Math 120c
Lloyd Kilford

Coursework 2
Due: 18th April 2003

Note: E, F, K, L, M are all fields.

1. Suppose that $E = F(\alpha)$ is a finite extension of odd degree. Show that $E = F(\alpha^2)$.

2. Let E and F be finite extensions of K, both contained in an extension M/K. Show that

$$[EF : K] \subseteq [E : K][F : K],$$

   and that we have equality here if $([E : K], [F : K]) = 1$.

3. Let $f \in K[x]$ be a polynomial of degree n, with splitting field L/K. Show that $[L : K]$ divides $n!$.

4. What is the splitting field L of $x^5 - 11$ over $\mathbf{Q}$? What is $[L : \mathbf{Q}]$?

5. Let L/K be an algebraic extension. Let R be a ring such that

$$K \subseteq R \subseteq L.$$

   Show that R is a field. Is this still true if L/K is not an algebraic extension?

6. Let K have characteristic $p > 0$. Show that the extension $K(\alpha)/K$ is separable iff

$$K(\alpha) = K(\alpha^{p^n}) \text{ for all positive } n.$$