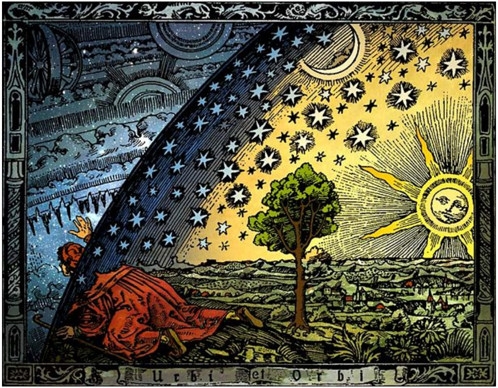
Can you hear the shape of the cosmos?

Matilde Marcolli

Revolution Books, Berkeley, 2011

Cosmology



Questions:

- Past and future evolution of the universe
- Geometry of the universe (closed, open, curved, flat)
- Forces and matter (gravity, quantum gravity, particle physics, dark matter, dark energy)

General relativity and the expansion of the universe

Einstein's general relativity:

- Not space and time but spacetime
- Gravitational fields curves spacetime
- Causality (causally connected regions, finite speed of light)
- Universe evolves: expands or constracts
  (but... cosmological constant)

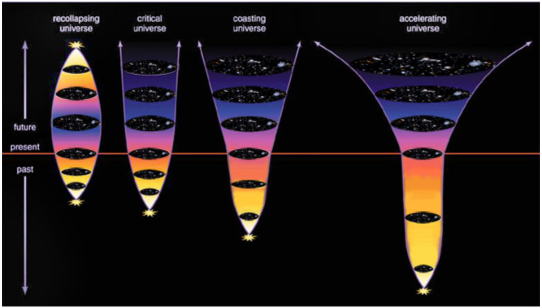
Einstein field equations for the gravitational field

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$S = \int_X \left(\frac{1}{2\kappa}(R - 2\lambda) + \mathcal{L}\right) dv, \qquad \text{with } \kappa = 8\pi G/c^4$$

Einstein–Hilbert action

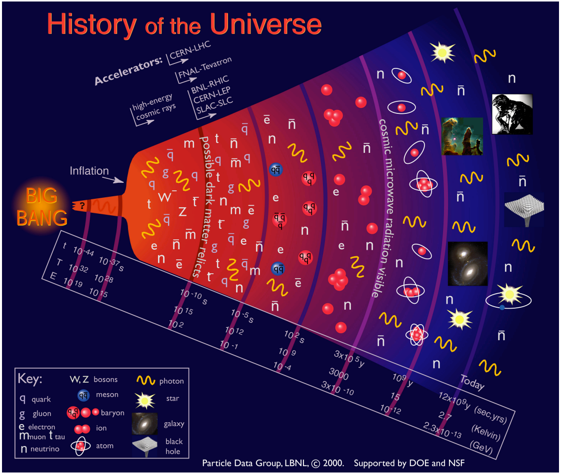
Expansion of the universe



- Evidence from the red-shift of galaxies: Hubble (1929), more distant galaxies receding faster
- Extrapolating backward: Big Bang cosmology. Evidence: Penzias and Wilson discovered microwave background radiation (1964)
- Future destiny: contraction, expansion, accelerated expansion (matter and energy content)

Inflation in the early universe

Why is the universe so homogeneous: distant zones not causally connected, unless... very accelerated expansion in very early universe (inflation)



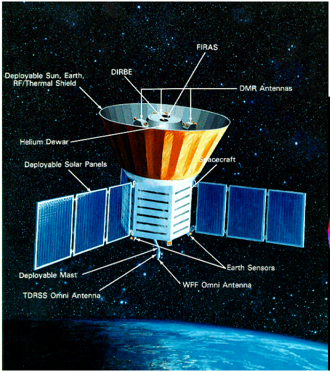
The Cosmic Microwave Background

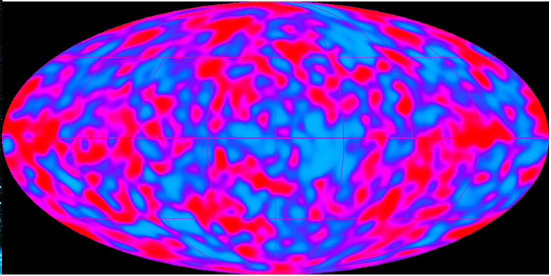
- Last scattering surface: cannot directly *see* earlier universe, but... can infer by what traces left on the CMB
- CMB fluctuations extremely faint: one part in 100,000 (of 2.73 degree Kelvin average temperature)

Observing the CMB sky

- COBE satellite (1989)
- WMAP satellite (2001)
- Planck satellite (2009)

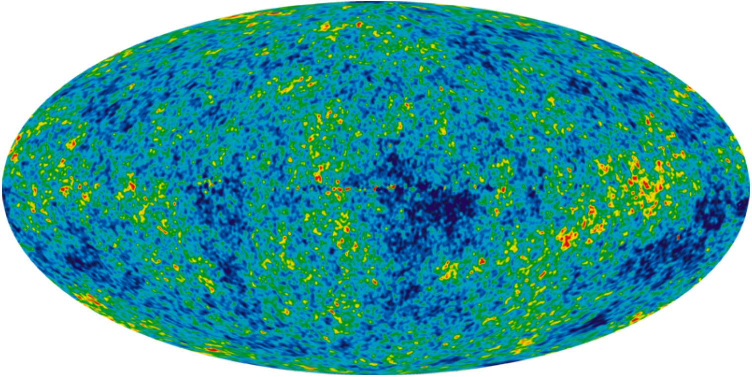
COBE's CMB sky



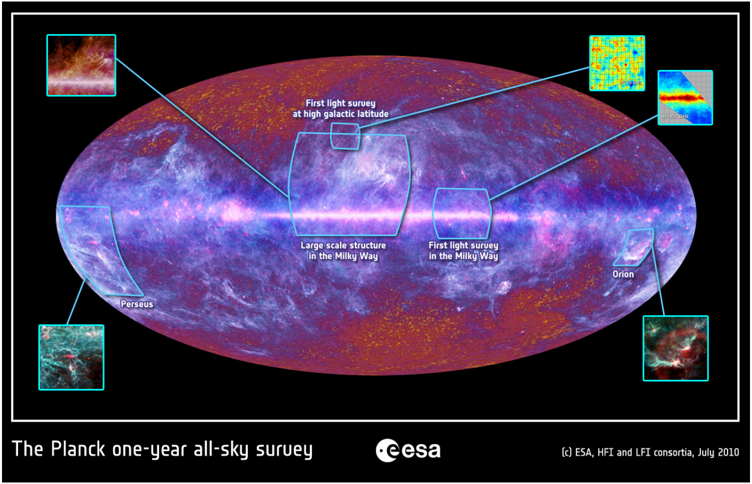


Anisotropies of the CMS
George Smoot and John Mather, Physics Nobel Prize 2006

WMAP's CMB sky

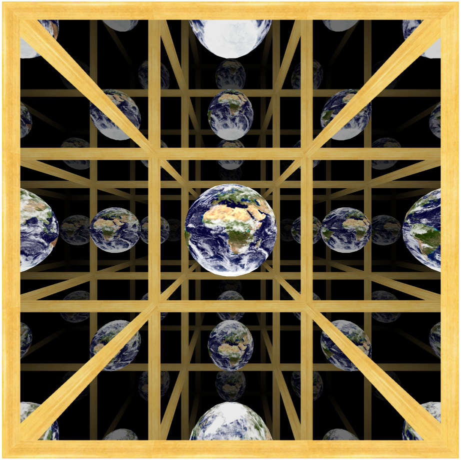


Planck's CMB sky: in the making



The shape of space question

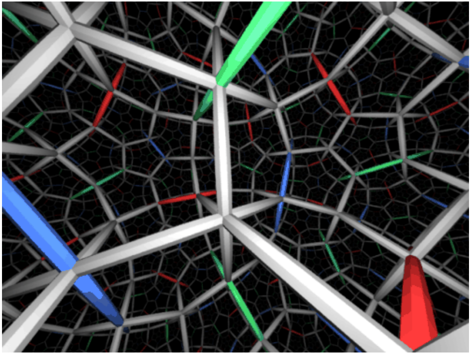
General relativity described curvature of space, but many different "shapes" with same curvature... problem of cosmic topology



J.Weeks "The shape of space"

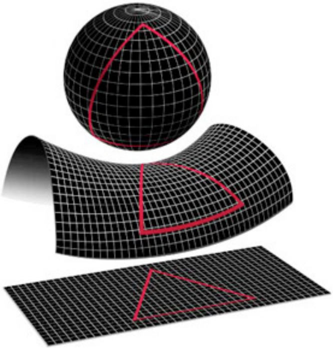
Question of cosmic topology

General relativity does not distinguish: can find something in the CMB that distinguishes them?



Why look for topology in the CMB?

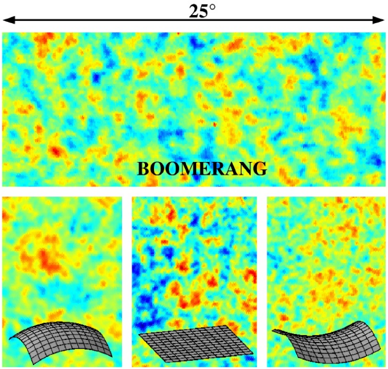
Because it has a lot to say about geometry (curvature)

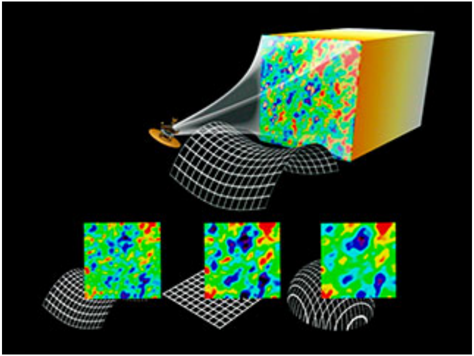


Curvature from sum of internal angles of a triangle

Measuring curvature in the CMB

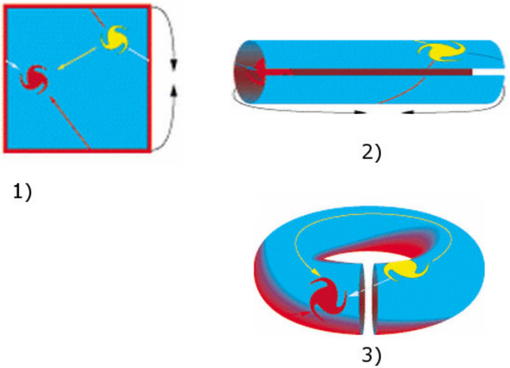
Boomerang experiment (balloon) and WMAP satellite





Space is flat! (or nearly so: slightly positively curved also possible)

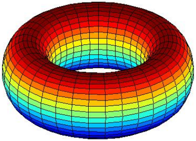
What about topology then? Multiple images effects



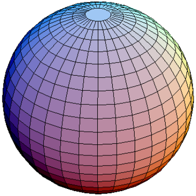
search for matching circles in the CMB sky

Cosmic topologies

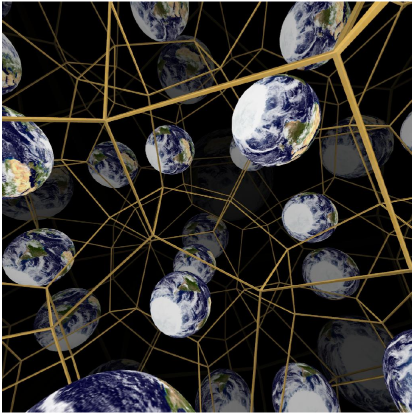
Flat spaces: 3-dimensional tori; Bieberbach manifolds (half-turn space, quarter turn space, etc)



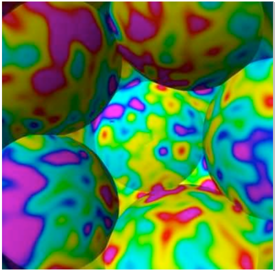
Spherical manifolds: 3-dimensional sphere; spherical space form (dodecahedral space, octahedral space, etc)

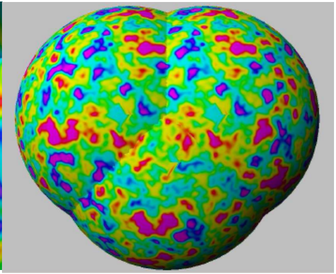


Inside a spherical geometry with dodecahedral space topology



Statistical approach: compare simulated CMB skies



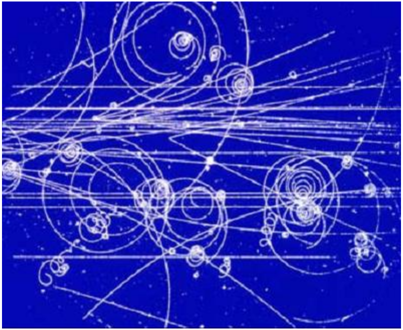


French cosmology school: Luminet, Lehoucq, Lachièze-Rey, Riazuelo, Uzan, etc.

This did not give conclusive evidence, so try different methods

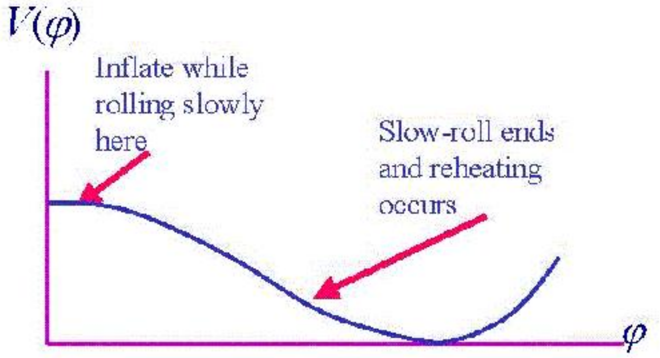
Matter and forces in the cosmos

Behavior of matter and energy in different cosmic topologies: elementary particles coupled to gravity



Question: would cosmic inflation happen differently in different topologies? Can particle physics predict that?

The slow-roll inflation mechanism



- There are very strong constraints on the slow-roll potential from the observational data of the Cosmic Microwave Background

Let's get some action

- Action functional: determines all interactions of matter, energy, and gravity
- Slow roll potential part of it: determine shape of inflation
- Different possible models of gravity coupled to matter

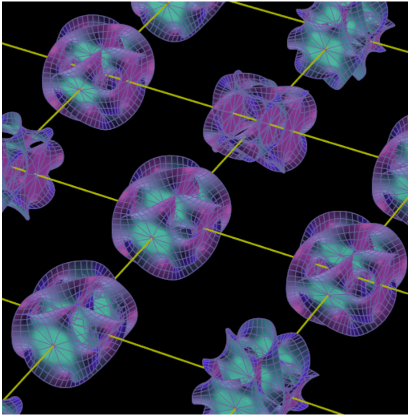
Least action principle: trajectories of motions (of particle, fields, etc) minimize energy (action)

$\Rightarrow$ Geometric ways to construct action functionals

Compare behavior of elementary particles on different geometries/topologies

Matter and gravity from (noncommutative) geometry

Spacetime (4-dim) with extra dimensions (noncommutative)



weirder than extra dimensions of string theory

Gravity on this space = gravity + matter on ordinary spacetime

The universe as a drum: the spectral action

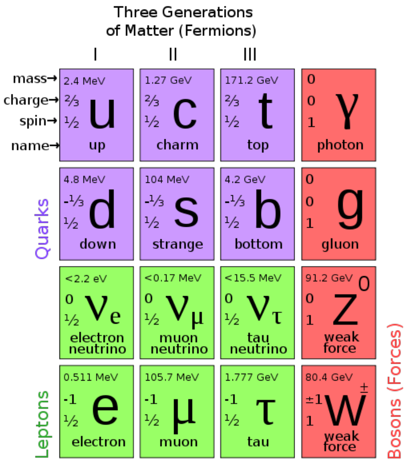
- A drum vibrates on certain characteristic *frequencies*: spectrum of frequencies
- Spaces also have a spectrum of frequencies they vibrate on
- Spectral action sums up all the frequencies of vibration of the space (spacetime + extra dimensions)

$$S = \mathrm{Tr}(f(D/\Lambda)) = \sum_k m_k f(\lambda_k/\Lambda)$$

Spectral action and gravity coupled to matter asymptotic formula

$$\mathrm{Tr}(f(D/\Lambda)) \sim \sum_{k \in \mathrm{DimSp}} f_k \Lambda^k \int\!\!\!\!\!\!- |D|^{-k} + f(0)\zeta_D(0) + o(1)$$

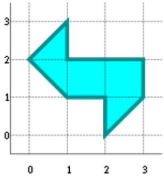
$\Rightarrow$ Gravity + Standard Model of elementary particles

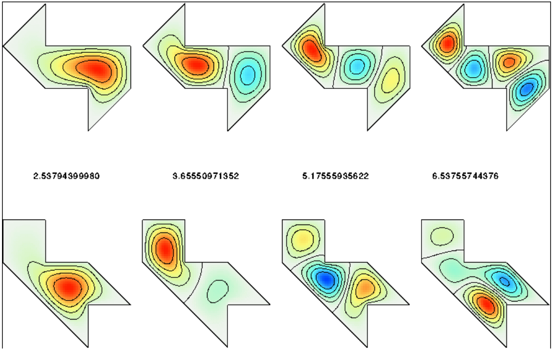


Can you hear the shape of a drum?

Warning: knowing frequencies not enough to determine shape

Gordon, Webb and Wolpert example

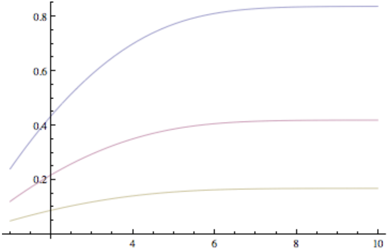




The spectral action and cosmic topology

But the spectral action detects cosmic topology: different shapes of slow-roll potential (Marcolli–Pierpaoli–Teh, 2010)

different spherical geometries:



different flat geometries: