

**Math 120c - Spring 2004**  
**Lloyd Kilford**

**Final Examination**  
**Due: 2004**

You may use the notes for this course (including photocopied notes), your solutions to course homeworks, and the course textbooks (Lang, Hungerford, Milne's notes on Class Field Theory) for reference. Please provide references for any quoted theorems. You may not use the Internet or other books for reference. Please do not discuss this exam with other students or faculty without contacting me first.

There are no time restrictions on this examination, except the final deadline.

Each question is worth an equal number of marks, and partial credit will be given for incomplete answers.

Hand this exam in to my office (Sloan 358) or to my pigeonhole in a sealed envelope before 6pm on Friday June 4th 2004.

Let  $G$  be a finite group.

1. Let  $n$  be a positive integer. Prove that the entries of the character table of the symmetric group are integers. (Hint: prove the more general theorem that if  $g \in G$  has order  $n$ ,  $g^i$  is conjugate to  $g$  for all  $(i, n) = 1$ , then  $\chi(g)$  is an integer for all characters  $\chi$  of  $G$ . You will also need to prove that certain sums of roots of unity are integers).
2. Write down various methods to find characters of a group  $G$ . Find the character table of  $S_6$ . Why is  $S_n$  not a simple group, for any integer  $n \geq 3$ ?
3. Question XVII.15 of Lang (Conjugation action).
4. Exercise 5.11 from Milne's notes on Class Field Theory.
5. (a) Let  $K$  be a field. Is the algebra  $K[x]$ , considered as a module over itself, semisimple?  
(b) Now assume that the characteristic of  $K \neq 2$ . For  $k \in K$ , we define  $A_k$  to be the algebra  $K[x]/(x^2 - k)$ . Prove that  $A_k$  (as a module over itself) is simple unless  $k$  is a square in  $K$ , and semisimple unless  $k = 0$ .