Math 160a - Fall 2002
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Homework set 5
Due: 7th November 2002

1. Let a, b and c be squarefree integers which are relatively prime and not all of the same sign. Prove that the quadratic form $ax^2+by^2+cz^2$ represents 0 over $\mathbf{Q}$ if and only if we have that $-bc$ is a square modulo a, $-ac$ is a square modulo b and $-ab$ is a square modulo c.

2. Let p be a prime such that $p \equiv 1 \mod 4$. Show that the quadratic form $x^2 + y^2 - pz^2$ has a solution over $\mathbf{Q}$. Use the Davenport-Cassels theorem to show that p can be written as the sum of two square integers.

3. Let p be a prime such that $p \equiv 1 \mod 8$ or $p \equiv 3 \mod 8$. Show that the quadratic form $x^2 + 2y^2 - pz^2$ has a solution over $\mathbf{Q}$. As before, use the Davenport-Cassels theorem to show that $p = a^2 + 2b^2$ for some $a, b \in \mathbf{Z}$. Comment briefly on why the Davenport-Cassels theorem as stated in class would not suffice to show the existence of *integral* solutions if we were considering the quadratic form $x^2 + 3y$.

4. Prove that any positive definite binary integral quadratic form of rank 2 and discriminant 1 is equivalent over $\mathbf{Z}$ to $x^2 + y^2$.

5. Let a be an integer, and let f be the quadratic form $2x^2+2xy+3ayz+z^2$. For which values of a is f positive definite, negative definite, or indefinite?

6. Give an example of two quadratic forms over $\mathbf{Z}_p$ which both have discriminant p^2 but which are not isomorphic *over* $\mathbf{Z}_p$. Can this happen for discriminant p?