Math 120c - Spring 2003-2004
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Homework set 4
Due: 5th May 2004

Let L and K be fields.

1. Let μ_n be the group of n^{th} roots in the field K. Prove that

$$H^2(\operatorname{Gal} L/K, \mu_n) \equiv \operatorname{Br}(K)[n].$$

   (Hint: think about exact sequences).

2. Prove that $\operatorname{Br}(\cdot)$ is a functor from the category of fields to the category of abelian groups.

3. Let $i, j : L \to K$ be field homomorphisms and let i_* and j_* be the induced maps from Br(L) to Br(K). Define F by

$$F = \{x \in K : i(x) = j(x)\}.$$

   Assume that $K/i(F)$ is a Galois extension. Prove that $i_* = j_*$.

4. Let D be a finite-dimensional division algebra over K. Suppose that $a, b \in D$ have the same minimum polynomial over K. Prove that there exists $x \in D$ such that $xbx^{-1} = a$. (Hint: think about the Noether-Skolem theorem).

5. Let R be a ring, and let n be a positive integer. Show that

$$J(M_n(R)) = M_n(J(R)).$$