

**Math 120b - Winter 2003-2004**

**Lloyd Kilford**

**Homework set 6**

**Due: 27th February 2004**

Let  $L$  be an extension of the field  $K$ .

1. Show that, with a straight-edge, a pair of compasses and an angle-trisector, one can construct the regular 7-gon. Indicate clearly where the angle-trisector is used.
2. Show that  $\bigcup_{n=0}^{\infty} \mathbf{F}_{p^{n!}}$  is an algebraic closure of  $\mathbf{F}_p$ .
3. Suppose that  $K$  has characteristic 0. Show that  $F(X^2) \cap F(X^2 - X) = F$ , where the intersection occurs inside  $F(X)$ . You may wish to find automorphisms of order 2 which fix  $F(X^2)$  and  $F(X^2 - X)$ , and then consider the order of the composition of the automorphisms.
4. Let  $f(X) = X^5 + aX + b$ , with  $a, b \in \mathbf{Q}$ . Show that the Galois group of  $f$  over  $\mathbf{Q}$  is the dihedral group  $D_5$  if and only if:
  - (a)  $f$  is irreducible over  $\mathbf{Q}$ ,
  - (b) the discriminant of  $f$  is a square, and
  - (c) the equation  $f(X) = 0$  is solvable by radicals.
5. Suppose that  $L$  and  $K$  are finite. Show that the norm function  $N_{E/F} : E^\times \rightarrow F^\times$  is surjective.