1. Let $a$, $b$ and $c$ be squarefree integers which are relatively prime and not all of the same sign. Prove that the quadratic form $ax^2 + by^2 + cz^2$ represents 0 over $\mathbb{Q}$ if and only if we have that $-bc$ is a square modulo $a$, $-ac$ is a square modulo $b$ and $-ab$ is a square modulo $c$.

2. Let $p$ be a prime such that $p \equiv 1 \mod 4$. Show that the quadratic form $x^2 + y^2 - pz^2$ has a solution over $\mathbb{Q}$. Use the Davenport-Cassels theorem to show that $p$ can be written as the sum of two square integers.

3. Let $p$ be a prime such that $p \equiv 1 \mod 8$ or $p \equiv 3 \mod 8$. Show that the quadratic form $x^2 + 2y^2 - pz^2$ has a solution over $\mathbb{Q}$. As before, use the Davenport-Cassels theorem to show that $p = a^2 + 2b^2$ for some $a, b \in \mathbb{Z}$. Comment briefly on why the Davenport-Cassels theorem as stated in class would not suffice to show the existence of integral solutions if we were considering the quadratic form $x^2 + 3y$.

4. Prove that any positive definite binary integral quadratic form of rank 2 and discriminant 1 is equivalent over $\mathbb{Z}$ to $x^2 + y^2$.

5. Let $a$ be an integer, and let $f$ be the quadratic form $2x^2 + 2xy + 3ayz + z^2$. For which values of $a$ is $f$ positive definite, negative definite, or indefinite?

6. Give an example of two quadratic forms over $\mathbb{Z}_p$ which both have discriminant $p^2$ but which are not isomorphic over $\mathbb{Z}_p$. Can this happen for discriminant $p$?