Coursework 1 for Elliptic Curves class

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You are encouraged to use computer software to help with these results. You should attach either a printout or a description of any computations that you do.

One possibly useful site is at

http://www.math.mcgill.ca/connell/public/apecs/

which has a MAPLE package dealing with elliptic curves.

PARI-GP is a free software program which does arithmetic on elliptic curves. Their website is

http://www.parigp-home.de/

1. Transform the curve given by

$$X^2 - 2Y^2 + Z^2 = 0,\ Y^2 - 2Z^2 + T^2 = 0$$

into canonical Weierstrass form.

Find the torsion points on this curve.

2. Show that the cubic

$$F(X, Y, Z) := a_1X^3 + a_2Y^3 + a_3Z^3 + dXYZ = 0$$

is non-singular if and only if $27a_1a_2a_3 + d^3 \neq 0$.

Prove that the following formulae hold:

(a) The third point $t = (t_1, t_2, t_3)$ of intersection of the cubic and the tangent at (x_1, x_2, x_3) is given by

$$t_i := x_i(a_{i+1}x_{i+1}^3 - a_{i+2}x_{i+2}^3),\ \text{for } i = 1, 2, 3.$$

(b) If x and y are two distinct points on the curve F then the third point of intersection z of the line through x and y is given by

$$z_i = x_i^2y_{i+1}y_{i+2} - y_i^2x_{i+1}x_{i+2}.$$

[In these formulae we take the subscripts modulo 3.]

3. Find 10 distinct points on the curve $X^3+Y^3+7Z^3$; one point is $(2,-1,-1)$. Find another curve of the form $X^3+Y^3+\alpha Z^3$ with more than 1 point on it.