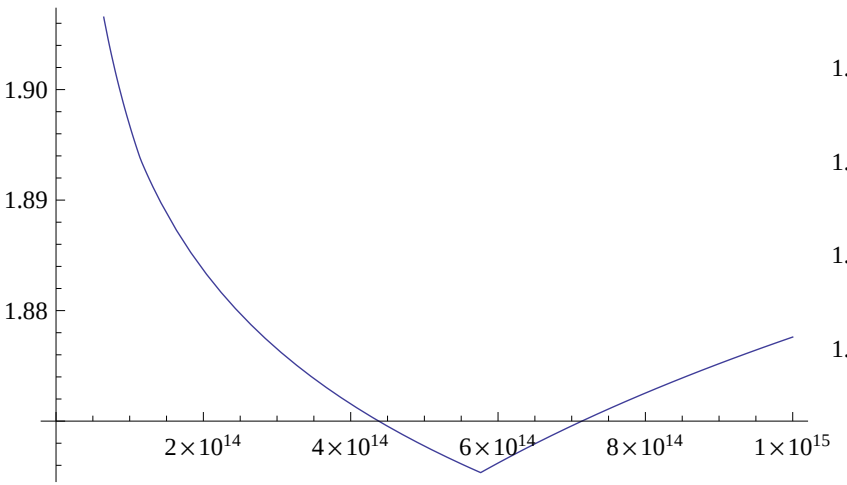
Note: different normalization from [CCM] and 5/3 factor included in g_1^2

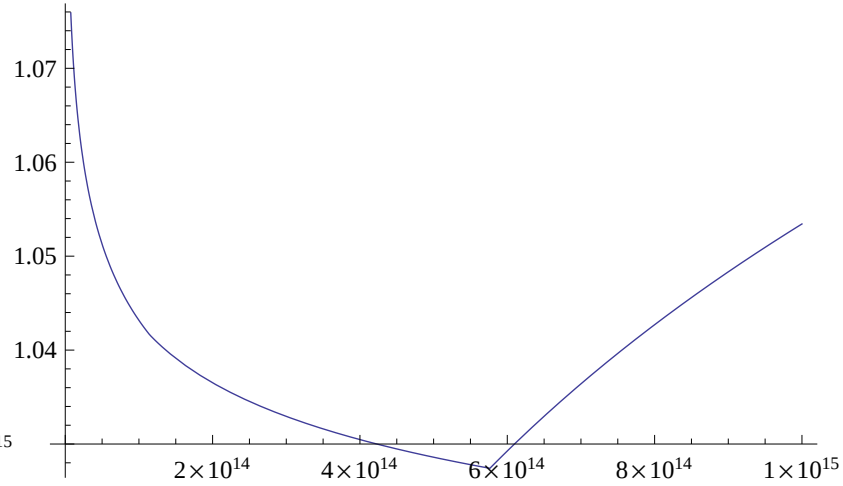
Method of AKLRS: non-degenerate spectrum of Majorana masses, different effective field theories in between the three see-saw scales:

- RGE from unification Λ_{unif} down to first see-saw scale (largest eigenvalue of M)
- Introduce $Y_\nu^{(3)}$ removing last row of Y_ν in basis where M diagonal and $M^{(3)}$ removing last row and column.
- Induced RGE down to second see-saw scale
- Introduce $Y_\nu^{(2)}$ and $M^{(2)}$, matching boundary conditions
- Induced RGE down to first see-saw scale
- Introduce $Y_\nu^{(1)}$ and $M^{(1)}$, matching boundary conditions
- Induced RGE down to electoweak energy Λ_{ew}

Use effective field theories $Y_\nu^{(N)}$ and $M^{(N)}$ between see-saw scales

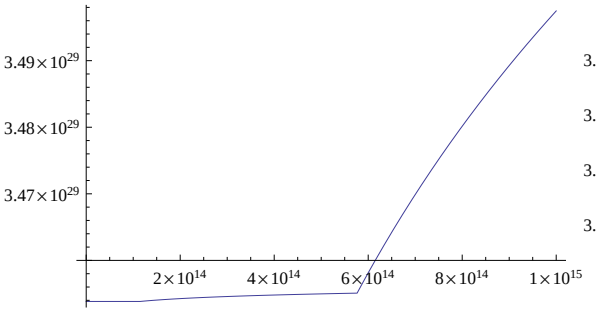
Running of coefficients $\mathfrak{a}, \mathfrak{b}$ with RGE

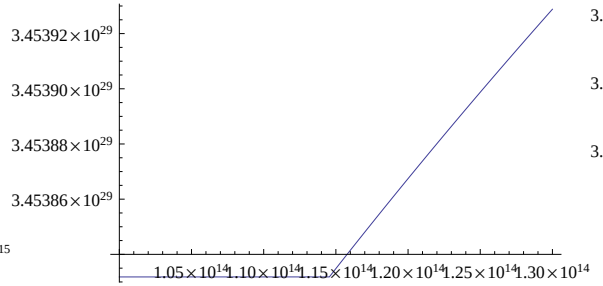


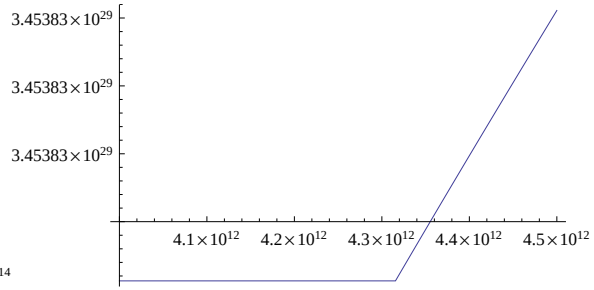


Coefficients $\mathfrak{a}$ and $\mathfrak{b}$ near the top see-saw scale

Running of coefficient $\mathfrak{c}$ with RGE

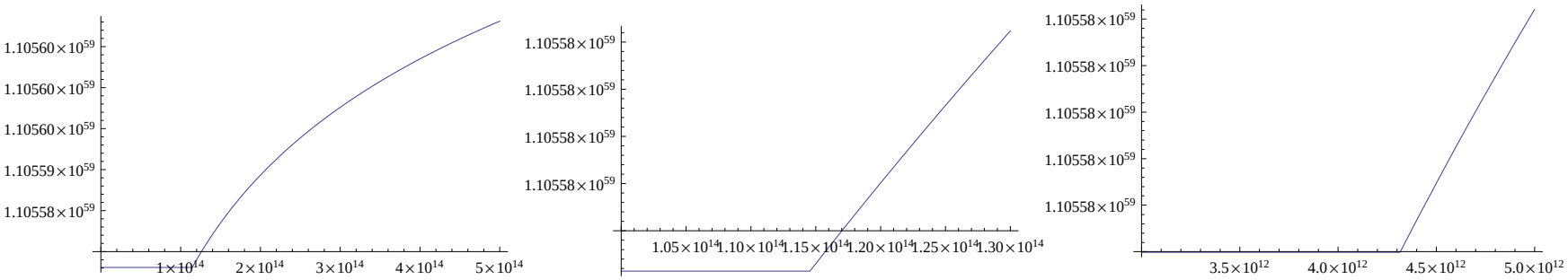






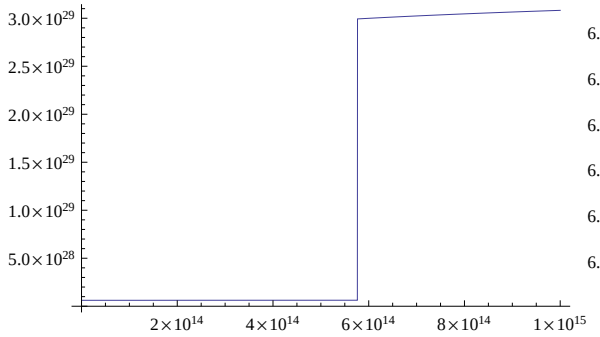
Effect of the three see-saw scales

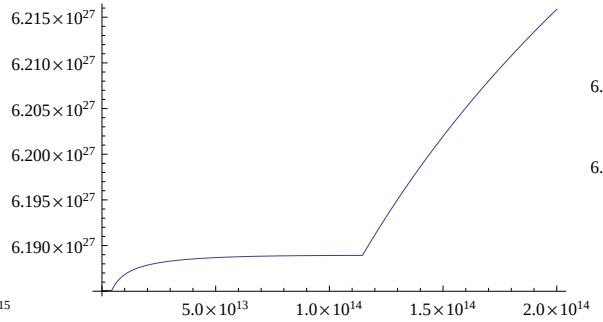
Running of coefficient $\mathfrak{d}$ with RGE

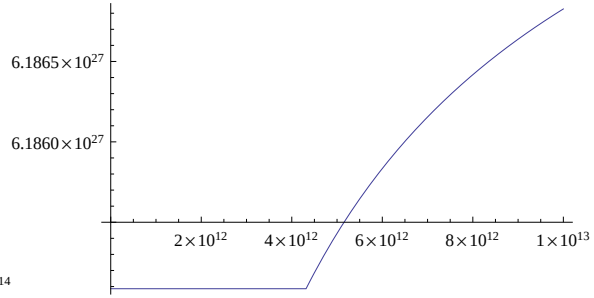


Effect of the three see-saw scales

Running of coefficient $\mathfrak{e}$ with RGE







Effect of the three see-saw scales

Cosmology timeline

- Planck epoch: $t \leq 10^{-43}\, s$ after the Big Bang (unification of forces with gravity, quantum gravity)
- Grand Unification epoch: $10^{-43}\, s \leq t \leq 10^{-36}\, s$ (electroweak and strong forces unified; Higgs)
- Electroweak epoch: $10^{-36}\, s \leq t \leq 10^{-12}\, s$ (strong and electroweak forces separated)
- Inflationary epoch: possibly $10^{-36}\, s \leq t \leq 10^{-32}\, s$

- NCG SM preferred scale at unification; RGE running between unification and electroweak $\Rightarrow$ info on inflationary epoch.
- Remark: Cannot extrapolate to modern universe, nonperturbative effects in the spectral action

Cosmological implications of the NCG SM

- Linde's hypothesis (antigravity in the early universe)
- Primordial black holes and gravitational memory
- Gravitational waves in modified gravity
- Gravity balls
- Varying effective cosmological constant
- Higgs based slow-roll inflation
- Spontaneously arising Hoyle-Narlikar in EH backgrounds

Effective gravitational constant

$$G_{\mathrm{eff}} = \frac{\kappa_0^2}{8\pi} = \frac{3\pi}{192 f_2 \Lambda^2 - 2 f_0 \mathfrak{c}(\Lambda)}$$

Effective cosmological constant

$$\gamma_0 = \frac{1}{4\pi^2}(192 f_4 \Lambda^4 - 4 f_2 \Lambda^2 \mathfrak{c}(\Lambda) + f_0 \mathfrak{d}(\Lambda))$$

Conformal non-minimal coupling of Higgs and gravity

$$\frac{1}{16\pi G_{\rm eff}}\int R\sqrt{g}d^4x - \frac{1}{12}\int R\,|H|^2\sqrt{g}d^4x$$

Conformal gravity

$$\frac{-3f_0}{10\pi^2}\int C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}\sqrt{g}d^4x$$

$C^{\mu\nu\rho\sigma}$ = Weyl curvature tensor (trace free part of Riemann tensor)

An example: $G_{\rm eff}(\Lambda_{ew}) = G$ (at electroweak phase transition $G_{\rm eff}$ is already modern universe Newton constant)
$1/\sqrt{G} = 1.22086 \times 10^{19}$ GeV $\Rightarrow f_2 = 7.31647 \times 10^{32}$

$$G_{\rm eff}^{-1}(\Lambda) \sim \frac{96 f_2 \Lambda^2}{24\pi^2}$$

Term $\mathfrak{e}/\mathfrak{a}$ lower order
Dominant terms in the spectral action:

$$\Lambda^2 \left(\frac{1}{2\tilde{\kappa}_0^2} \int R\sqrt{g} d^4x - \tilde{\mu}_0^2 \int |H|^2 \sqrt{g} d^4x \right)$$

$\tilde{\kappa}_0 = \Lambda \kappa_0$ and $\tilde{\mu}_0 = \mu_0/\Lambda$, where $\mu_0^2 \sim \frac{2 f_2 \Lambda^2}{f_0}$

Detectable by gravitational waves:
Einstein equations $R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = \kappa_0^2 T^{\mu\nu}$

$$g_{\mu\nu} = a(t)^2 \begin{pmatrix} -1 & 0 \\ 0 & \delta_{ij} + h_{ij}(x) \end{pmatrix}$$

trace and traceless part of $h_{ij} \Rightarrow$ Friedmann equation

$$-3\left(\frac{\dot{a}}{a}\right)^2 + \frac{1}{2}\left(4\left(\frac{\dot{a}}{a}\right)\dot{h} + 2\ddot{h}\right) = \frac{\tilde{\kappa}_0^2}{\Lambda^2}\, T_{00}$$

$\Lambda(t) = 1/a(t)$ (f_2 large) Inflationary epoch: $a(t) \sim e^{\alpha t}$
NCG model solutions:

$$h(t) = \frac{3\pi^2 T_{00}}{192 f_2 \alpha^2} e^{2\alpha t} + \frac{3\alpha}{2} t + \frac{A}{2\alpha} e^{-2\alpha t} + B$$

Ordinary cosmology:

$$(\frac{4\pi G T_{00}}{\alpha} + \frac{3\alpha}{2}) t + \frac{A}{2\alpha} e^{-2\alpha t} + B$$

Radiation dominated epoch: $a(t) \sim t^{1/2}$
NCG model solutions:

$$h(t) = \frac{4\pi^2 T_{00}}{288 f_2} t^3 + B + A \log(t) + \frac{3}{8} \log(t)^2$$

Ordinary cosmology:

$$h(t) = 2\pi G T_{00} t^2 + B + A \log(t) + \frac{3}{8} \log(t)^2$$

Same example, special case:

$$R \sim \frac{2\tilde{\kappa}_0^2 \tilde{\mu}_0^2 \mathfrak{a} f_0}{\pi^2} \sim 1 \quad \text{and} \quad H \sim \sqrt{\mathfrak{a} f_0}/\pi$$

Leaves conformally coupled matter and gravity

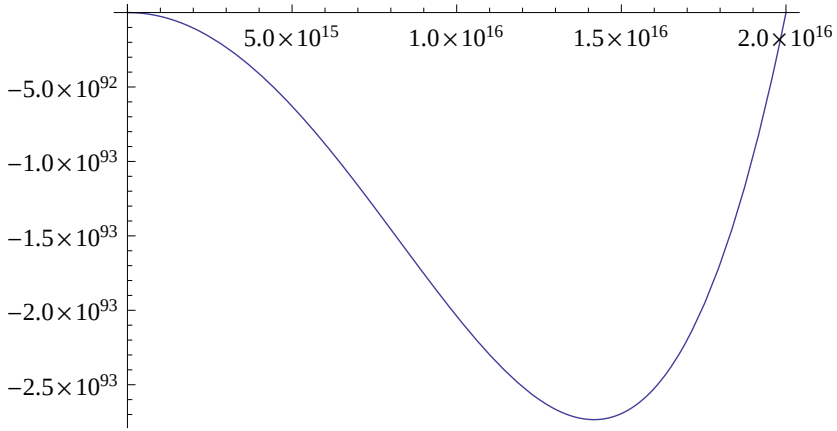
$$\begin{aligned} S_c = \quad & \alpha_0 \int C_{\mu\nu\rho\sigma}\, C^{\mu\nu\rho\sigma} \sqrt{g}\, d^4x + \frac{1}{2} \int |DH|^2 \sqrt{g}\, d^4x \\ & -\xi_0 \int R\, |H|^2 \sqrt{g}\, d^4x + \lambda_0 \int |H|^4 \sqrt{g}\, d^4x \\ & + \frac{1}{4} \int (G^i_{\mu\nu}\, G^{\mu\nu i} + F^\alpha_{\mu\nu}\, F^{\mu\nu\alpha} + B_{\mu\nu}\, B^{\mu\nu}) \sqrt{g}\, d^4x. \end{aligned}$$

A Hoyle-Narlikar type cosmology, normally suppressed by dominant Einstein–Hilbert term, arises when $R \sim 1$ and $H \sim v$.

Cosmological term controlled by additional parameter f_4, vanishing condition:

$$f_4 = \frac{(4 f_2 \Lambda^2 \mathfrak{c} - f_0 \mathfrak{d})}{192 \Lambda^4}$$

Example: vanishing at unification $\gamma_0(\Lambda_{unif}) = 0$



Running of $\gamma_0(\Lambda)$: possible inflationary mechanism

The λ_0-ansatz

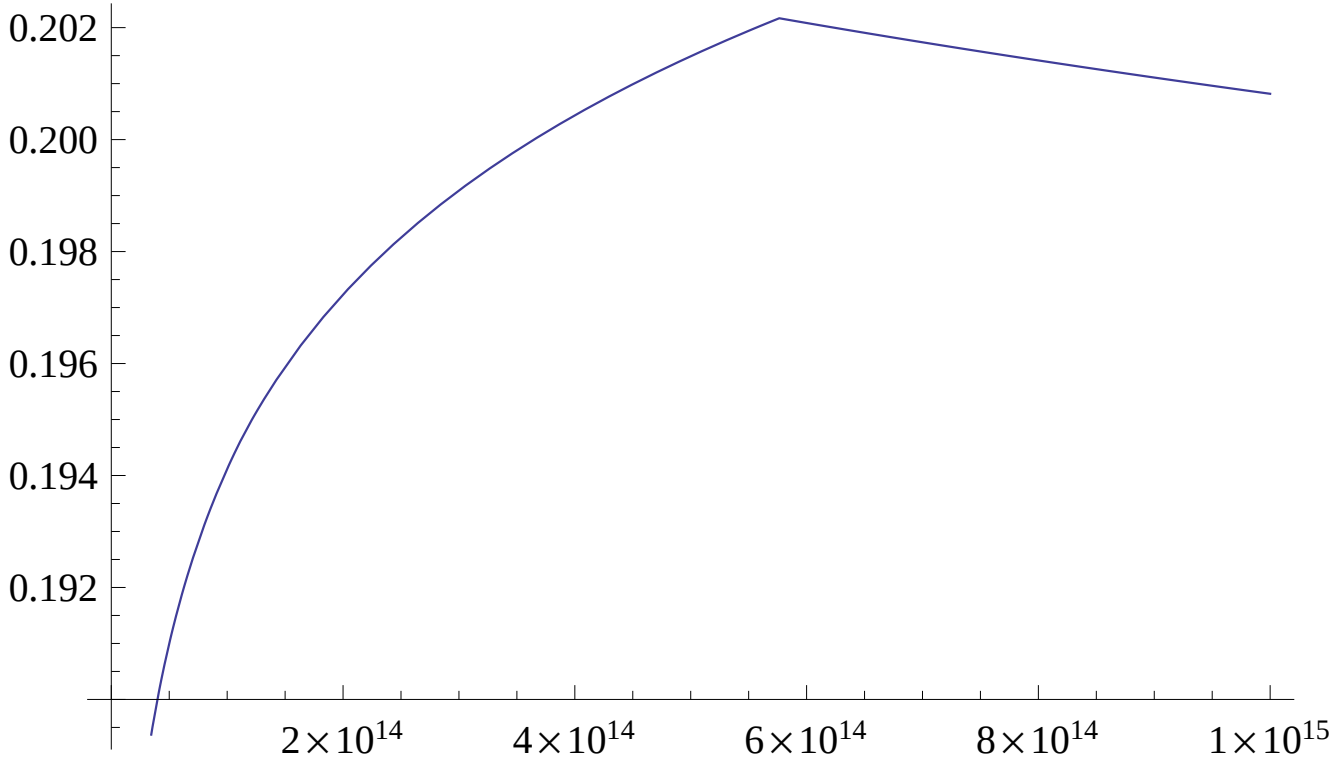
$$\lambda_0|_{\Lambda=\Lambda_{unif}} = \lambda(\Lambda_{unif})\frac{\pi^2\mathfrak{b}(\Lambda_{unif})}{f_0\mathfrak{a}^2(\Lambda_{unif})},$$

- Run like $\lambda(\Lambda)$ but change boundary condition to $\lambda_0|_{\Lambda=\Lambda_{unif}}$
- Run like

$$\lambda_0(\Lambda) = \lambda(\Lambda)\frac{\pi^2\mathfrak{b}(\Lambda)}{f_0\mathfrak{a}^2(\Lambda)}$$

For most of our cosmological estimates no serious difference

The running of $\lambda_0(\Lambda)$ near the top see-saw scale



Tongue-in-cheek remark:

- Higgs mass estimate in [CCM] from low energy limit of λ (running with RGE of minimal SM)

$$\sqrt{2\lambda}\frac{2M}{g} \sim 170\,\mathrm{GeV}$$

  Higgs vacuum $2M/g \sim 246$ GeV

- Estimate using the ansatz for $\lambda_0(\Lambda)$:

$$\sqrt{2\lambda_0}\frac{2M}{g} < 158\,\mathrm{GeV}$$

Tevatron collaboration: projected window of exclusion for the Higgs starts at 158 GeV

Linde's hypothesis antigravity in the early universe

- A.D. Linde, *Gauge theories, time-dependence of the gravitational constant and antigravity in the early universe*, Phys. Letters B, Vol.93 (1980) N.4, 394–396

Based on a conformal coupling

$$\frac{1}{16\pi G}\int R\sqrt{g}d^4x - \frac{1}{12}\int R\,\phi^2\sqrt{g}d^4x$$

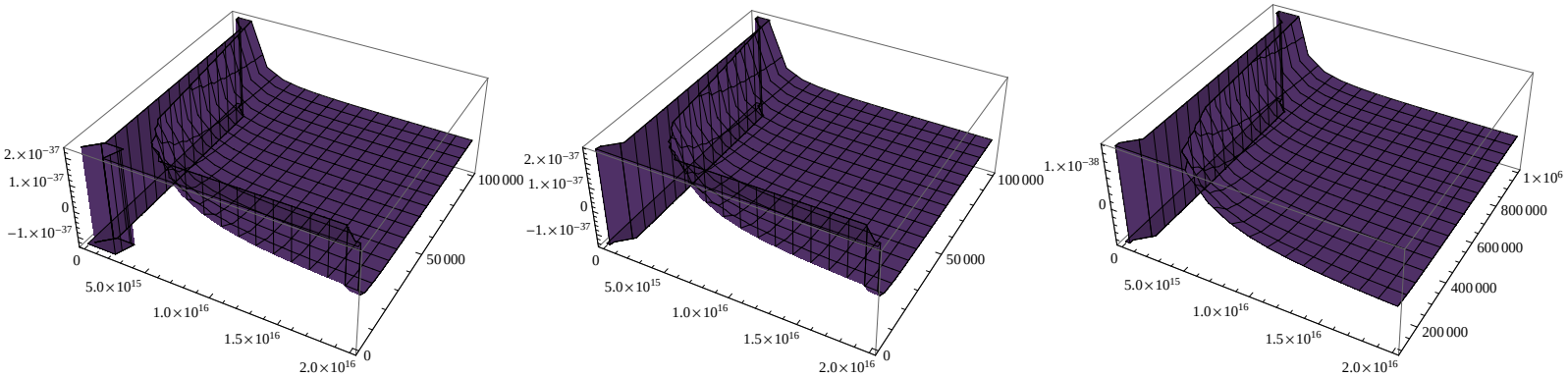
giving an effective

$$G_{\rm eff}^{-1} = G^{-1} - \frac{4}{3}\pi\phi^2$$

In the NCG SM model two sources of negative gravity

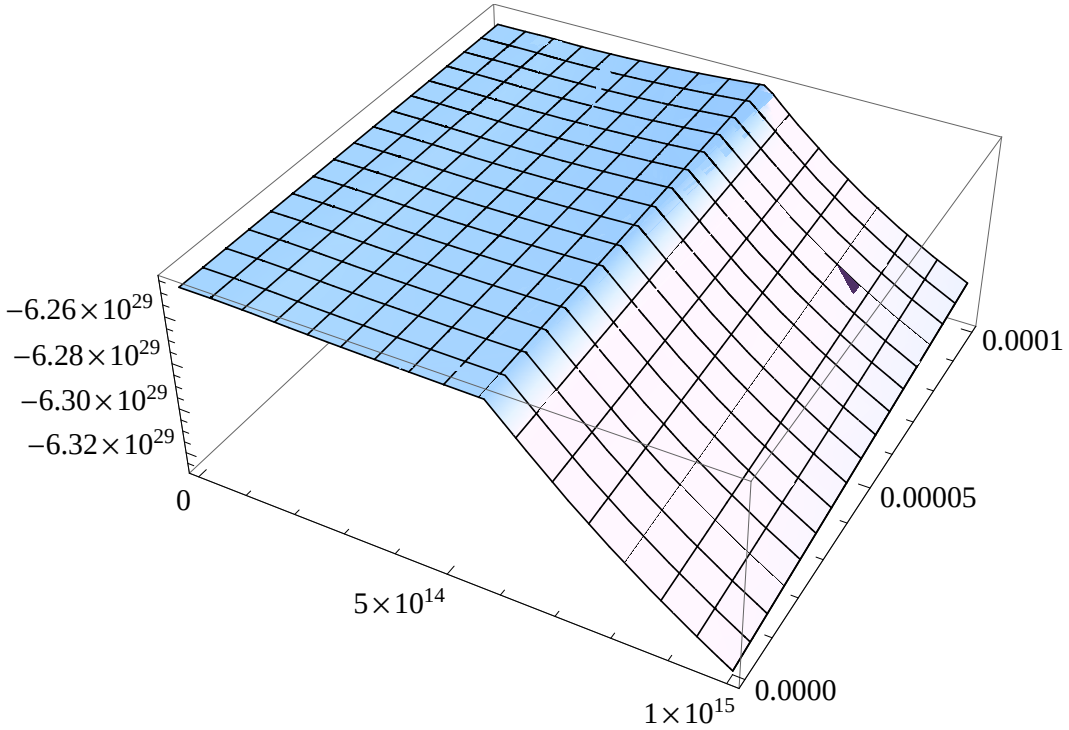
- Running of $G_{\rm eff}(\Lambda)$
- Conformal coupling to the Higgs field

The effective gravitational constant surface

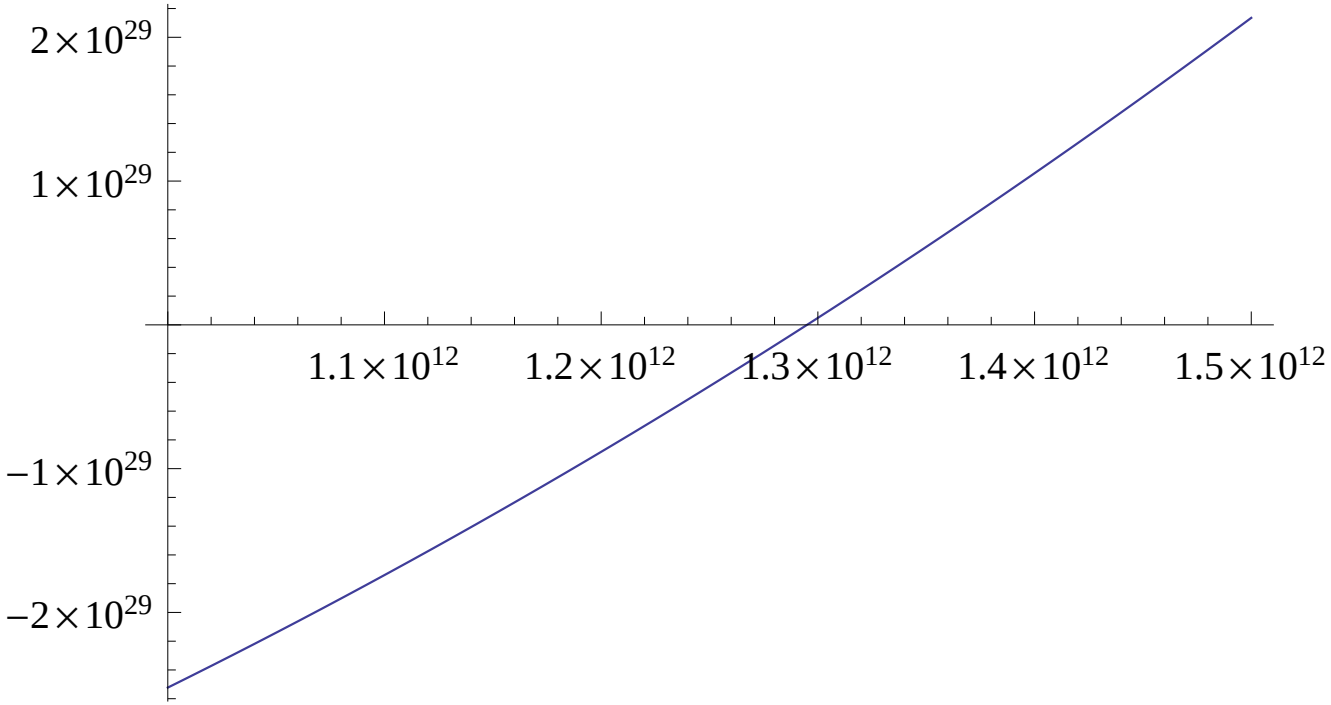


The surface $G_{\mathrm{eff}}(\Lambda, f_2)$

Effective $G_{\text{eff}}^{-1}(\Lambda, f_2)$ near the top see-saw scale



Example: fixing $G_{\rm eff}(\Lambda_{unif}) = G$ (as in [CCM] but different RGE)



gives an example of negative gravity regime with conformal gravity becoming dominant near sign change of $G_{\rm eff}(\Lambda)^{-1}$ at $\sim 10^{12}$ GeV

Gravity balls (or "Space Balls")

$$G_{\mathrm{eff},H} = \frac{G_{\mathrm{eff}}}{1 - \frac{4\pi}{3} G_{\mathrm{eff}} |H|^2}$$

combines running of G_{eff} with Linde mechanism
Suppose f_2 such that $G_{\mathrm{eff}}(\Lambda) > 0$

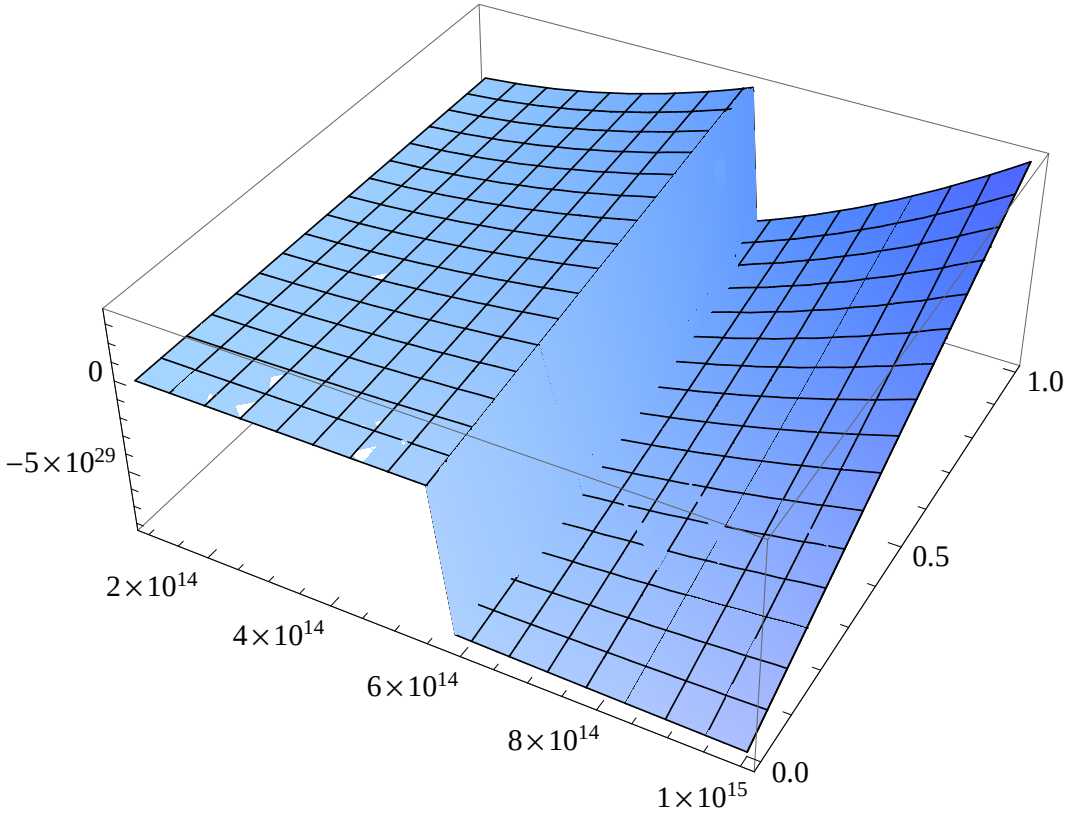
$$\begin{cases} G_{\mathrm{eff},H} < 0 \quad \text{for } |H|^2 > \dfrac{3}{4\pi G_{\mathrm{eff}}(\Lambda)}, \\ G_{\mathrm{eff},H} > 0 \quad \text{for } |H|^2 < \dfrac{3}{4\pi G_{\mathrm{eff}}(\Lambda)}. \end{cases}$$

Unstable and stable equilibrium for H:

$$\ell_H(\Lambda, f_2) := \frac{\mu_0^2}{2\lambda_0}(\Lambda) = \frac{2\frac{f_2\Lambda^2}{f_0} - \frac{\mathfrak{e}(\Lambda)}{\mathfrak{a}(\Lambda)}}{\lambda(\Lambda)\frac{\pi^2\mathfrak{b}(\Lambda)}{f_0\mathfrak{a}^2(\Lambda)}} = \frac{(2f_2\Lambda^2\mathfrak{a}(\Lambda) - f_0\mathfrak{e}(\Lambda))\mathfrak{a}(\Lambda)}{\pi^2\lambda(\Lambda)\mathfrak{b}(\Lambda)}$$

(with λ_0-ansatz)

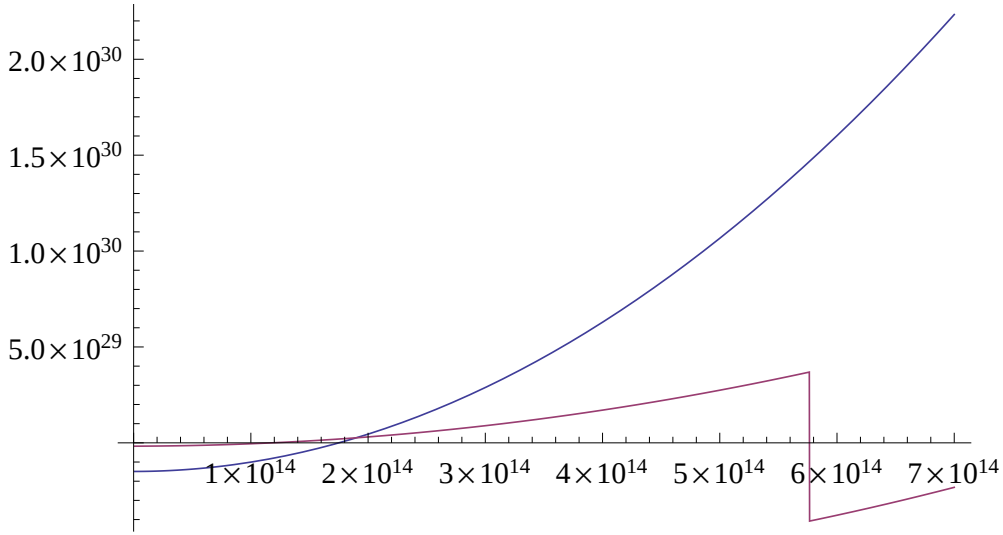
Surface $\ell_H(\Lambda, f_2)$ near the top see-saw scale



Negative gravity regime where

$$\ell_H(\Lambda, f_2) > \frac{3}{4\pi G_{\rm eff}(\Lambda, f_2)}$$

An example of transition to negative gravity



Gravity balls: regions where $|H|^2 \sim 0$ unstable equilibrium (positive gravity) surrounded by region with $|H|^2 \sim \ell_H(\Lambda, f_2)$ stable (negative gravity): possible model of dark energy

Primordial black holes (Zeldovich–Novikov, 1967)

- I.D. Novikov, A.G. Polnarev, A.A. Starobinsky, Ya.B. Zeldovich, *Primordial black holes*, Astron. Astrophys. 80 (1979) 104–109
- J.D. Barrow, *Gravitational memory?* Phys. Rev. D Vol.46 (1992) N.8 R3227, 4pp.

Caused by: collapse of overdense regions, phase transitions in the early universe, cosmic loops and strings, inflationary reheating, etc
Gravitational memory: if gravity balls with different $G_{\mathrm{eff},H}$ primordial black holes can evolve with different $G_{\mathrm{eff},H}$ from surrounding space

Evaporation of PBHs by Hawking radiation

$$\frac{d\mathcal{M}(t)}{dt} \sim -(G_{\rm eff}(t)\mathcal{M}(t))^{-2}$$

with Hawking temperature $T = (8\pi G_{\rm eff}(t)\mathcal{M}(t))^{-1}$.
In terms of energy:

$$\mathcal{M}^2\, d\mathcal{M} = \frac{1}{\Lambda^2 G_{\rm eff}^2(\Lambda, f_2)} d\Lambda$$

Without gravitational memory:

$$\mathcal{M}(\Lambda, f_2) = \sqrt[3]{\mathcal{M}^3(\Lambda_{in}) - \frac{2}{3\pi^2}\int_{\Lambda}^{\Lambda_{in}} \frac{(192 f_2 x^2 - 2 f_0 \mathfrak{c}(x))^2}{x^3}\, dx}$$

With gravitational memory:

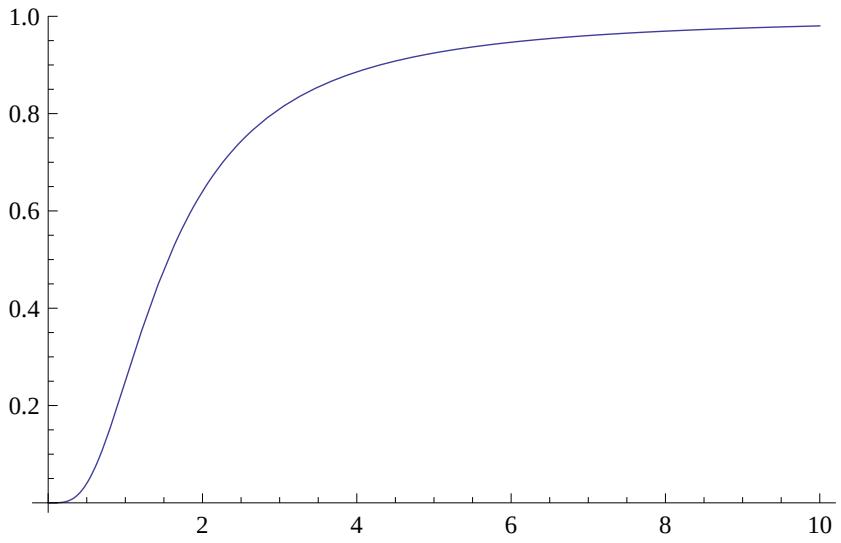
$$\mathcal{M}(\Lambda, f_2) = \sqrt[3]{\mathcal{M}^3(\Lambda_{in}) - \frac{2}{3\pi^2}\int_{\Lambda}^{\Lambda_{in}} \frac{(1 - \frac{4\pi}{3} G_{\rm eff}(x)|H|^2)^2}{x^3 G_{\rm eff}(x)^2}\, dx}$$

Evaporation of PBHs linked to γ-ray bursts

Higgs based slow-roll inflation

dSHW A. De Simone, M.P. Hertzberg, F. Wilczek, *Running inflation in the Standard Model*, hep-ph/0812.4946v2

Minimal SM and non-minimal coupling of Higgs and gravity.
Non-conformal coupling $\xi_0 \neq 1/12$, running of ξ_0
Effective Higgs potential: inflation parameter $\psi = \sqrt{\xi_0}\kappa_0|H|$



inflationary period $\psi >> 1$, end of inflation $\psi \sim 1$, low energy regime $\psi << 1$

In the NCG SM have $\xi_0 = 1/12$ but same Higgs based slow-roll inflation due to κ_0 running (say $\kappa_0 > 0$)

$$\psi(\Lambda) = \sqrt{\xi_0(\Lambda)}\kappa_0(\Lambda)|H| = \sqrt{\frac{\pi^2}{96 f_2 \Lambda^2 - f_0 \mathfrak{c}(\Lambda)}}\,|H|$$

Einstein metric $g^E_{\mu\nu} = f(H) g_{\mu\nu}$, for $f(H) = 1 + \xi_0 \kappa_0 |H|^2$
Higgs potential

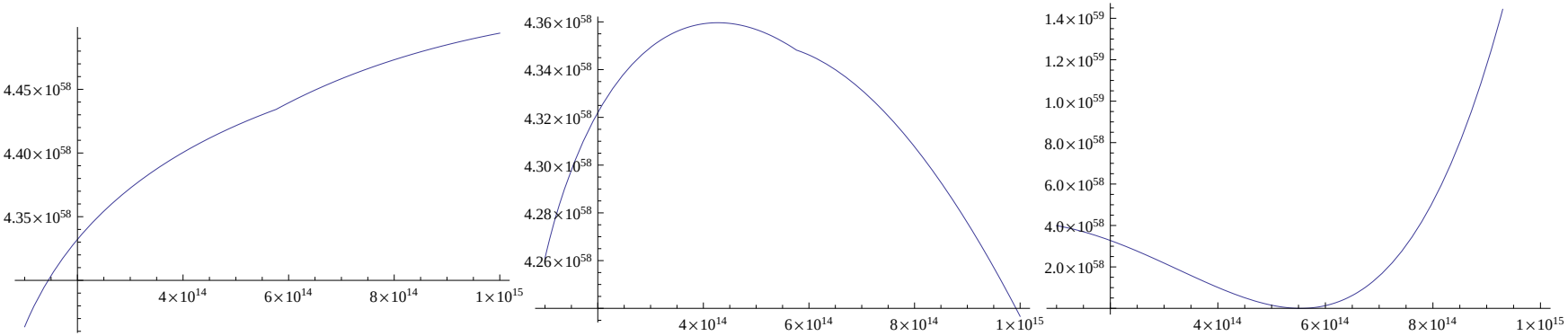
$$V_E(H) = \frac{\lambda_0 |H|^4}{(1 + \xi_0 \kappa_0^2 |H|^2)^2}$$

For $\psi >> 1$ approaches constant

$$V_E = \frac{\lambda_0(\Lambda)}{4\xi_0^2(\Lambda)\kappa_0^4(\Lambda)} = \frac{\lambda(\Lambda)\mathfrak{b}(\Lambda)(96 f_2 \Lambda^2 - f_0 \mathfrak{c}(\Lambda))^2}{4 f_0 \mathfrak{a}^2(\Lambda)}$$

usual quartic potential for $\psi << 1$

Running of V_E (for different values of f_2)



near the top see-saw scale

Slow roll parameters $V_E(s) = \frac{\lambda_0 s^4}{(1+\xi_0\kappa_0^2 s^2)^2}$

$$C(s) := \frac{1}{2(1+\xi_0\kappa_0^2 s^2)} + \frac{3}{2\kappa_0^2}\frac{(2\xi_0\kappa_0^2 s)^2}{(1+\xi_0\kappa_0^2 s^2)^2}$$

$$\epsilon(s) = \frac{1}{2\kappa_0^2}\left(\frac{V_E'(s)}{V_E(s)}\right)^2 C(s)^{-1} = \frac{16\kappa_0^2}{s^2+\xi_0\kappa_0^2(1+(\kappa_0^2)^2)s^4}$$

$$\eta(s) = \frac{1}{\kappa_0^2}\left(\frac{V_E''(s)}{V_E(s)}C(s)^{-1} - \frac{V_E'(s)}{V_E(s)}C(s)^{-3/2}\frac{d}{ds}C(s)^{1/2}\right)$$

$$= \frac{8(3+\xi_0\kappa_0^2 s^2(1-2\xi_0\kappa_0^2(s^2+12\kappa_0^2(-1+\xi_0\kappa_0^2 s^2))))}{\kappa_0^2(s+\xi_0\kappa_0^2(1+(\kappa_0^2)^2)s^3)^2}$$

Spectral index and tensor to scalar ratio

$$n_s = 1 + \frac{32(216+\kappa_0^2(6s^2-\kappa_0^2(432+12\kappa_0^2(2+3(\kappa_0^2)^2)s^2+(1+(\kappa_0^2)^2)s^4)))}{\kappa_0^2(12s+\kappa_0^2(1+(\kappa_0^2)^2)s^3)^2}$$

$$r = \frac{256\kappa_0^2}{s^2+\frac{\kappa_0^2}{12}(1+(\kappa_0^2)^2)s^4}$$

Currently under investigation

- Varying boundary conditions for the RGE flow
- Dark matter models with Majorana see-saw (Kusenko, Shaposhnikov–Tkachev)
- Other models of G_{eff} runnings (Buchbinder–Odintsov–Shapiro, Dou–Percacci, Reuter, Hamber–Williams)
- NCG SM with dilaton field
- Nonperturbative effects in the spectral action
- Quantum corrections from field vacua