Math 120b - Winter 2003-2004
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Homework set 3
Due: 6th February 2004

Let L be an extension of the field K.

1. Suppose that K has nonzero characteristic p, that L/K is finite, and that $p \nmid [L : K]$. Show that L is separable over K.

2. Let K be a finite field. Show that every element of K can be written as the sum of at most two squares in K.

3. Let $f \in \mathbf{Q}[x]$ be an irreducible polynomial of odd degree at least 3 over $\mathbf{Q}$. Let u be a root of f. Show that $\mathbf{Q}(u)/\mathbf{Q}$ is not a Galois extension.

4. Let $f \in K[x]$ be an irreducible polynomial. Find and prove a condition on the degree of f involving the characteristic of K for f to be separable.

5. Let $f \in \mathbf{Z}[x]$ be an irreducible quartic with Galois group S_4 over $\mathbf{Q}$. Let α be a root of f. Prove that there are no intermediate fields between $\mathbf{Q}(\alpha)$ and $\mathbf{Q}$. Is $\mathbf{Q}(\alpha)/\mathbf{Q}$ a Galois extension?

6. Show that $f = x^3 - 3x + 1$ is irreducible over $\mathbf{Q}$. What is its Galois group over $\mathbf{Q}$? Find four other cubic polynomials with the same Galois group over $\mathbf{Q}$.