

Coursework 3 for Elliptic Curves class

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1. The curve given by

$$D : X^2 - 2Y^2 + Z^2 = 0, Y^2 - 2Z^2 + T^2 = 0$$

can be transformed into the Weierstrass form

$$E : Y^2 = X^3 - X^2 - 4X + 4.$$

Find the rank of this curve (it has the point $(1, 0)$), and hence show that if

$$n_1 < n_2 < n_3 < n_4$$

are integers in arithmetic progression, then they are not all perfect squares.

You may assume that any torsion point on the curve E does not give a point other than

$$x^2 = y^2 = z^2 = t^2$$

on D .

2. Exercise III.3.8 from Silverman and Tate (on the rank of $y^2 = x^3 + px$ for certain primes p).