Noncommutative Geometry Models for Particle Physics and Cosmology

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The NCG standard model and cosmology

- CCM A. Chamseddine, A. Connes, M. Marcolli, *Gravity and the standard model with neutrino mixing*, Adv. Theor. Math. Phys. 11 (2007), no. 6, 991–1089.
- MP M. Marcolli, E. Pierpaoli, *Early universe models from noncommutative geometry*, arXiv:0908.3683
- MPT M. Marcolli, E. Pierpaoli, K. Teh, *The spectral action and cosmic topology*, coming soon to an eprint archive near you.

The noncommutative space $X \times F$ extra dimensions
product of 4-dim spacetime and finite NC space
The spectral action functional

$$\mathrm{Tr}(f(D_A/\Lambda)) + \frac{1}{2}\langle J\tilde{\xi}, D_A\tilde{\xi}\rangle$$

$D_A = D + A + \varepsilon' J A J^{-1}$ Dirac operator with inner fluctuations
$A = A^* = \sum_k a_k[D, b_k]$

- Action functional for gravity (modified gravity)
- Gravity on $X \times F$ = gravity coupled to matter on X

Spectral triples $(\mathcal{A}, \mathcal{H}, D)$:

- involutive algebra $\mathcal{A}$
- representation $\pi : \mathcal{A} \to \mathcal{L}(\mathcal{H})$
- self adjoint operator D on $\mathcal{H}$
- compact resolvent $(1 + D^2)^{-1/2} \in \mathcal{K}$
- $[a, D]$ bounded $\forall a \in \mathcal{A}$
- even $\mathbb{Z}/2$-grading $[\gamma, a] = 0$ and $D\gamma = -\gamma D$
- real structure: antilinear isom $J : \mathcal{H} \to \mathcal{H}$ with $J^2 = \varepsilon$, $JD = \varepsilon' DJ$, and $J\gamma = \varepsilon'' \gamma J$

n	0	1	2	3	4	5	6	7
ε	1	1	-1	-1	-1	-1	1	1
ε'	1	-1	1	1	1	-1	1	1
ε''	1		-1		1		-1	

- bimodule: $[a, b^0] = 0$ for $b^0 = Jb^* J^{-1}$
- order one condition: $[[D, a], b^0] = 0$

Ansatz for the NC space F

$$\mathcal{A}_{LR} = \mathbb{C} \oplus \mathbb{H}_L \oplus \mathbb{H}_R \oplus M_3(\mathbb{C})$$

(or more general) $\Rightarrow$ everything else follows by *computation*

- Representation: $\mathcal{M}_F$ sum of all inequiv irred odd $\mathcal{A}_{LR}$-bimodules (fix N generations) $\mathcal{H}_F = \oplus^N \mathcal{M}_F$ fermions
- Algebra $\mathcal{A}_F = \mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$: order one condition
- F zero dimensional but KO-dim 6
- $J_F =$ matter/antimatter, $\gamma_F =$ L/R chirality
- Classification of Dirac operators (moduli spaces)

Dirac operators and Majorana mass terms

$$D(Y) = \begin{pmatrix} S & T^* \\ T & \bar{S} \end{pmatrix}, \qquad S = S_1 \oplus (S_3 \otimes 1_3), \qquad T = Y_R : |\nu_R\rangle \to J_F\,|\nu_R\rangle$$

$$S_1 = \begin{pmatrix} 0 & 0 & Y^*_{(\uparrow 1)} & 0 \\ 0 & 0 & 0 & Y^*_{(\downarrow 1)} \\ Y_{(\uparrow 1)} & 0 & 0 & 0 \\ 0 & Y_{(\downarrow 1)} & 0 & 0 \end{pmatrix}$$

$$S_3 = \begin{pmatrix} 0 & 0 & Y^*_{(\uparrow 3)} & 0 \\ 0 & 0 & 0 & Y^*_{(\downarrow 3)} \\ Y_{(\uparrow 3)} & 0 & 0 & 0 \\ 0 & Y_{(\downarrow 3)} & 0 & 0 \end{pmatrix}$$

Yukawa matrices: Dirac masses and mixing angles in $\mathrm{GL}_{N=3}(\mathbb{C})$

$Y_e = Y_{(\downarrow 1)}$ (charged leptons)

$Y_\nu = Y_{(\uparrow 1)}$ (neutrinos)

$Y_d = Y_{(\downarrow 3)}$ (d/s/b quarks)

$Y_u = Y_{(\uparrow 3)}$ (u/c/t quarks)

$M = Y_R^t$ Majorana mass terms symm matrix

Moduli space of Dirac operators on finite NC space F

$$\mathcal{C}_3 \times \mathcal{C}_1$$

- $\mathcal{C}_3$ = pairs $(Y_{(\downarrow 3)}, Y_{(\uparrow 3)})$ modulo W_j unitary matrices:

$$Y'_{(\downarrow 3)} = W_1 \, Y_{(\downarrow 3)} \, W_3^* \, , \;\; Y'_{(\uparrow 3)} = W_2 \, Y_{(\uparrow 3)} \, W_3^*$$

$G = \mathrm{GL}_3(\mathbb{C})$ and $K = U(3)$: $\quad \mathcal{C}_3 = (K \times K)\backslash(G \times G)/K$
$\dim_{\mathbb{R}} \mathcal{C}_3 = 10 = 3 + 3 + 4$ (eigenval, coset 3 angles 1 phase)

- $\mathcal{C}_1$ = triplets $(Y_{(\downarrow 1)}, Y_{(\uparrow 1)}, Y_R)$ with Y_R symmetric modulo

$$Y'_{(\downarrow 1)} = V_1 \, Y_{(\downarrow 1)} V_3^* \, , \;\; Y'_{(\uparrow 1)} = V_2 \, Y_{(\uparrow 1)} \, V_3^* \, ,$$

$$Y'_R = V_2 \, Y_R \, \bar{V}_2^*$$

$\pi : \mathcal{C}_1 \to \mathcal{C}_3$ surjection forgets Y_R fiber symm matrices mod $Y_R \mapsto \lambda^2 Y_R$
$\dim_{\mathbb{R}}(\mathcal{C}_3 \times \mathcal{C}_1) = 31$ (dim fiber 12-1=11)

Parameters of νMSM

- three coupling constants
- 6 quark masses, 3 mixing angles, 1 complex phase
- 3 charged lepton masses, 3 lepton mixing angles, 1 complex phase
- 3 neutrino masses
- 11 Majorana mass matrix parameters
- QCD vacuum angle

Moduli space of Dirac operators on F $\Rightarrow$ geometric form of all the Yukawa and Majorana parameters

Product geometry $(C^\infty(X), L^2(X,S), D_X) \cup (\mathcal{A}_F, \mathcal{H}_F, D_F)$

- $\mathcal{A} = C^\infty(X) \otimes \mathcal{A}_F = C^\infty(X, \mathcal{A}_F)$
- $\mathcal{H} = L^2(X,S) \otimes \mathcal{H}_F = L^2(X, S \otimes \mathcal{H}_F)$
- $D = D_X \otimes 1 + \gamma_5 \otimes D_F$

Inner fluctuations of the Dirac operator

$$D \to D_A = D + A + \varepsilon' J A J^{-1}$$

A self-adjoint operator

$$A = \sum a_j [D, b_j], \quad a_j, b_j \in \mathcal{A}$$

Fields content of the model

• Bosons: inner fluctuations $A = \sum_j a_j [D, b_j]$

- In M direction: $U(1)$, $SU(2)$, and $SU(3)$ gauge bosons

- In F direction: Higgs field $H = \varphi_1 + \varphi_2 j$

Fields content of the model

- Fermions: basis of $\mathcal{H}_F$

$$|\uparrow\rangle \otimes \mathbf{3}^0, \quad |\downarrow\rangle \otimes \mathbf{3}^0, \quad |\uparrow\rangle \otimes \mathbf{1}^0, \quad |\downarrow\rangle \otimes \mathbf{1}^0$$

Gauge group $SU(\mathcal{A}_F) = U(1) \times SU(2) \times SU(3)$
(up to fin abelian group)

- Hypercharges: adjoint action of $\mathrm{U}(1)$ (in powers of $\lambda \in \mathrm{U}(1)$)

	$\uparrow \otimes \mathbf{1}^0$	$\downarrow \otimes \mathbf{1}^0$	$\uparrow \otimes \mathbf{3}^0$	$\downarrow \otimes \mathbf{3}^0$
$\mathbf{2}_L$	-1	-1	$\frac{1}{3}$	$\frac{1}{3}$
$\mathbf{2}_R$	0	-2	$\frac{4}{3}$	$-\frac{2}{3}$

$\Rightarrow$ Correct hypercharges to the fermions

Action functional

$$\mathrm{Tr}(f(D_A/\Lambda)) + \frac{1}{2}\langle J\tilde{\xi}, D_A\tilde{\xi}\rangle$$

Fermion part: antisymmetric bilinear form $\mathfrak{A}(\tilde{\xi})$ on

$$\mathcal{H}^+ = \{\xi \in \mathcal{H} \,|\, \gamma\xi = \xi\}$$

$\Rightarrow$ nonzero on Grassmann variables
Euclidean functional integral $\Rightarrow$ Pfaffian

$$Pf(\mathfrak{A}) = \int e^{-\frac{1}{2}\mathfrak{A}(\tilde{\xi})} D[\tilde{\xi}]$$

(avoids Fermion doubling problem of previous models based on symmetric $\langle \xi, D_A\xi\rangle$ for NC space with KO-dim=0)

Explicit computation gives part of SM Larangian with

- $\mathcal{L}_{Hf}$ = coupling of Higgs to fermions
- $\mathcal{L}_{gf}$ = coupling of gauge bosons to fermions
- $\mathcal{L}_f$ = fermion terms

Asymptotic formula for the spectral action (Chamseddine–Connes)

$$\mathrm{Tr}(f(D/\Lambda)) \sim \sum_{k \in \mathrm{DimSp}} f_k \Lambda^k \int\!\!\!\!- |D|^{-k} + f(0)\zeta_D(0) + o(1)$$

for large Λ with $f_k = \int_0^\infty f(v)v^{k-1}dv$ and integration given by residues of zeta function $\zeta_D(s) = \mathrm{Tr}(|D|^{-s})$; DimSp poles of zeta functions

At low energies: only nonperturbative form of the spectral action

$$\mathrm{Tr}(f(D_A/\Lambda))$$

The asymptotic expansion of the spectral action from [CCM]

$$
\begin{aligned}
S = \ & \frac{1}{\pi^2}(48\, f_4\, \Lambda^4 - f_2\, \Lambda^2\, \mathfrak{c} + \frac{f_0}{4}\, \mathfrak{d}) \int \sqrt{g}\, d^4x \\
+ \ & \frac{96\, f_2\, \Lambda^2 - f_0\, \mathfrak{c}}{24\pi^2} \int R\, \sqrt{g}\, d^4x \\
+ \ & \frac{f_0}{10\, \pi^2} \int (\frac{11}{6}\, R^*\, R^* - 3\, C_{\mu\nu\rho\sigma}\, C^{\mu\nu\rho\sigma})\, \sqrt{g}\, d^4x \\
+ \ & \frac{(-2\, \mathfrak{a}\, f_2\, \Lambda^2 \ + \ \mathfrak{e}\, f_0)}{\pi^2} \int |\varphi|^2\, \sqrt{g}\, d^4x \\
+ \ & \frac{f_0 \mathfrak{a}}{2\, \pi^2} \int |D_\mu \varphi|^2\, \sqrt{g}\, d^4x \\
- \ & \frac{f_0 \mathfrak{a}}{12\, \pi^2} \int R\, |\varphi|^2\, \sqrt{g}\, d^4x \\
+ \ & \frac{f_0 \mathfrak{b}}{2\, \pi^2} \int |\varphi|^4\, \sqrt{g}\, d^4x \\
+ \ & \frac{f_0}{2\, \pi^2} \int (g_3^2\, G^i_{\mu\nu}\, G^{\mu\nu i} + g_2^2\, F^\alpha_{\mu\nu}\, F^{\mu\nu\alpha} + \frac{5}{3}\, g_1^2\, B_{\mu\nu}\, B^{\mu\nu})\, \sqrt{g}\, d^4x,
\end{aligned}
$$

Parameters:

- f_0, f_2, f_4 free parameters, $f_0 = f(0)$ and, for $k > 0$,

$$f_k = \int_0^\infty f(v) v^{k-1} dv.$$

- $\mathfrak{a}, \mathfrak{b}, \mathfrak{c}, \mathfrak{d}, \mathfrak{e}$ functions of Yukawa parameters of SM+r.h.ν

$$\begin{aligned}
\mathfrak{a} &= \mathrm{Tr}(Y_\nu^\dagger Y_\nu + Y_e^\dagger Y_e + 3(Y_u^\dagger Y_u + Y_d^\dagger Y_d)) \\
\mathfrak{b} &= \mathrm{Tr}((Y_\nu^\dagger Y_\nu)^2 + (Y_e^\dagger Y_e)^2 + 3(Y_u^\dagger Y_u)^2 + 3(Y_d^\dagger Y_d)^2) \\
\mathfrak{c} &= \mathrm{Tr}(MM^\dagger) \\
\mathfrak{d} &= \mathrm{Tr}((MM^\dagger)^2) \\
\mathfrak{e} &= \mathrm{Tr}(MM^\dagger Y_\nu^\dagger Y_\nu).
\end{aligned}$$

Normalization and coefficients

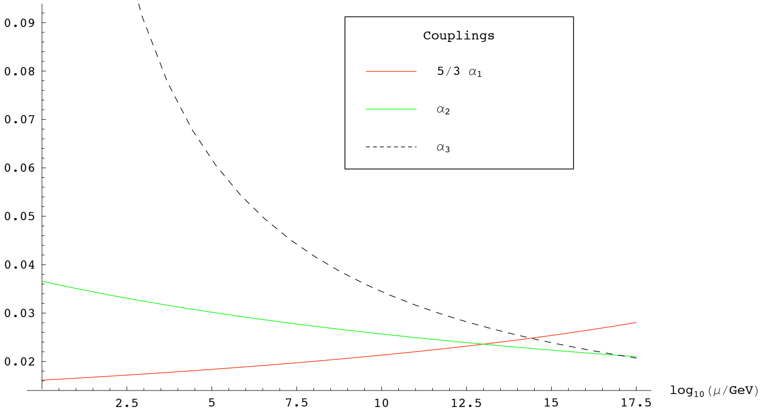
$$
\begin{aligned}
S = & \frac{1}{2\kappa_0^2} \int R \sqrt{g}\, d^4x + \gamma_0 \int \sqrt{g}\, d^4x \\
+ & \alpha_0 \int C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} \sqrt{g}\, d^4x + \tau_0 \int R^* R^* \sqrt{g}\, d^4x \\
+ & \frac{1}{2} \int |DH|^2 \sqrt{g}\, d^4x - \mu_0^2 \int |H|^2 \sqrt{g}\, d^4x \\
- & \xi_0 \int R\, |H|^2 \sqrt{g}\, d^4x + \lambda_0 \int |H|^4 \sqrt{g}\, d^4x \\
+ & \frac{1}{4} \int (G^i_{\mu\nu} G^{\mu\nu i} + F^\alpha_{\mu\nu} F^{\mu\nu\alpha} + B_{\mu\nu} B^{\mu\nu}) \sqrt{g}\, d^4x,
\end{aligned}
$$

Energy scale: Unification ($10^{15} - 10^{17}$ GeV)

$$
\frac{g^2 f_0}{2\pi^2} = \frac{1}{4}
$$

Preferred energy scale, unification of coupling constants

But warning of new physics: coupling constants do not really meet



RGE running (minimal SM) for coupling constants at 1-loop

Coefficients

$$\frac{1}{2\kappa_0^2} = \frac{96 f_2 \Lambda^2 - f_0 \mathfrak{c}}{24\pi^2} \qquad \gamma_0 = \frac{1}{\pi^2}\left(48 f_4 \Lambda^4 - f_2 \Lambda^2 \mathfrak{c} + \frac{f_0}{4}\mathfrak{d}\right)$$

$$\alpha_0 = -\frac{3 f_0}{10\pi^2} \qquad \tau_0 = \frac{11 f_0}{60\pi^2}$$

$$\mu_0^2 = 2\frac{f_2 \Lambda^2}{f_0} - \frac{\mathfrak{e}}{\mathfrak{a}} \qquad \xi_0 = \frac{1}{12}$$

$$\lambda_0 = \frac{\pi^2 \mathfrak{b}}{2 f_0 \mathfrak{a}^2}$$

Renormalization group equations

• In [CCM] RGE of minimal SM: predictions

- Higgs mass ~ 170 GeV

- mass relation at unification (top quark mass via RGE)

$$\sum_\sigma (m_\nu^\sigma)^2 + (m_e^\sigma)^2 + 3\,(m_u^\sigma)^2 + 3\,(m_d^\sigma)^2 = 8\,M_W^2$$

• In [MP] RGE for SM with right handed neutrinos + Majorana (from unification energy 2×10^{16} GeV to electroweak scale 10^2 GeV)

AKLRS S. Antusch, J. Kersten, M. Lindner, M. Ratz, M.A. Schmidt *Running neutrino mass parameters in see-saw scenarios*, JHEP 03 (2005) 024.

1-loop RGE equations: $\Lambda \frac{df}{d\Lambda} = \beta_f(\Lambda)$

$$16\pi^2 \ \beta_{g_i} = b_i \, g_i^3 \quad \text{with } (b_{SU(3)}, b_{SU(2)}, b_{U(1)}) = (-7, -\frac{19}{6}, \frac{41}{10})$$

$$16\pi^2 \ \beta_{Y_u} = Y_u(\frac{3}{2} Y_u^\dagger Y_u - \frac{3}{2} Y_d^\dagger Y_d + \mathfrak{a} - \frac{17}{20} g_1^2 - \frac{9}{4} g_2^2 - 8 g_3^2)$$

$$16\pi^2 \ \beta_{Y_d} = Y_d(\frac{3}{2} Y_d^\dagger Y_d - \frac{3}{2} Y_u^\dagger Y_u + \mathfrak{a} - \frac{1}{4} g_1^2 - \frac{9}{4} g_2^2 - 8 g_3^2)$$

$$16\pi^2 \ \beta_{Y_\nu} = Y_\nu(\frac{3}{2} Y_\nu^\dagger Y_\nu - \frac{3}{2} Y_e^\dagger Y_e + \mathfrak{a} - \frac{9}{20} g_1^2 - \frac{9}{4} g_2^2)$$

$$16\pi^2 \ \beta_{Y_e} = Y_e(\frac{3}{2} Y_e^\dagger Y_e - \frac{3}{2} Y_\nu^\dagger Y_\nu + \mathfrak{a} - \frac{9}{4} g_1^2 - \frac{9}{4} g_2^2)$$

$$16\pi^2 \ \beta_M = Y_\nu Y_\nu^\dagger M + M (Y_\nu Y_\nu^\dagger)^T$$

$$16\pi^2 \ \beta_\lambda = 6\lambda^2 - 3\lambda(3 g_2^2 + \frac{3}{5} g_1^2) + 3 g_2^4 + \frac{3}{2}(\frac{3}{5} g_1^2 + g_2^2)^2 + 4\lambda\mathfrak{a} - 8\mathfrak{b}$$

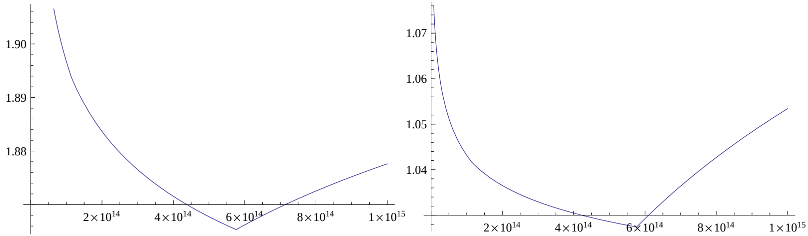
Note: different normalization from [CCM] and 5/3 factor included in g_1^2

Method of AKLRS: non-degenerate spectrum of Majorana masses, different effective field theories in between the three see-saw scales:

- RGE from unification Λ_{unif} down to first see-saw scale (largest eigenvalue of M)
- Introduce $Y_\nu^{(3)}$ removing last row of Y_ν in basis where M diagonal and $M^{(3)}$ removing last row and column.
- Induced RGE down to second see-saw scale
- Introduce $Y_\nu^{(2)}$ and $M^{(2)}$, matching boundary conditions
- Induced RGE down to first see-saw scale
- Introduce $Y_\nu^{(1)}$ and $M^{(1)}$, matching boundary conditions
- Induced RGE down to electoweak energy Λ_{ew}

Use effective field theories $Y_\nu^{(N)}$ and $M^{(N)}$ between see-saw scales

Running of coefficients $\mathfrak{a}, \mathfrak{b}$ with RGE



Coefficients $\mathfrak{a}$ and $\mathfrak{b}$ near the top see-saw scale

Similar runnings for coefficients $\mathfrak{c}$, $\mathfrak{d}$, $\mathfrak{e}$

Strong dependence on initial conditions at unification!

Cosmology timeline

- Planck epoch: $t \leq 10^{-43}\, s$ after the Big Bang (unification of forces with gravity, quantum gravity)
- Grand Unification epoch: $10^{-43}\, s \leq t \leq 10^{-36}\, s$ (electroweak and strong forces unified; Higgs)
- Electroweak epoch: $10^{-36}\, s \leq t \leq 10^{-12}\, s$ (strong and electroweak forces separated)
- Inflationary epoch: possibly $10^{-36}\, s \leq t \leq 10^{-32}\, s$

- NCG SM preferred scale at unification; RGE running between unification and electroweak Very Early Universe $\Rightarrow$ info on inflationary epoch.

- Remark: Cannot extrapolate to modern universe, nonperturbative effects in the spectral action: requires nonperturbative spectral action

Cosmological implications of the NCG SM

- Linde's hypothesis (antigravity in the early universe)
- Primordial black holes and gravitational memory
- Gravitational waves in modified gravity
- Gravity balls
- Varying effective cosmological constant
- Higgs based slow-roll inflation
- Spontaneously arising Hoyle-Narlikar in EH backgrounds

Effects in the very early universe: inflation mechanisms

Effective gravitational constant

$$G_{\text{eff}} = \frac{\kappa_0^2}{8\pi} = \frac{3\pi}{192 f_2 \Lambda^2 - 2 f_0 \mathfrak{c}(\Lambda)}$$

Effective cosmological constant

$$\gamma_0 = \frac{1}{4\pi^2}(192 f_4 \Lambda^4 - 4 f_2 \Lambda^2 \mathfrak{c}(\Lambda) + f_0 \mathfrak{d}(\Lambda))$$

Conformal non-minimal coupling of Higgs and gravity

$$\frac{1}{16\pi G_{\rm eff}} \int R \sqrt{g} d^4x - \frac{1}{12} \int R\,|H|^2 \sqrt{g} d^4x$$

Conformal gravity

$$\frac{-3f_0}{10\pi^2} \int C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} \sqrt{g} d^4x$$

$C^{\mu\nu\rho\sigma}$ = Weyl curvature tensor (trace free part of Riemann tensor)

Effective gravitational constant and gravitational waves:
Einstein equations $R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = \kappa_0^2 T^{\mu\nu}$

$$g_{\mu\nu} = a(t)^2 \begin{pmatrix} -1 & 0 \\ 0 & \delta_{ij} + h_{ij}(x) \end{pmatrix}$$

trace and traceless part of $h_{ij} \Rightarrow$ Friedmann equation

$$-3 \left(\frac{\dot{a}}{a}\right)^2 + \frac{1}{2} \left(4 \left(\frac{\dot{a}}{a}\right) \dot{h} + 2\ddot{h}\right) = \frac{\tilde{\kappa}_0^2}{\Lambda^2} T_{00}$$

$\Lambda(t) = 1/a(t)$ (f_2 large) Inflationary epoch: $a(t) \sim e^{\alpha t}$
NCG model solutions:

$$h(t) = \frac{3\pi^2 T_{00}}{192 f_2 \alpha^2} e^{2\alpha t} + \frac{3\alpha}{2} t + \frac{A}{2\alpha} e^{-2\alpha t} + B$$

Ordinary cosmology:

$$(\frac{4\pi G T_{00}}{\alpha} + \frac{3\alpha}{2}) t + \frac{A}{2\alpha} e^{-2\alpha t} + B$$

Radiation dominated epoch: $a(t) \sim t^{1/2}$
NCG model solutions:

$$h(t) = \frac{4\pi^2 T_{00}}{288 f_2} t^3 + B + A\log(t) + \frac{3}{8}\log(t)^2$$

Ordinary cosmology:

$$h(t) = 2\pi G T_{00} t^2 + B + A\log(t) + \frac{3}{8}\log(t)^2$$

Find choices of f_2 parameter, for constant curvature spaces $R \sim 1$
Dominant terms in the spectral action:

$$\Lambda^2 \left(\frac{1}{2\tilde{\kappa}_0^2} \int R \sqrt{g} d^4x - \tilde{\mu}_0^2 \int |H|^2 \sqrt{g} d^4x \right)$$

$\tilde{\kappa}_0 = \Lambda \kappa_0$ and $\tilde{\mu}_0 = \mu_0/\Lambda$, where $\mu_0^2 \sim \frac{2f_2\Lambda^2}{f_0}$
But near see-saw scale emergent conformally coupled matter and gravity

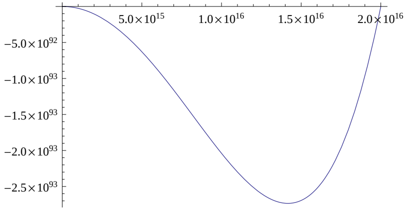
$$\begin{aligned} S_c = \; & \alpha_0 \int C_{\mu\nu\rho\sigma} \, C^{\mu\nu\rho\sigma} \sqrt{g} \, d^4x + \frac{1}{2} \int |DH|^2 \sqrt{g} \, d^4x \\ & - \xi_0 \int R \, |H|^2 \sqrt{g} \, d^4x + \lambda_0 \int |H|^4 \sqrt{g} \, d^4x \\ & + \frac{1}{4} \int (G^i_{\mu\nu} \, G^{\mu\nu i} + F^\alpha_{\mu\nu} \, F^{\mu\nu\alpha} + B_{\mu\nu} \, B^{\mu\nu}) \sqrt{g} \, d^4x. \end{aligned}$$

A Hoyle-Narlikar type cosmology, normally suppressed by dominant Einstein–Hilbert term

Cosmological term controlled by additional parameter f_4, vanishing condition:

$$f_4 = \frac{(4 f_2 \Lambda^2 \mathfrak{c} - f_0 \mathfrak{d})}{192 \Lambda^4}$$

Example: vanishing at unification $\gamma_0(\Lambda_{unif}) = 0$



Running of $\gamma_0(\Lambda)$: possible inflationary mechanism

The λ_0-ansatz

$$\lambda_0|_{\Lambda=\Lambda_{unif}} = \lambda(\Lambda_{unif})\frac{\pi^2\mathfrak{b}(\Lambda_{unif})}{f_0\mathfrak{a}^2(\Lambda_{unif})},$$

- Run like $\lambda(\Lambda)$ but change boundary condition to $\lambda_0|_{\Lambda=\Lambda_{unif}}$
- Run like
$$\lambda_0(\Lambda) = \lambda(\Lambda)\frac{\pi^2\mathfrak{b}(\Lambda)}{f_0\mathfrak{a}^2(\Lambda)}$$

For most of our cosmological estimates no serious difference, but can lower Higgs mass estimate to ~ 158 GeV

Linde's hypothesis antigravity in the early universe

- A.D. Linde, *Gauge theories, time-dependence of the gravitational constant and antigravity in the early universe*, Phys. Letters B, Vol.93 (1980) N.4, 394–396

Based on a conformal coupling

$$\frac{1}{16\pi G}\int R\,\sqrt{g}\,d^4x - \frac{1}{12}\int R\,\phi^2\sqrt{g}\,d^4x$$

giving an effective

$$G_{\rm eff}^{-1} = G^{-1} - \frac{4}{3}\pi\phi^2$$

In the NCG SM model two sources of negative gravity

- Running of $G_{\rm eff}(\Lambda)$
- Conformal coupling to the Higgs field

Gravity balls (or "Space Balls") $G_{\text{eff},H} = G_{\text{eff}}(1 - \frac{4\pi}{3} G_{\text{eff}} |H|^2)^{-1}$
combines running of G_{eff} with Linde mechanism
Suppose f_2 such that $G_{\text{eff}}(\Lambda) > 0$

$$\begin{cases} G_{\text{eff},H} < 0 & \text{for } |H|^2 > \dfrac{3}{4\pi G_{\text{eff}}(\Lambda)}, \\ G_{\text{eff},H} > 0 & \text{for } |H|^2 < \dfrac{3}{4\pi G_{\text{eff}}(\Lambda)}. \end{cases}$$

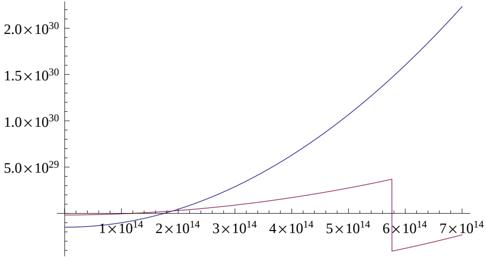
Near equilibrium for H:

$$\ell_H(\Lambda, f_2) := \frac{\mu_0^2}{2\lambda_0}(\Lambda) = \frac{2\frac{f_2 \Lambda^2}{f_0} - \frac{\mathfrak{e}(\Lambda)}{\mathfrak{a}(\Lambda)}}{\lambda(\Lambda)\frac{\pi^2 \mathfrak{b}(\Lambda)}{f_0 \mathfrak{a}^2(\Lambda)}} = \frac{(2 f_2 \Lambda^2 \mathfrak{a}(\Lambda) - f_0 \mathfrak{e}(\Lambda))\mathfrak{a}(\Lambda)}{\pi^2 \lambda(\Lambda)\mathfrak{b}(\Lambda)}$$

(with λ_0-ansatz)
Negative gravity regime where

$$\ell_H(\Lambda, f_2) > \frac{3}{4\pi G_{\text{eff}}(\Lambda, f_2)}$$

An example of transition to a negative gravity phase



Gravity balls: regions where $|H|^2 \sim 0$ unstable equilibrium (positive gravity) surrounded by region with $|H|^2 \sim \ell_H(\Lambda, f_2)$ stable (negative gravity): possible model of dark energy

Primordial black holes (Zeldovich–Novikov, 1967)

- I.D. Novikov, A.G. Polnarev, A.A. Starobinsky, Ya.B. Zeldovich, *Primordial black holes*, Astron. Astrophys. 80 (1979) 104–109
- J.D. Barrow, *Gravitational memory?* Phys. Rev. D Vol.46 (1992) N.8 R3227, 4pp.

Caused by: collapse of overdense regions, phase transitions in the early universe, cosmic loops and strings, inflationary reheating, etc
Gravitational memory: if gravity balls with different $G_{\text{eff},H}$ primordial black holes can evolve with different $G_{\text{eff},H}$ from surrounding space

Evaporation of PBHs by Hawking radiation

$$\frac{d\mathcal{M}(t)}{dt} \sim -(G_{\rm eff}(t)\mathcal{M}(t))^{-2}$$

with Hawking temperature $T = (8\pi G_{\rm eff}(t)\mathcal{M}(t))^{-1}$.
In terms of energy:

$$\mathcal{M}^2\, d\mathcal{M} = \frac{1}{\Lambda^2 G_{\rm eff}^2(\Lambda, f_2)} d\Lambda$$

With gravitational memory:

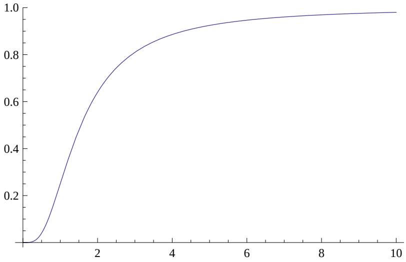
$$\mathcal{M}(\Lambda, f_2) = \sqrt[3]{\mathcal{M}^3(\Lambda_{in}) - \frac{2}{3\pi^2}\int_{\Lambda}^{\Lambda_{in}} \frac{(1 - \frac{4\pi}{3} G_{\rm eff}(x)|H|^2)^2}{x^3 G_{\rm eff}(x)^2}\, dx}$$

Evaporation of PBHs linked to γ-ray bursts

Higgs based slow-roll inflation

dSHW A. De Simone, M.P. Hertzberg, F. Wilczek, *Running inflation in the Standard Model*, hep-ph/0812.4946v2

Minimal SM and non-minimal coupling of Higgs and gravity.
Non-conformal coupling $\xi_0 \neq 1/12$, running of ξ_0
Effective Higgs potential: inflation parameter $\psi = \sqrt{\xi_0}\kappa_0|H|$



inflationary period $\psi >> 1$, end of inflation $\psi \sim 1$, low energy regime $\psi << 1$

In the NCG SM have $\xi_0 \equiv 1/12$ but same Higgs based slow-roll inflation due to κ_0 running

Slow roll parameters for a slow roll potential

$$V_E(x) = \frac{\lambda_0 x^4}{(1+\xi_0 \kappa_0^2 x^2)^2}$$

Spectral index and tensor to scalar ratio

$$n_s = 1 + \frac{32(216 + \kappa_0^2(6x^2 - \kappa_0^2(432 + 12\kappa_0^2(2+3(\kappa_0^2)^2)x^2 + (1+(\kappa_0^2)^2)x^4)))}{\kappa_0^2(12x + \kappa_0^2(1+(\kappa_0^2)^2)x^3)^2}$$

$$r = \frac{256\kappa_0^2}{x^2 + \frac{\kappa_0^2}{12}(1+(\kappa_0^2)^2)x^4}$$

Cosmological models for the not-so-early-universe?

Need to work with non-perturbative form of the spectral action

Can to for specially symmetric geometries!

The spectral action and the question of cosmic topology

(with E. Pierpaoli and K. Teh)

Spatial sections of spacetime closed 3-manifolds $\neq S^3$?

- Cosmologists search for signatures of topology in the CMB
- Model based on NCG distinguishes cosmic topologies?

Yes! the non-perturbative spectral action predicts different models of slow-roll inflation

Poisson summation formula

$$\sum_{n\in\mathbb{Z}} h(x+\lambda n) = \frac{1}{\lambda}\sum_{n\in\mathbb{Z}} \exp\left(\frac{2\pi i n x}{\lambda}\right)\ \widehat{h}(\frac{n}{\lambda})$$

$\lambda \in \mathbb{R}_+^*$ and $x \in \mathbb{R}$ with

$$\widehat{h}(x) = \int_{\mathbb{R}} h(u)\, e^{-2\pi i u x}\, du$$

Idea: write $\mathrm{Tr}(f(D/\Lambda))$ as sums over lattices

- Need explicit spectrum of D with multiplicities
- Need to write as a union of arithmetic progressions $\lambda_{n,i}$, $n \in \mathbb{Z}$
- Multiplicities polynomial functions $m_{\lambda_{n,i}} = P_i(\lambda_{n,i})$

$$\mathrm{Tr}(f(D/\Lambda)) = \sum_i \sum_{n\in\mathbb{Z}} P_i(\lambda_{n,i}) f(\lambda_{n,i}/\Lambda)$$

The standard topology S^3 (Chamseddine–Connes)

Dirac spectrum $\pm a^{-1}(\frac{1}{2}+n)$ for $n \in \mathbb{Z}$, with multiplicity $n(n+1)$

$$\mathrm{Tr}(f(D/\Lambda)) = (\Lambda a)^3 \widehat{f}^{(2)}(0) - \frac{1}{4}(\Lambda a)\widehat{f}(0) + O((\Lambda a)^{-k})$$

with $\widehat{f}^{(2)}$ Fourier transform of $v^2 f(v)$ 4-dimensional Euclidean $S^3 \times S^1$

$$\mathrm{Tr}(h(D^2/\Lambda^2)) = \pi \Lambda^4 a^3 \beta \int_0^\infty u\, h(u)\, du - \frac{1}{2}\pi \Lambda a \beta \int_0^\infty h(u)\, du + O(\Lambda^{-k})$$

$$g(u,v) = 2P(u)\, h(u^2(\Lambda a)^{-2} + v^2(\Lambda\beta)^{-2})$$

$$\widehat{g}(n,m) = \int_{\mathbb{R}^2} g(u,v) e^{-2\pi i(xu+yv)}\, du\, dv$$

A slow roll potential from non-perturbative effects

perturbation $D^2 \mapsto D^2 + \phi^2$ gives potential $V(\phi)$ scalar field coupled to gravity

$$\mathrm{Tr}(h((D^2+\phi^2)/\Lambda^2)) = \pi\Lambda^4\beta a^3 \int_0^\infty uh(u)du - \frac{\pi}{2}\Lambda^2\beta a \int_0^\infty h(u)du$$

$$+\pi\Lambda^4\beta a^3\,\mathcal{V}(\phi^2/\Lambda^2) + \frac{1}{2}\Lambda^2\beta a\,\mathcal{W}(\phi^2/\Lambda^2)$$

$$\mathcal{V}(x) = \int_0^\infty u(h(u+x) - h(u))du, \qquad \mathcal{W}(x) = \int_0^x h(u)du$$

Slow roll parameters Minkowskian Friedmann metric on $S \times \mathbb{R}$

$$ds^2 = a(t)^2 ds_S^2 - dt^2$$

accelerated expansion $\frac{\ddot{a}}{a} = H^2(1-\epsilon)$ Hubble parameter

$$H^2(\phi)\left(1 - \frac{1}{3}\epsilon(\phi)\right) = \frac{8\pi}{3m_{Pl}^2} V(\phi)$$

m_{Pl} Planck mass

$$\epsilon(\phi) = \frac{m_{Pl}^2}{16\pi}\left(\frac{V'(\phi)}{V(\phi)}\right)^2$$

inflation phase $\epsilon(\phi) < 1$

$$\eta(\phi) = \frac{m_{Pl}^2}{8\pi}\left(\frac{V''(\phi)}{V(\phi)}\right) - \frac{m_{Pl}^2}{16\pi}\left(\frac{V'(\phi)}{V(\phi)}\right)^2$$

second slow-roll parameter $\Rightarrow$ measurable quantities

$$n_s = 1 - 6\epsilon + 2\eta \quad r = 16\epsilon$$

spectral index and tensor-to-scalar ratio

Slow-roll parameters from spectral action $S = S^3$

$$\epsilon(x) = \frac{m_{Pl}^2}{16\pi} \left(\frac{h(x) - 2\pi(\Lambda a)^2 \int_x^\infty h(u)du}{\int_0^x h(u)du + 2\pi(\Lambda a)^2 \int_0^\infty u(h(u+x) - h(u))du} \right)^2$$

$$\eta(x) = \frac{m_{Pl}^2}{8\pi} \frac{h'(x) + 2\pi(\Lambda a)^2 h(x)}{\int_0^x h(u)du + 2\pi(\Lambda a)^2 \int_0^\infty u(h(u+x) - h(u))du}$$

$$-\frac{m_{Pl}^2}{16\pi} \left(\frac{h(x) - 2\pi(\Lambda a)^2 \int_x^\infty h(u)du}{\int_0^x h(u)du + 2\pi(\Lambda a)^2 \int_0^\infty u(h(u+x) - h(u))du} \right)^2$$

In Minkowskian Friedmann metric $\Lambda(t) \sim 1/a(t)$
Also independent of β (artificial Euclidean compactification)

The quaternionic space $SU(2)/Q8$ (quaternion units $\pm 1, \pm \sigma_k$)

Dirac spectrum

$$\frac{3}{2} + 4k \quad \text{with multiplicity} \quad 2(k+1)(2k+1)$$

$$\frac{3}{2} + 4k + 2 \quad \text{with multiplicity} \quad 4k(k+1)$$

Polynomial interpolation of multiplicities

$$P_1(u) = \frac{1}{4}u^2 + \frac{3}{4}u + \frac{5}{16}$$

$$P_2(u) = \frac{1}{4}u^2 - \frac{3}{4}u - \frac{7}{16}$$

Spectral action

$$\mathrm{Tr}(f(D/\Lambda)) = \frac{1}{8}(\Lambda a)^3 \widehat{f}^{(2)}(0) - \frac{1}{32}(\Lambda a)\widehat{f}(0) + O(\Lambda^{-k})$$

(1/8 of action for S^3) with $g_i(u) = P_i(u) f(u/\Lambda)$:

$$\mathrm{Tr}(f(D/\Lambda)) = \frac{1}{4}\left(\widehat{g}_1(0) + \widehat{g}_2(0)\right) + O(\Lambda^{-k})$$

from Poisson summation $\Rightarrow$ Same slow-roll parameters

The dodecahedral space Poincaré homology sphere S^3/Γ
binary icosahedral group 120 elements
Dirac spectrum: eigenvalues of S^3 different multiplicities $\Rightarrow$
generating function

$$F_+(z) = \sum_{k=0}^{\infty} m(\frac{3}{2}+k, D) z^k \quad F_-(z) = \sum_{k=0}^{\infty} m(-(\frac{3}{2}+k), D) z^k$$

$$F_+(z) = -\frac{16(710647 + 317811\sqrt{5}) G^+(z)}{(7+3\sqrt{5})^3 (2207 + 987\sqrt{5}) H^+(z)}$$

$$G^+(z) = 6z^{11}+18z^{13}+24z^{15}+12z^{17}-2z^{19}-6z^{21}-2z^{23}+2z^{25}+4z^{27}+3z^{29}+z^{31}$$

$$H^+(z) = -1-3z^2-4z^4-2z^6+2z^8+6z^{10}+9z^{12}+9z^{14}+4z^{16}-4z^{18}-9z^{20} -9z^{22}-6z^{24}-2z^{26}+2z^{28}+4z^{30}+3z^{32}+z^{34}$$

$$F_-(z) = -\frac{1024(5374978561 + 2403763488\sqrt{5}) G^-(z)}{(7+3\sqrt{5})^8 (2207 + 987\sqrt{5}) H^-(z)}$$

$$G^-(z) = 1+3z^2+4z^4+2z^6-2z^8-6z^{10}-2z^{12}+12z^{14}+24z^{16}+18z^{18}+6z^{20}$$

$$H^-(z) = -1-3z^2-4z^4-2z^6+2z^8+6z^{10}+9z^{12}+9z^{14}+4z^{16}-4z^{18}-9z^{20} -9z^{22}-6z^{24}-2z^{26}+2z^{28}+4z^{30}+3z^{32}+z^{34}$$

Polynomial interpolation of multiplicities: 60 polynomials $P_i(u)$

$$\sum_{j=0}^{59} P_j(u) = \frac{1}{2}u^2 - \frac{1}{8}$$

Spectral action: functions $g_j(u) = P_j(u) f(u/\Lambda)$

$$\mathrm{Tr}(f(D/\Lambda)) = \frac{1}{60}\sum_{j=0}^{59} \widehat{g}_j(0) + O(\Lambda^{-k})$$

$$= \frac{1}{60}\int_{\mathbb{R}} \sum_j P_j(u) f(u/\Lambda) du + O(\Lambda^{-k})$$

by Poisson summation $\Rightarrow 1/120$ of action for S^3

Same slow-roll parameters

The lens spaces $\mathcal{L}_N = SU(2)/\mathbb{Z}_N$, $N \geq 3$

$$\begin{pmatrix} \omega & 0 \\ 0 & \omega^{-1} \end{pmatrix}, \quad \text{with} \quad \omega^N = 1$$

Positive spectrum, part of arithmetic progressions with multiplicities interpolation

$$P_0^+(u) = \frac{2}{N}u^2 + \frac{2}{N}u + \frac{1}{2N}$$

$$P_1^+(u) = \frac{2}{N}u^2 - \frac{1}{2N}$$

$$P_j^+(u) = \frac{2}{N}u^2 + \frac{2-2j+N}{N}u + \frac{1-2j+N}{2N}, \quad j = 2, 3, \ldots, N-1$$

Negative spectrum

$$P_0^-(u) = \frac{2}{N}u^2 + \frac{2}{N}u + \frac{1}{2N}$$

$$P_1^-(u) = \frac{2}{N}u^2 + \frac{4}{N}u + \frac{3}{2N}$$

$$P_j^-(u) = \frac{2}{N}u^2 + \frac{2+2j-N}{N}u + \frac{1+2j-N}{2N}, \quad j = 2, 3, \ldots, N-1$$

$$P^-(u) = -2u - 1$$

Spectral action for $\mathcal{L}_N$

$$\mathrm{Tr}(f(|D|/\Lambda)) = \mathrm{Tr}(f_+(|D|/\Lambda)) + \mathrm{Tr}(f_-(|D|/\Lambda))$$

$$\mathrm{Tr}(f(|D|/\Lambda)) = \frac{1}{N}\left(4\Lambda^3 \widehat{f}_+^{(2)}(0) + 2\Lambda^2 \widehat{f}_+^{(1)}(0)\right) + O(\Lambda^{-k})$$

$$\mathrm{Tr}(h(D^2/\Lambda^2)) = 2\pi\Lambda^4 a^3\beta \int_0^\infty u\, h(u)\, du + 2\Lambda^3 a^2 \beta \int_0^\infty u^{1/2} h(u)\, du + O(\Lambda^{-k})$$

$$\mathrm{Tr}(h((D^2+\phi^2)/\Lambda^2)) = \mathrm{Tr}(h(D^2/\Lambda^2)) + 2\pi\Lambda^4 a^3 \beta \mathcal{V}(\phi^2/\Lambda^2) + 2\Lambda^3 a^2 \beta \mathcal{Z}(\phi^2/\Lambda^2)$$

$$\mathcal{V}(x) = \int_0^\infty u\,(h(u+x) - h(u))\, du \quad \mathcal{Z}(x) = \int_0^\infty u^{1/2}\,(h(u+x) - h(u))\, du$$

Different slow-roll potential and parameters

$$V(x) = 2\pi\Lambda^4 a^3 \beta\, \mathcal{V}(\phi^2/\Lambda^2) + 2\Lambda^3 a^2 \beta\, \mathcal{Z}(\phi^2/\Lambda^2)$$

A modified gravity model based on the spectral action cannot rule out most likely cosmic topology candidates (dodecahedral, quaternionic) but can rule out less symmetric ones like lens spaces: predicts different behavior of cosmological inflation!