An Opto-Mechanical Microwave-Rate Oscillator

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Diameter of a human hair
Opto excited Vibration: Explanation

- Experimental setup for modal spectroscopy
- Experimental results:
  1) Stokes and Anti-Stokes lines (device vibrations)
  2) Selective excitation of many spectral lines
  3) Going above GHz to microwave rate vibration
  4) Line split (perturbation induced)
  5) Continuum

Start with casting this story into a set of equations
Dynamic Back-Action
(V. Braginsky 1985)

\[
\begin{aligned}
\frac{d^2 x}{dt^2} + \left( \frac{\Omega}{Q_{\text{mech}}} \right) \frac{dx}{dt} + \Omega^2 x &= \frac{F_{\text{rad}}}{m} = \frac{2\pi n}{cm} |A|^2 \\
\frac{dA}{dt} &= i \sqrt{\frac{c \omega_0}{2\pi n R Q_c}} B - \left( \frac{\omega_0}{2Q_{\text{total}}} + i\Delta\omega \right) A \\
\frac{\Delta\omega}{\omega_0} &= + \frac{\omega_0}{R} x \\
|A|^2 &= P_{\text{cav}} \\
|B|^2 &= P_{\text{in}} \\
\end{aligned}
\]

Mechanical Gain/Loss

\[
\begin{aligned}
P_{\text{cav}} &= P_{\text{cav}}^{(0)} - P_{\text{cav}}^{(0)} \left( \frac{4\beta}{\beta^2 + Q_{\text{tot}}^{-2}} \right) \frac{x}{R} + P_{\text{cav}}^{(0)} \left( \frac{16Q_{\text{tot}}^{-1}\beta}{(\beta^2 + Q_{\text{tot}}^{-2})^2} \right) \frac{1}{R\omega_0} \int dx \\
\beta &= \frac{2\Delta\omega_0}{\omega_0} \\
\gamma &= \gamma_0 (1 - P / P_{\text{threshold}}) \\
\gamma_0 &= \frac{\Omega}{Q_{\text{mech}}} 
\end{aligned}
\]
Theory

Experimental Observation of Dynamical Effect

CLEO 2005

Chaos: Non-periodic dynamic.


CLEO 2006

First observation of oscillations

Carmon, **PRL** **94**, 223902 (2005).


Kippenberg **PRL** **95**, 033901 (2005).

CLEO 2007

Today:

Spectroscopy at >GHz freq

- Continuum
- Line split.
- Stokes and Anti-Stokes lines


Cooling, CLEO Thu 8AM


Schliesser et al, **PRL** **97**, 243905 (2006)

Harris et. al **Rev. of Sci. Ins.** **78** 013107 (2007)

Corbitt et al. **PRL** **98**, (2007)

Measurement of weak forces in Physics Experiments (1977)

"The oscillations will increase its amplitude and then become stationary”

Braginsky & Manukin

"Damping increases with intensity”.
Resonance enhancement

\[ I = \frac{N \times \text{(source power)}}{\text{Area}} \]

\[ N = 100,000 \]
\[ P = 1 \text{ mWatt} \]
\[ \text{Area} = 1 \mu m^2 \]

\[ I = 10^{10} \text{ Watt} / cm^2 \]

Multiple passes of the acoustical (electro-magnetic) wave enhance the mechanical (optical) amplitude.

Swing
Experimental setup

Frequency comparable with the \((\text{velocity of sound})/(\text{cavity size})\)

- **Cavity was not designed to vibrate**
- **Effect came as an unexpected surprise**

Experimental results

(1) Stokes and anti-Stokes lines

Period doubling and continuum

Low power
Periodic oscillations

Medium power
(2) Period doubling
(Still periodic)

High power
Turns into a (3) continuum

4) Selectively exciting different modes

- By tuning the photon lifetime to be comparable with the acoustical period
- Modal spectroscopy: deduce from the spectral line of the scattered light on the mechanical vibrational mode.

**Spectroscopy:**

Perturbation $\rightarrow$ fine split of spectral line

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### Theory

<table>
<thead>
<tr>
<th>Calculated modes</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Mode 1" /> 30 $\mu$m</td>
</tr>
<tr>
<td><img src="image2.png" alt="Mode 2" /> 112.32 MHz</td>
</tr>
<tr>
<td><img src="image3.png" alt="Mode 3" /> 112.58 MHz</td>
</tr>
<tr>
<td><img src="image4.png" alt="Mode 4" /> 28.5 $\mu$m</td>
</tr>
</tbody>
</table>

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### Experiment

Results

5) Line split (perturbation induced)

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## Comparison between light-matter and light-structure interaction

<table>
<thead>
<tr>
<th></th>
<th><strong>Raman</strong></th>
<th><strong>Opto-excited vibration</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Origin</strong></td>
<td>Molecular vibration freq.</td>
<td>Cavity vibration freq</td>
</tr>
<tr>
<td><strong>Frequency</strong></td>
<td>THz</td>
<td>MHz to GHz</td>
</tr>
<tr>
<td></td>
<td>(matter)</td>
<td>(structure)</td>
</tr>
<tr>
<td><strong>Line split</strong></td>
<td>Magnetic perturbation</td>
<td>Eccentricity perturbation</td>
</tr>
<tr>
<td><strong>High power</strong></td>
<td>Spectral continuum</td>
<td>Spectral continuum</td>
</tr>
<tr>
<td><strong>Period doubling</strong></td>
<td>Can we learn?</td>
<td>√</td>
</tr>
</tbody>
</table>

shown that in every case in which light is scattered by the molecules in dust-free liquids or gases, the diffuse radiation of the ordinary kind, having the same wave-length as the incident beam, is accompanied by a modified scattered radiation of degraded frequency.

The new type of light scattering discovered by us naturally requires very powerful illumination for its observation. In our experiments, a beam of sunlight was converged successively by a telescope objective of 18 cm. aperture and 230 cm. focal length, and by a second lens of 5 cm. focal length. At the focus of the second lens was placed the scattering material, which is either a liquid (carefully purified by repeated distillation in vacuo) or its dust-free vapour. To detect the presence of a modified scattered radiation, the method of complementary light-filters was used. A blue-violet filter, when coupled with a yellow-green filter and placed in the incident light, completely extinguished the track of the light through the liquid or vapour. The reappearance of the track when the yellow filter is transferred to a place between it and the observer’s eye is proof of the existence of a modified scattered radiation. Spectroscopic confirmation is also available.

Some sixty different common liquids have been examined in this way, and every one of them showed the effect in greater or less degree. That the effect is a true scattering and not a fluorescence is indicated in the first place by its feebleness in comparison with the ordinary scattering, and secondly by its polarisation, which is in many cases quite strong and comparable with the polarisation of the ordinary scattering. The investigation is naturally much more difficult in the case of gases and vapours, owing to the excessive feebleness of the effect. Nevertheless, when the vapour is of sufficient density, for example with ether or amylene, the modified scattering is readily demonstrable.

C. V. RAMAN.
K. S. KRISHNAN.

Atlantic the salmon generally leaves such colonisation to the trout, and itself forms fresh-water colonies only in exceptional circumstances, either in very large lakes with abundance of fishes, or in rivers or lakes with such quantities of parr-food that it is tempted to prolong the parr life. In America, when there are no trout, the salmon form fresh-water colonies more readily.

C. TATE REGAN.

British Museum (Natural History),
S.W.7, Mar. 17.

Anomalous Groups in the Periodic System of Elements.

In a paper which will shortly appear in the Rend. Accad. Lincei, I have calculated the distribution of the electrons in a heavy atom. The electrons were considered as forming an atmosphere of completely degenerated gas held in proximity to the nucleus by the attraction of the nuclear charge screened by the electrons. Formulae were given for the density of the electrons and the potential as functions of the distance $r$ from the nucleus.

In continuation of the previous work, I have applied the same method to the study of the formation of anomalous groups in the periodic system of elements. From the density of the electrons and their velocity distribution, one can easily calculate how many electrons have a given angular momentum in their motion about the nucleus, that is, how many electrons have a given azimuthal quantum number $l$.

It is known, for example, that the formation of the group of the rare earths corresponds to the bounding of electrons in $4f$ orbits, that is, to the presence in the atom of electrons with $l = 4$. Now it follows from the theory that electrons with $l = 4$ exist in the normal state only for atoms with atomic number $z \geq 55$. This agrees well with the empirical result that the group of the rare earths begins at $z = 58$ (cerium).
Conclusions

• **Mode spectroscopy, Experimental demonstration**
  - Selective excitation of different mechanical modes
  - High order mechanical modes vibration at microwave rates (>GHz)
  - Deducing on the mechanical vibrational mode from the optical spectral lines of the scattered light (*Spectral Signature*)
  - Line split
  - Doubling
  - Continuum

• **No external feedback or modulation**
  - Intrinsic cavity properties.

• **Different geometries (tori and spheres)**
  - Behaviour is relevant to different types of resonators

• **Sustained trend in**
  - Miniaturization (k~area/length, small=soft)
  - Dissipation reduction (mechanical- and optical-resonance enhancement)

Similar vibrations in other platforms at even higher frequencies are expected

energy of the input light is tuned to one microsphere as it vibrates from an oblate

And with others studying optical effects in systems such as crystals, these sorts of optoelectrical devices could be put to use elsewhere.