

Math 120c - Spring 2003-2004
Lloyd Kilford

Homework set 2
Due: 16th April 2004

Let R be a ring, with $1 \in R$.

1. When is R a simple module over itself?
2. (a) List all non-isomorphic semisimple rings with 81 elements.
(b) Which of these are commutative?
3. Let R be a left Artinian ring, and let I be an ideal of R . Assume that $I^2 = (0)$ implies that $I = (0)$ for all I . Prove that R is semisimple.
4. Exercise XVII.10 of Lang (semisimplicity for vector spaces).
5. Let G be a group and let H be a subgroup of G of finite index. Let K be a field such that the characteristic does not divide $[G : H]$.
Let V be a KG -module with a submodule W and let U_0 be a KH -module such that $V = W \oplus U_0$.
Show that there exists a KG -submodule U of V such that $V = W \oplus U$.
6. Let K be a field and define

$$S := \left\{ \begin{pmatrix} a & 0 \\ b & c \end{pmatrix} : a, b, c \in K \right\}.$$

How many simple modules does S have? Is S a semisimple ring?