Math 120b - Winter 2003-2004
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Homework set 2
Due: 30th January 2004

Let L be an extension of the field K.

For question 1, you may assume the Fundamental Theorem of Galois Theory, as stated in the lectures.

1. Let ζ_{11} be a primitive 11^{th} root of unity. What are the intermediate fields between $\mathbf{Q}(\zeta_{11})$ and $\mathbf{Q}$, and what are the Galois groups of these fields over $\mathbf{Q}$? Draw the diagram showing all of these.

2. Let char $K = p \neq 0$ and let $L = K(u, v)$ where

$$u^p, v^p \in K \text{ and } [L : K] = p^2.$$

   Show that L/K is not a simple extension, and write down infinitely many intermediate fields.

3. What is the splitting field of $x^p - x - a$ over $\mathbf{F}_p$, where $a \in \mathbf{F}_p^\times$. By proving that $\alpha \mapsto \alpha + 1$ is an automorphism, or otherwise, show that the Galois group is cyclic.

4. Show that an algebraically closed field has infinite cardinality.

5. Show that if the splitting field of a cubic polynomial over $\mathbf{Q}$ is cyclic of order 3 then the three roots of the cubic are real.

6. Find a primitive element for the extension $\mathbf{Q}(\sqrt{3}, \sqrt{5}, \sqrt{7})/\mathbf{Q}$.