Math 160a - Fall 2002
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Homework set 4
Due: 31st October 2002

1. Classify all quadratic forms over $\mathbf{R}$ up to equivalence.

2. Let $a \in \mathbf{Q}_p^*$. Show that the quadratic forms

$$x_1^2 + x_2^2 + x_3^2 + x_4^2 \text{ and } a(x_1^2 + x_2^2 + x_3^2 + x_4^2)$$

are equivalent over $\mathbf{Q}_p$. Are they equivalent over $\mathbf{R}$ if $a \in \mathbf{R}^*$?

3. Let $a \in \mathbf{Q}_p$, and suppose that

$$\begin{aligned} |a-1|_p &\leq p^{-1} \text{ if } p \neq 3 \\ |a-1|_3 &\leq 3^{-2}. \end{aligned}$$

Using Hensel's Lemma (or otherwise) show that $a \in (\mathbf{Q}_p)^3$.

4. For which primes p do the following forms represent 0 over $\mathbf{Q}_p$?

$$2x_1^2 + 3x_2^2 + 5x_3^2 \text{ and } x_1^2 + 11x_2^2 + x_3^2 + 13x^4.$$

5. Let k be a field of characteristic not equal to 2, and let $Q(x, y)$ be a binary quadratic form over k. Show that the set of values $Q(k)$ taken by Q is closed under taking products if and only if Q represents 1.