Exploit Broadcast Advantage in Wireless Networks

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Joint work with Lijun Chen and Tracey Ho
Outline

1. Wireless Networks, Multicast, and Network Coding
2. Cross-Layer Design with Broadcast Advantage
3. Distributed Hypergraph Matching
4. Experimental Results
Wireless Networks

- Wireless networks have significantly impacted the world.
- Can be classified as
  - Cellular Networks
  - Wireless Sensor Networks
  - Wireless Ad-hoc Networks
- Play important roles in
  - Military communication,
  - Commercial communication,
  - Education ....
Challenging problems:
- Broadcast interference,
- Distributed control ...

Broadcast advantage:
- Every transmission by a node can be received by all nodes that lie within its communication range.
- Possible power saving and throughput improvement especially with broadcasting and multicasting.

Question: How to exploit the broadcast advantage in a distributed fashion?
Multicast

- Multicast delivers information to a group of receivers simultaneously.
- It uses each link of the network only once, and creates copies only when the links to the receivers split.
- Useful in applications such as teleconferencing and audio/video broadcasting.
Network Coding

- Conventional packet networks: each node’s functions are limited to the forwarding or replication of received packets.

- Network coding:
  - Each node is allowed to perform algebraic operations on received packets.
  - Necessary to achieve multicast capacity in some networks.
  - Complexity benefits.
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Directed Hypergraph

Hypergraph:
- The network is modelled as a directed hypergraph $\mathcal{H} = (\mathcal{N}, \mathcal{A})$.
- A hyperarc is a pair $(i, J)$, e.g., $a = (1, \{2, 3\})$.
- Each hyperarc $(i, J)$ represents a broadcast link from node $i$ to nodes in $J$.

Variables:
- $g_{iJ}^{\text{mst}}$: virtual flow from source $s$ to sink $t$ over $(i, J)$ and is intended to node $j \in J$, e.g., $g_{1a2}^{14}$.
- $f_{ij}^m$: physical flow of session $m$ on $(i, J)$.
- $r_{ij}$: achievable rate on $(i, J)$.
Problem Formulation

Network resource allocation problem:

- Maximize $\sum_{m,s} U_{ms}(x^{ms})$
- Subject to:
  - Flow conservation of $g_{ij}^{ms}$
  - Network coding: $\sum_{s,j} g_{ij}^{ms} \leq f_{ij}^m$
  - Rate constraint: $\sum_m f_{ij}^m \leq r_{ij}$, $(r_{ij}) \in \text{Co}(P, S))$

Solution:

- Dual decomposition.
- Introduce dual variable $q_{ij}^{ms}$ at each node, which can be interpreted as queue length.
Each source adjusts its sending rate according to the aggregate queue lengths

\[ x^{ms}(\tau + 1) = U'_{ms} \left( \sum_t q^{mst}_s(\tau) \right) \]
For each \((i, J)\), the session with the maximum aggregate queue difference is chosen, i.e., \(\hat{m} = \arg \max_m \sum_t \max_{s,j \in J} \left[ q_{imst}^{mst} - q_{jmst}^{mst} \right]^{+}\).

A random linear combination of packets from sources in \(\hat{m}\) is broadcast to all nodes in \(J\) at the rate of \(r_{iJ}\).
Link Scheduling and Queue Length Update

Link Scheduling:

- Define \( w_{iJ} = \max_m \sum_t \max_{s,j \in J} \left[ q_i^{mst} - q_j^{mst} \right]^+ \).
- Link scheduling problem:

\[
\max_{r,P} \sum_{(i,J) \in A} w_{iJ} r_{iJ}, \text{ s.t. } (r_{iJ}) \in \text{Co}(r(P, S))
\]

Queue Length Update:

- At the end of each time slot, each node passes its queue length to all its neighbors for next time slot rate control, scheduling and network coding.
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Hypergraph Matching Under the Primary Interference Model

- Primary interference model:
  - Each node is equipped with only a single transceiver. Hyperarcs that share a common node cannot be active simultaneously. CDMA or FDMA is used.

Hypergraph matching

A set of hyperarcs with no pair incident to the same node.

- Link scheduling becomes finding a maximum weighted hypergraph matching in $\mathcal{H}$ with weight $w_{ij} r_{ij}$.
Hypergraph matching is NP-complete [Lovasz86].

Global greedy algorithm:
- Adds a globally maximum weight edge into the matching.
- Hard to be decentralized.

Local greedy algorithm:
- Adds a locally heaviest hyperedge into the hypergraph matching at each step.

Locally heaviest hyperedge
A hyperedge is locally heaviest if its weight is at least as large as the weight of all adjacent hyperedges.

- Linear-time complexity and fully decentralized.
- Achieve an approximation ratio \(\max\left\{\frac{1}{K}, \frac{1}{\kappa}\right\}\), where \(K\) is the maximum cardinality of the hyperedges and \(\kappa = \max_{m \in M} |T_m| + 1\).
Greedy Algorithms (Cont.)

- Local greedy algorithm sometimes performs not well.
  - Some matched nodes may not contribute much to this locally heaviest hyperedge.
  - When these nodes are matched in other hyperedges, they may contribute more.

- Improved greedy algorithm
  - Use the average hyperedge weight, i.e., $\bar{w}_e = w_e/|e|$.
  - Achieve the same approximation ratio but perform better in practice.
Randomized Algorithm

- Randomized algorithm to find a maximal hypergraph matching.

Algorithm

1. Each unmatched node $i$ attempts to transmit with probability $p_i$.
2. If $i$ attempts to transmit, it sends a matching request to each neighbor with probability $1/2$.
3. If node $i$ does not transmit and it receives several matching requests from its neighbors, it chooses one of them uniformly at random and sends a matched message.
4. For all the nodes that decide to transmit, the nodes that reply with a matched messages are added into the hypergraph matching.
5. Continue until no hyperedge can be found.

- The expected running time is $O(\log |\mathcal{E}|)$.
- Stabilizes the system for any rate vector from $\frac{1}{K}\Lambda$. 

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Path-loss exponent: 1.

All nodes have unity signal power and identical noise power 0.1.
Rate Evolution

Number of Iterations vs Rate of Source $s_1$

- Maximum Weighted Hypergraph Matching
- Local Greedy Matching Algorithm 1
- Local Greedy Matching Algorithm 2
- Randomized Algorithm
- Maximum Weighted Graph Matching
- Local Greedy Graph Matching

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### Table: Comparison of Different Algorithms in the Wireless Butterfly Network.

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Throughput gain increases as the number of sinks increases.
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Summary

- Distributed optimization with broadcast advantage.
- Distributed low complexity hypergraph matching algorithms for link scheduling.
- Ongoing work:
  - Extension to data gathering in wireless sensor networks.
  - New hypergraph matching algorithms
  - Extension to other interference models.