Math 120c - Spring 2003-2004
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Homework set 5
Due: 19th May 2004

1. Let G be the group generated by the permutations

$$\begin{aligned} a &= (1\ 2\ 3\ 4\ 5\ 6\ 7)(7\ 8\ 9\ 10\ 11\ 12) \\ b &= (1\ 7\ 4\ 10)(2\ 12\ 5\ 9)(3\ 11\ 6\ 8). \end{aligned}$$

   Prove that G has exactly 12 elements and that

$$a^6 = 1,\ a^3 = b^2,\ b^{-1}ab = a^{-1}.$$

   Hence prove that G has six conjugacy classes, and give the character table for G.

2. Exercise IX.6.9 of Hungerford (the question of Dickson).

3. Let H be an abelian subgroup of the finite group G. Let ρ be an irreducible complex representation of G. Show that the dimension of ρ is $\leq [G : H]$.

4. Let n be a positive integer. We say that a field K is n-*closed* if there is no proper extension L of K such that $[L : K]$ divides n.

   Let K be an n-closed field. Prove that the order of every element of $Br(K)$ is prime to n.

5. A K-algebra A is called *cyclic* if there is a strictly maximal subfield L of A such that $\mathrm{Gal}(L/K)$ is cyclic.

   Find a cyclic division algebra D such that $M_n(D)$ is not cyclic for all $n > 1$. (Hint: think about the quaternions).