

Coursework 2 for Elliptic Curves class

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1. Let $B \in \mathbf{Z}$ be sixth power free. Fill in the details of the argument attached to show that the torsion points of the curve

$$E : Y^2 = X^3 + B$$

are given by

- (a) If $B = C^2$, then the points $(0, \pm C)$ are points of order 3.
- (b) If $B = D^3$, then the point $(-D, 0)$ has order 2.
- (c) If $B = 1$, then the points $(2, \pm 3)$ have order 6.
- (d) If $B = -432$, then the points $(12, \pm 36)$ have order 3.

Explain briefly what happens if B is not sixth power free.

2. Prove that there is a positive rational solution to $x^3 + y^3 = a$ if and only if there is a positive rational solution to $x^3 - y^3 = a$.