Math 160a - Fall 2002
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Homework set 3
Due: 24th October 2002

1. Let $a \in \mathbf{Z}$. Prove that a is a perfect square if and only if $a \mod p$ is a perfect square for all p.

2. Find a rational number r such that $|r-5|_3 < \frac{1}{3}$, $|r-1|_5 < 1$ and $|r+2|_\infty < \frac{1}{20}$.

3. Let $\alpha \in \mathbf{Q}_p^*$, where $p \neq 2$. Prove that there exist four numbers $\alpha_1, \alpha_2, \alpha_3, \alpha_4 \in \mathbf{Q}_p$ such that exactly one of the numbers $\alpha\alpha_1, \alpha\alpha_2, \alpha\alpha_3, \alpha\alpha_4$ has a square root in $\mathbf{Q}_p$.

4. Show that the Hilbert symbol $(2, -3)_5$ is 1 by exhibiting a solution to the quadratic form $f(z, x, y)$ in the definition of the Hilbert Symbol.

5. Let $m \in \mathbf{Z}_p$. Show that the expansion of m stabilises (that is, there exists an integer k such that $\varepsilon_k(m) = \varepsilon_{k+N}(m)$ for all $N > 0$) if and only if m is a positive rational number which has no powers of p in its denominator.

6. Choose a prime l. Show that $-l$ is a square in $\mathbf{Q}_p$ for infinitely many primes p. You may assume Dirichlet's theorem on primes in arithmetic progression.