

Math 120b - Winter 2003-2004
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Homework set 7
Due: 5th March 2004

Let L be an extension of the field K .

1. Which Galois extensions of \mathbf{Q} of degree 10 are solvable by radicals?
2. (a) Show that equivalence of representations of a group G over K is an equivalence relation.
(b) Let ρ and σ be equivalent representations of a group G over K . Show that if ρ is reducible then σ is reducible.
(c) Let $G = C_{13}$ (cyclic group of order 13). Find a representation of G which is neither reducible nor irreducible.
(d) Let $\rho : G \rightarrow \mathrm{GL}_2(K)$ be a representation. Show that the map $g \mapsto \det(\rho(g))$ is a representation.
3. Let L/K be a Galois extension of degree p^n , where p is prime. Show that there exist intermediate fields of degree p and p^{n-1} over K .
4. Find sufficient conditions on $a, b, c \in \mathbf{Z}$ such that $\mathbf{Q}(\sqrt{a + b\sqrt{c}})/\mathbf{Q}$ is a cyclic extension of degree 4.
5. Let t be transcendental over \mathbf{Q} . Show that $\mathbf{Q}(t, \sqrt{t^3 - t})$ is not a purely transcendental extension of \mathbf{Q} .