LIGO-T890017-00-R

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List of 3/1/89

Light Scattering and Proposed Baffle Configuration for the LIGO

Second Draft, 11 January 1989

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ABSTRACT

When light, hitting a mirror, scatters out of the main beam of an interferometric gravitational-wave detector, then makes its way back into the beam via reflection, scattering, and/or diffraction off the LIGO vacuum pipe, baffles, or mirrors, it produces a slight phase shift in the main beam's light. Wiggle of the main beam and vibrations of the vacuum pipe and baffles cause this phase shift to oscillate, simulating a gravitational wave. The dominant contributions to this gravitational-wave noise are here computed; and those computations are used to suggest constraints on the design of the baffles for the LIGO. If these constraints are followed, there is no reason to expect serious problems with light scattering in the LIGO, even in very advanced detectors of the highest currently projected sensitivities. By contrast, without baffles there would be severe and perhaps insurmountable light-scattering noise.

Contents

* Supported in part by the National Science Foundation Grants AST-8514911 and PHY-8803557

I. Overview

This document is divided into three main sections: Section II presents the recommended constraints on baffle design that come out of the scattering calculations. Section III presents the results of the scattering calculations: the spectral density $\tilde{h}^2(f)$ of the gravitational-wave noise due to the dominant scattering processes, and a set of simple formulae that can be used to compute scattering noise from these and other processes. Section IV sketches the details of the scattering calculations.

II. Recommended Constraints on Baffle Design and Other LIGO Features.

1. Baffle Spacing and Height

The most severe scattering noise in the absence of baffles comes from light that leaves one mirror and travels down the vacuum pipe, reflecting many times off the pipe walls, until it reaches the other mirror. It is essential that the baffles stop all such light.

The larger the angle θ that a light ray makes to the pipe's central axis, the greater the number of reflections it will undergo in traveling from one end of the pipe to the other, and the greater will be the power lost from the ray to scattering and absorption at each reflection. Correspondingly, there is a maximum angle θ_o such that for $\theta > \theta_o$ light is strongly attenuated in traveling from one end of the pipe to the other, while for $\theta < \theta_o$ it is weakly attenuated. The turn on of attenuation as θ increases through θ_o is very sharp; see Eq. (2.3) below. The baffles must be designed to stop (almost) all light with angles $\theta < \theta_o$; light with $\theta > \theta_o$ will get adequately attenuated without the help of baffles. If the pipe is corrugated, then θ_o will be approximately equal to the "Rayleigh angle" below which the tops of the corrugations look like an excellent mirror:

$$\theta_o \sim \theta_R = \left(\frac{2\lambda}{a}\right)^{1/2} = 0.6\times10^{-2}\left(\frac{\lambda}{0.4\,\mu\text{m}}\right)^{1/2}\left(\frac{1.5\,\text{cm}}{a}\right)^{1/2} . \qquad (2.1)$$

Here λ is the wavelength of light used, $2\pi a$ is the wavelength of the corrugations, and it is assumed that the corrugations have an amplitude (half the peak-to-trough distance) of a. If the pipe is not corrugated, then θ_o will be somewhat larger than (2.1). Rai Weiss (private communication) shows that for stainless steel and for the least attenuated polarization component of the light, and for angles $\theta \ll \pi/2$, the fraction of the photons lost in each bounce is $0.55\,\theta$ and correspondingly, the fraction that survive is $1-0.55\,\theta$. Since the total number of reflections in going from one end of the pipe to the other is no less than $L\theta/2R$ where L is the length of the pipe and R is its radius, the fraction of the photons that survive the entire trip is

$$(1-0.55\,\theta)^{L\theta/2R} \cong \exp\left(-\frac{L\theta^2}{3.6R}\right) , \qquad (2.2)$$

which cuts off very sharply at

$$\theta_o = 4\sqrt{3.6R/L} = 0.1 . \qquad (2.3)$$

For $\theta = 0.1$ the attenuation is 10^8 in energy (10^4 in amplitude, which is the relevant thing for scattering noise $\tilde{h}$); for $\theta = 0.05$ the attenuation is only 10^2 in energy (10 in amplitude). For further details see a forthcoming report by Weiss. It will be important in the design of the baffles to have a fairly accurate estimate of θ_o. Such an estimate and methods of minimizing θ_o are being developed by Weiss.

These considerations, together with a calculation of the dominant scattering noise effects in the presence of baffles (Secs. III.C and IV.C below) produce the following recommendation for constraints on the heights and spacings of the baffles:

Recommendation 1. Consider any point at the end of the pipe, at which the center of a mirror might be

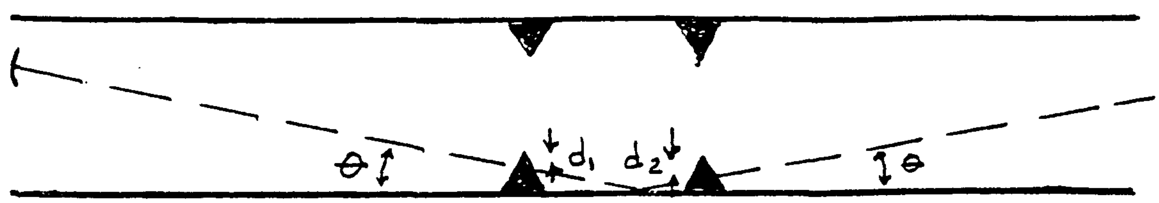

Fig. 2.1. The encounter of a ray that has angle θ with a pair of baffles. The distances d_1 and d_2 must satisfy $d_1+d_2 \geq 12\delta H$.

placed. Consider any straight ray eminating from that point in any direction that makes an angle $\theta < \theta_o$ with the pipe's central axis (Fig. 2.1). Follow that ray down the pipe, letting it penetrate straight through any baffles it encounters and reflecting it specularly off the wall at each wall encounter. The baffles must be so designed that the ray at least twice — once near the first end of the pipe and once near the second end — encounters a pair of baffles in the manner of Fig. 2.1; and in each of these encounters the points at which the ray penetrates the baffles must be at distances d_1 and d_2 below the baffle tops such that $d_1+d_2 \geq 2\delta H$. Here δH is a *height safety factor* that appears in the diffraction calculations. Those calculations suggest that $\delta H = 1$ cm is a reasonable value. Of course, rays that never encounter the pipe are not subject to the above constraint; and rays that encounter it only once are required to have only one encounter of the type shown in Fig. 2.1. No baffles should be closer to any detector mirror than 10 meters. [*Note*: The requirement for two encounters of the type of Fig. 2.1, one near each end of the pipe, requires for its implementation twice as many baffles as would be needed if one such encounter were sufficient. If the cost of the baffles is inordinately high we could consider relaxing the requirement back to one such encounter, and insist that it occur near the corner mirror. For a discussion of the dangers inherent in such a decision, see Sec. III.C.]

The following is a suggestion of how to implement this recommendation; see Fig. 2.2: Begin at the plane of the corner mirror and move into the vacuum pipe some (arbitrary) small distance l_1 — but a distance of at least 10 meters and at least as large as (the closest an interferometer beam is likely to come to the pipe wall)/θ_o. At this location place the first baffle. Thereafter, for a distance $2R/\theta_o$ down the pipe place baffles with heights and spacings governed by the following law: If H_n is the height of baffle n and s_n is the spacing between baffles n and $n+1$, then

$$H_{n+1}+H_n \geq \theta_o s_n + 2\delta H \ . \tag{2.4a}$$

(Provision is made here for the possibility that one might want to vary the baffles' spacings so as to put the baffles at particularly convenient locations, and correspondingly one might want to vary the baffle height.) This first series of baffles, which terminates after a distance $2R/\theta_o$, is designed to intercept in the manner of Fig. 2.1 all rays with $\theta = \theta_o$. Thereafter, moving on down the pipe, the baffle spacing can gradually increase: Baffle n at a distance d_n from the first baffle, and baffle $n+1$ at $d_{n+1} = l_n + s_n$ must have heights and separations given by

$$H_n + H_{n+1} \geq \frac{s_n}{d_n} 2R + 2\delta H \ . \tag{2.4b}$$

Once the middle of the pipe has been reached, this sequence of baffles stops. Then, if the requirement is

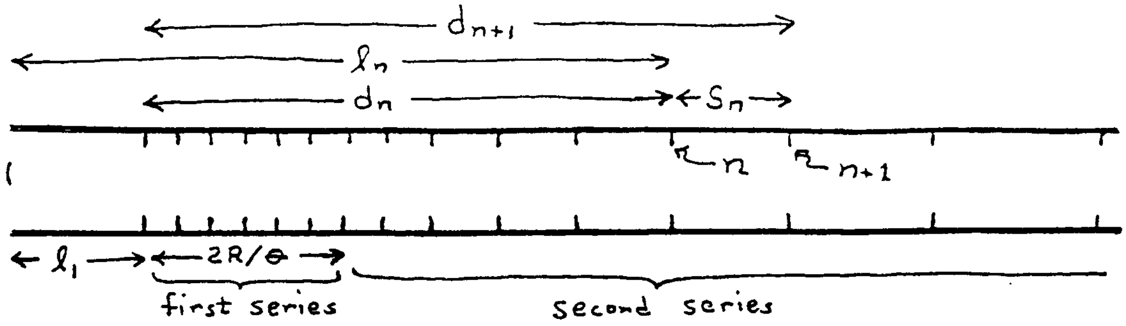

Fig. 2.2 A possible implementation of Recommendation 1 on baffle heights and spacings.

relaxed to only one encounter of the type of Fig. 2.1, no baffles are needed in the outer half of the pipe. If, however, the recommendation of two such encounters is followed, then one must begin once more at the outer end of the pipe and build two sequences of baffles in the same manner as above [Eq. (2.4a) for a distance $2R$, then (2.4b) to pipe's middle].

The total number of baffles in this implementation is readily shown to be, if baffles are put in both ends of the pipe as recommended,

$$N_b = \frac{2R}{H-\delta H}\left[1+\ln\left(\frac{L\theta_o}{4R}\right)\right], \qquad (2.5)$$

where H is the mean baffle height and δH is the height safety factor. For a smooth pipe with $\theta_o = 0.1$ this formula gives $N_b \simeq 146[5\text{ cm}/(H-\delta H)]$. For a corrugated pipe with θ_o=0.01 it gives $N_b \simeq 90[5\text{ cm}/(H-\delta H)]$. If we are willing to live dangerously and baffle only the inner half of the pipe, the number of baffles is reduced to $N_b \simeq 73[5\text{ cm}/(H-\delta H)]$ for a smooth pipe and $N_b \simeq 45[5\text{ cm}/(H-\delta H)]$ for a corrugated pipe.

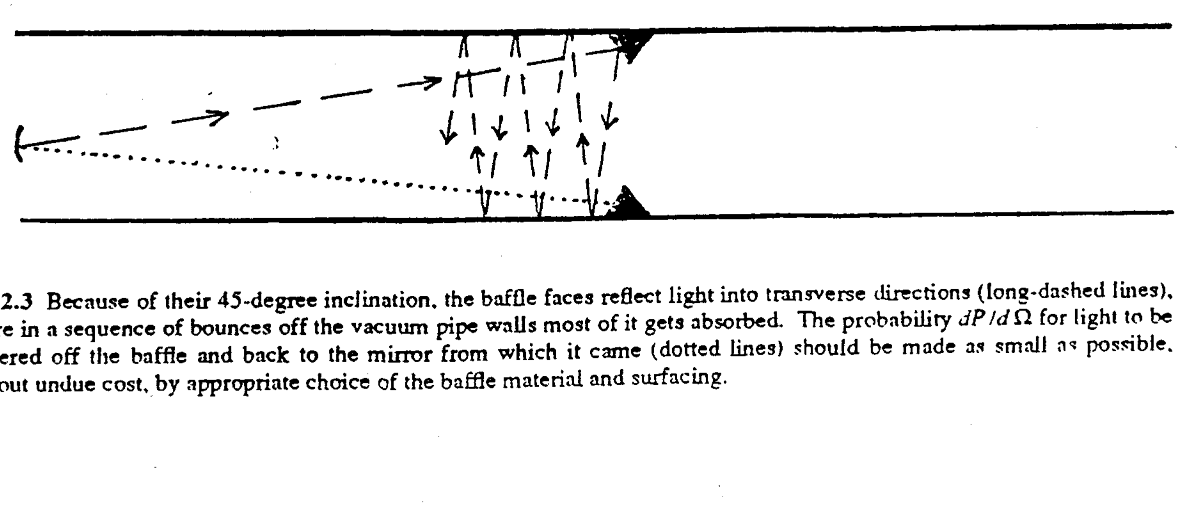

Fig. 2.3 Because of their 45-degree inclination, the baffle faces reflect light into transverse directions (long-dashed lines), where in a sequence of bounces off the vacuum pipe walls most of it gets absorbed. The probability $dP/d\Omega$ for light to be scattered off the baffle and back to the mirror from which it came (dotted lines) should be made as small as possible, without undue cost, by appropriate choice of the baffle material and surfacing.

2. Shape and Reflectivity of the Faces of the Baffles.

The faces of the baffles should be highly reflecting and should be inclined at an angle of about 45 degrees to the vacuum pipe wall. This will enable them to reflect most of the scattered light into transverse directions, thereby preventing it from subsequently reaching either mirror, except by one or more scatterings; see Fig. 2.3. Stated more precisely:

Recommendation 2. (i) The scattering probability $dP/d\Omega \equiv d\sigma/d\Omega dA$ is defined to be the cross section $d\sigma$ for a unit area dA of baffle to scatter light into a unit solid angle $d\Omega$. This scattering probability should be made as small as possible, without undue expense, for light that comes in from a mirror (hitting the baffle surface at 45 degrees to its normal), and is scattered back toward that same mirror. (ii) The baffle shape and tolerances on its shape should be chosen to guarantee that the total amount of light that is reflected (specularly) off the baffle and that subsequently makes its way back to the mirror by any route whatsoever (e.g., via the dashed lines of Fig. 2.4) is less than the amount that is scattered directly off the baffle and back to the mirror (dotted lines in Fig. 2.3). [*Note*: This is especially important for baffles near the ends of the pipe, and less so for baffles farther from the ends; see the discussion of baffle scattering in Sec. III.D.]

This probably will not be difficult to achieve. It will constrain, for example, the sharpness of the baffle edges and the precise angle of the baffle faces to the vacuum wall (e.g. 45.5 degrees versus 46 degrees) and tolerances on that angle.

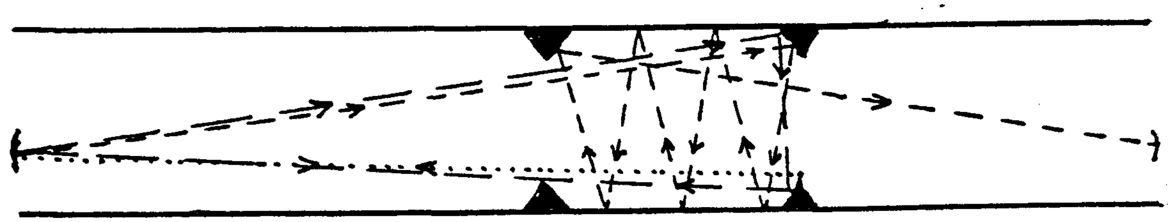

Fig. 2.4 Several routes by which light that reflects (rather than scatters) off a baffle can subsequently make its way back to a mirror. The baffle shape and tolerances should be specified so as to guarantee that the total amount of light that gets back to the mirrors by such routes is less than the amount that is scattered directly back from the baffles (dotted route in Fig. 2.3).

3. Shape of the baffle edges.

Cadez, Saulson, Weiss, and I have discussed at various times the desirability of making the edges of the baffles jagged, i.e. giving them quasi-random variations in height with amplitudes of a few millimeters and wavelengths as short as is convenient. The purpose of this jaggedness is to randomize the phase of light that diffracts off the baffle edge, thereby lessening the gravitational-wave noise that results from such light. It turns out that jaggedness has a significant beneficial effect only in the case of mirrors that are within a distance $\sim\sqrt{\lambda L} = 4$ cm of the center of the pipe's cross section (see Sec. IV.E for details); and even then the randomness of phases introduced by irregularities in the mirrors, the photodiode, and the shape of the vacuum pipe might be sufficient to protect adequately against coherent effects without the additional help of jaggedness. However, it is not certain that this is the case. Therefore, I am led to the following recommendation:

Recommendation 3. I recommend that the edges of the baffles be made jagged rather than smooth. The jaggedness should consist of height variations with rms peak-to-peak amplitudes

$$\Delta H \geq \frac{\lambda L}{2R} = 2\,\mathrm{mm}\left(\frac{\lambda}{1.2\,\mu\mathrm{m}}\,\frac{L}{4\,\mathrm{km}}\,\frac{60\,\mathrm{cm}}{R}\right). \qquad (2.6)$$

Here λ is the longest wavelength of light expected to be used in the LIGO. The height variations can be random or regular; it does not matter which. What is important is (i) that they have the shortest wavelength that can be achieved without great effort, preferably $\ll 1$ cm; and (ii) that they contain as little longer wavelength component as can be achieved without great effort. [*Note*: If future calculations suggest that one should worry about phase coherence for mirrors far from the center of the pipe, as well for those close to the center, then to protect against it the rms amplitude ΔH of height variations should be 3 times larger than (2.6).]

4. Straightness and Roundness of the vacuum pipe.

It is *not* desirable for the pipe to be perfectly straight and round. The reason is that for a straight, round pipe and for mirrors near the pipe's center, light that scatters off one mirror and then propagates to the other mirror via an even number of reflections off the pipe wall will get focused and amplified in the reflections by a net factor that can be as large as $2R/\sqrt{\lambda L} \simeq 30$ in amplitude (1000 in energy). To prevent this, there must be fluctuations in the straightness at least, and if convenient also in the roundness (Sec. IV.B). On the other hand, if the fluctuations are too great, then they will amplify certain types of scattering noise (Sec. III.C). This leads to the following recommendation:

Recommendation 4. A "longitudinal curve" on the vacuum pipe's wall is defined to be the intersection of that wall with any plane containing the straight central axis of the pipe. At any point on the wall, define the angle μ_o to be the angle between the longitudinal curve through that point and the central axis of the pipe. Then there should be fluctuations in μ_o with rms amplitude σ_{μ_o} in the range

$$10^{-4}\,\text{radians} < \sigma_{\mu_o} < 10^{-3}\,\text{radians}\,. \tag{2.7a}$$

Accompanying these angular fluctuations there should be fluctuating linear offsets ξ_o of the pipe's central axis with rms amplitude

$$\sigma_{\xi_o} \gtrsim 1\,\text{cm}\,. \tag{2.7b}$$

For detailed justifications of these limits see Secs. III.C, IV.B, and IV.E.

5. Beam Wiggle.

In the design and development of detectors for the LIGO, account should be taken of the effect of beam wiggle on scattering. This might be the most serious factor constraining the allowable beam wiggle — especially if only one end of the pipe is baffled rather than both ends. For details see the paragraph following Eqs. (3.25).

6. Computer Simulations of Scattering

I am not sufficiently confident of my scattering calculations to justify basing the final LIGO baffle design solely on them.

Recommendation 5. I recommend that computer simulations be carried out to check my calculations.

7. Paramenters of Final Baffle Design

Recommendation 6. The parameters of the final baffle design should be so chosen as to keep smaller than about 1/10 the standard quantum limit (3.2) the noise levels (3.25)—(3.33) — at least in the case where a mode cleaner is used on the interferometer output, and if possible also without a mode cleaner. If the general design of the baffles differs from that recommended here, then for the chosen design the noise levels for the processes in Secs. III.B—III.E should be recomputed, and the design should be adjusted to keep them below about 1/10 of (3.2).

III. Results of Scattering Calculations

The formulas for scattering noise in this section are probably accurate to within a factor 10 and perhaps better. This seems accurate enough for our purposes — and higher accuracy would require far more effort than I have expended. In most of the formulas factors of order unity (2's, π's, etc. have been kept because they occasionally combine to give a net multiplicative factor of order 10 or larger. However, the precise combinations of 2's, π's, etc. that are quoted are almost certainly not correct.

The formulas are given for Fabry-Perot interferometers. I have not analyzed scattering noise in Michelson interferometers in detail, except for an old (May 1987) calculation of one rather unimportant scattering process. It would be worthwhile for somebody to extend this report's analysis to Michelsons.

The parameters that will appear in the scattering formulas are listed here in alphabetical order for ease of reference:

B — effective number of bounces that the light beam makes in the arms of the interferometer; defined more precisely by the equation preceding (3.7) below; for a simple interferometer or a light-recycled interferometer optimized to gravitational waves with frequency f, this quantity is (see page 424 of Ref. 5)

$$B = \frac{c}{2\pi f L} = \frac{4\times 10^3}{f/10\,\mathrm{Hz}} \;. \tag{3.1}$$

We shall use this value in our numerical estimates. For an interferometer with resonant recycling B can be larger than this. Using that larger B would produce smaller levels $\tilde{h}(f)$ of scattering noise than those quoted in this report.

d_n — distance of baffle n from the first baffle at the end of the pipe nearest to baffle n; approximately equal to the smaller of l_n and $L-l_n$.

$\dfrac{d\sigma}{dx\,d\theta'}$ — the differential cross section for diffracting light off a baffle, either in the case of ordinary diffraction or the case of diffraction-aided reflection; see Eq. (3.9) for the precise definition of this cross section; see Eqs. (3.10) and (3.15) & (3.16) for formulas for the cross section.

$\dfrac{d\sigma}{dx}$ — the total cross section for diffracting light off a baffle into angles larger than the incoming angle. See Eqs. (3.11) and (3.17) for formulas in the case of ordinary diffraction and diffraction-aided reflection, respectively.

$\dfrac{d\sigma}{d\Omega dA}$ — the scattering probability for the baffle surfaces, i.e. the cross section $d\sigma$ for a unit area dA of baffle surface to scatter light into a unit solid angle $d\Omega$. In this report this scattering probability is used only for incoming and outgoing light parallel to each other and at an angle of about 45 degrees to the baffle surface. For numerical estimates we use the conservative value 0.1 for this scattering probability.

f — the frequency of modulation of scattered light produced by its encounters with baffles and pipe walls.

$\tilde{h}(f)$ — square root of spectral density of noise in gravitational-wave units, i.e. $\tilde{h}(f) = \sqrt{S_h(f)}$ in the notation of Reference 5; units are "strain per root Hertz".

$\hbar = 1.054\times 10^{-27}$ erg sec — Planck's constant divided by 2π.

H — the mean height of the baffles' tops above the inner face of the vacuum pipe; for numerical estimates we use $H = 6$ cm.

H_n — the height of baffle n; in numerical estimates we shall use $H_n = H = 6$ cm.

δH — "height safety factor": the minimum height that a baffle should be above that required to intercept rays from mirrors; see Eqs. (2.4); the value suggested in this document is $\delta H = 1$ cm, and this is assumed in all numerical estimates.

ΔH — the maximum peak-to-valley variations of height of the baffles ("jaggedness"); assumed equal to 6 mm in numerical estimates, see Eq. (2.2)

l_n — the distance of baffle n from the closest interferometer mirror.

l_{Rn} — the "reduced distance" of a baffle pair, n and $n+1$, from the mirrors; given by $l_{Rn} \equiv l_n(L-l_{n+1})/L$ [where baffle n is presumed to be closer to the nearest mirror than baffle $n+1$]..

L — the length of an arm of the interferometer; assumed equal to 4 km in numerical estimates.

m — the mass of a mirror, assumed equal to 1000 kg in the numerical evaluation of quantum noise [Eq. (3.2) below].

N_b — the total number of baffles; see Eq. (2.5).

$P_{sc}(\theta)$ "scattering probability": the probability that a photon from the main beam gets scattered into a unit solid angle about a direction that makes an angle θ with the main beam. In this report it is assumed that $P_{sc}(\theta) = \alpha/\theta^2$, where $\alpha = 10^{-6}$.

$P_{rec}(\theta)$ — "recombination probability": the probability that a previously scattered photon impinging on a mirror at an angle θ to the mirror's normal will recombine with the main beam in such a way as to contribute to the scattering noise $\tilde{h}(f)$. This P_{rec} depends on whether a mode cleaner is used on the interferometer output. With a mode cleaner, P_{rec} is given by Eq. (3.5); without, by Eq. (3.6).

R — the radius of the vacuum pipe; assumed equal to 60 cm in numerical estimates.

Y_o — the distance from the center of the main beam to the mean position (smoothed over jaggedness) of the nearest baffle edge; assumed equal to 20 cm in numerical estimates when a small Y_o is the most dangerous.

s_n — the spacing between baffles n and $n+1$; assumed in numerical estimates to be given by Eqs. (2.4) with $R = 60$ cm, $H_n = 6$ cm for all n and $\delta H = 1$ cm; i.e. $s_n = 10\,\text{cm}/\theta_o$ for the first $120\,\text{cm}/\theta_o$ length of baffles near each end; thereafter $s_n = d_n/12$.

α — coefficient that appears in the approximate formula $P_{sc}(\theta) \equiv dP/d\Omega = \alpha/\theta^2$ [Eq. (3.4)] for the scattering of main-beam light off the interferometer's mirrors. In this formula $P_{sc}(\theta) = dP/d\Omega$ is the probability that a photon in the main beam will be scattered into a unit solid angle around a direction that makes an angle θ with the normal to the mirror. In numerical estimates we shall use a value $\alpha = 10^{-6}$ appropriate to supermirrors [Sec. III.A.1].

λ — the wavelength of light used in the interferometer; for numerical estimates we shall set $\lambda = 0.4\,\mu$m.

$\bar{\eta}$ — the photodiode efficiency, averaged over the spot made by the main beam on the photodiode: assumed equal to 0.9 in numerical estimates.

$\bar{\mu}$ — the value of an angle μ averaged over times long compared to the gravitational-wave period. This μ is defined as follows: A "longitudinal curve" on the vacuum pipe's inner wall is the intersection of that wall with any plane containing the central axis of the pipe. Consider a typical point at which light reflects off the vacuum pipe's inner wall. Then μ is the angle between the longitudinal curve at that typical reflection point and the line-of-sight direction between the two points where the reflecting ray originates and terminates. (Those origination and termination points are either on the mirrors or on the edges of diffracting baffles.) We sometimes shall refer to μ as the "slope" of the wall, and sometimes as its "angle".

$\tilde{\mu}(f)$ — the square root of the spectral density of fluctuations $\delta\mu(t)$ in the angle μ defined above. These fluctuations are due to (seismic-induced) acoustic oscillations of the vacuum pipe wall, plus — if only one end of the vacuum pipe or neither end is baffled — wiggle of the main beam; see the paragraph following Eqs. (3.25). We shall assume, for numerical estimates, that $\tilde{\mu}(f) = 10^{-10}\,\mathrm{Hz}^{-1/2}\times(10\,\mathrm{Hz}/f)$; see the passages following Eq. (3.22) for discussion of the reasons behind this value.

μ_o — the angle between a longitudinal curve on the vacuum pipe's inner wall and the straight-line central axis of the pipe.

σ_{μ_o} — rms value of the angle μ_o

σ_{ξ_o} — rms value of the lateral offset ξ_o of the pipe's central axis. This lateral offset, like μ_o, is produced by (desirable) crookedness in the way the pipe is laid during construction. For numerical estimates we shall use $\sigma_{\xi_o} = 1$ cm.

σ_H — the maximum wavelength of the jaggedness of the baffles, i.e. the wavelength above which the spectral density of the jaggedness is small.

σ_M — the mean wavelength for variations in the phase of scattered light along the edges of the baffles — variations produced by irregularities in the scattering mirrors; see Eq. (4.5d) and associated discussion

σ_{pd} — the mean wavelength for photodiode-irregularity-induced variations in phase along the edges of the baffles; see Eq. (4.11) and associated discussion.

$\tilde{\xi}(f)$ — the square root of the spectral density of fluctuational displacements of a typical point on a typical baffle — either longitudinal displacements or radial displacements. For numerical estimates we shall assume that the baffles are sufficiently well anchored that they experience displacements only of order the seismic noise; i.e. we shall assume $\tilde{\xi}(f) = 10^{-7}\,\mathrm{cm\,Hz}^{-1/2}\times(10\,\mathrm{Hz}/f)^2$, corresponding to the level of seismic noise at the likely LIGO sites.

θ — the angle between a ray of scattered light and the central axis of the main beam.

θ' — the angle (value of θ) into which light diffracts when it encounters a baffle.

θ_n — the minimum value of θ that a ray must have, when encountering baffles n and $n+1$, in order to avoid being caught by them; given by Eq. (3.13).

This section III is organized as follows: Sec. III.A presents a set of simple formulas which can be used to compute the noise due to a wide variety of scattering processes (scattering of the main beam off a mirror, diffraction of scattered light off baffles, reflection of scattered light off the pipe walls, recombination of scattered light with the main beam, ...). These formulas underlie most of the noise formulas of this report, and they can be used for other, future noise calculations. The need for baffles is elucidated in Section III.B by discussing the dominant scattering noise effect for a LIGO without baffles: scattering off a mirror, followed by reflections off the vacuum pipe wall, propagation to the other mirror, and recombination there with the main beam. The remaining sections discuss what appear to be the dominant scattering noise effects in the presence of baffles: scattering off one mirror, followed by a diffraction-aided reflection between a pair of baffles, followed by a series of reflections off the pipe wall and possibly a second diffraction-aided reflection, followed by recombination with the main beam at the second mirror (Sec. III.C); scattering off a mirror, followed by scattering off the baffles, propagation back to the mirror and recombination there with the main beam (Sec. III.D).

In discussing these scattering processes we shall need to compare their noise levels with the best sensitivities that are hoped for in the LIGO. As a measure of the best sensitivities we shall use the standard quantum limit

$$\tilde{h} = \left[\frac{8\hbar}{m(2\pi f L)^2} \right]^{1/2} = \frac{4\times10^{-24}}{\sqrt{\text{Hz}}} \frac{10\,\text{Hz}}{f} \tag{3.2}$$

[Eq. (121) of Ref. 5]. We shall regard scattering noise as acceptably small if it is at least one order of magnitude below this level. (The one-order-of-magnitude safety factor is to allow for inaccuracies in the scattering calculations.)

A. Formulas for Use in Intensity Analyses of Scattering

It is argued in various places in this document [see especially the summary and analysis in Sec. IV.E, the paragraph following Eq. (4.11), and Sec. IV.A.5] that by the time scattered light reaches the mirror or photodiode at which it will recombine with the interferometer's main beam to produce noise, the phases of light coming from different directions will be so scrambled that coherent superposition is relatively unimportant. As a result, computations of the noise due to scattered light can be carried out using intensity techniques in which the precise phase of any bit of light is ignored. In this section I give a number of useful formulas for such calculations; these formulas are derived in Sec. IV.A

1. Scattering Probability $P_{sc}(\theta)$

When the main beam of the interferometer hits a mirror, irregularities in the mirror scatter a bit of the main-beam light. We shall denote by $P_{sc}(\theta)$ the probability that a main-beam photon will get scattered into a unit solid angle in a direction that makes an angle θ with the specularly reflected main beam,

$$P_{sc} = \frac{d\ \text{Probability}}{d\Omega}\ . \tag{3.3}$$

The angles relevant to the LIGO are $\theta_o \gtrsim \theta \gtrsim 2Y_o/L$; i.e. 0.1 or 0.01 radians to 10^{-4} radians; i.e. 5 degrees or 30 minutes down to 0.3 minutes of arc. The most relevant scattering data I know of are: (i) measurements with a poor quality mirror by Michelle Stephens[1] in the range $0.011 \leq \theta \leq 0.027$, which are well fit by $P_{sc} \simeq 2.4\times10^{-3}(1-\mathcal{R})/\theta^2$, where $\mathcal{R} = 0.988$ is the reflectivity of the mirror that she used; and (ii) measurements on supermirrors in the range $XXX \lesssim \theta \lesssim XXX$ by Elson and Bennett[2], which are well fit by[3] $P_{sc}(\theta) = 1.5\times10^{-6}/\theta^2$. Assuming that P_{sc} is proportional to $1-\mathcal{R}$, there is reasonable agreement between Stephens' results and those of Elson and Bennett. Note, further, that the dependence on θ, $P_{sc} = \alpha/\theta^2$ with $\alpha \sim 10^{-6}$, puts equal amounts of power into equal increments of $\ln\theta$, and it leads to a total probability for scattering photons out of the main beam given by $P_{total} = \int P_{sc} 2\pi \sin\theta d\theta \simeq 2\pi\alpha \ln(\sqrt{L/\lambda}) = 7\times10^{-5}(\alpha/10^{-6})$. This is so close to the total losses of supermirrors ($\simeq 10^{-4}$) that there is no room for excess scattering, beyond that of the formula $P_{sc} = \alpha/\theta^2$, for angles θ below those at which the scattering has been measured. Thus, it seems reasonable to assume (and we shall do so throughout this report) that not only in the measured region but also down to the LIGO's smallest angles, which are 30 times smaller, the scattering probability is given by

$$P_{sc} = \frac{\alpha}{\theta^2}\ , \quad \text{with } \alpha = 10^{-6}\ . \tag{3.4}$$

The relationship of this scattering probability to the mirror's irregularities, and the spatial dependence of the phase of the scattered light are discussed in Sec. IV.A.2.

2. Probability $P_{rec}(\theta)$ for Recombination of Scattered Light with the Main Beam

After leaving the main beam, scattered light can interact with the walls of the vacuum pipe and with the baffles in a variety of ways to be discussed below, and then some of the scattered light can recombine with the main beam in two ways: By hitting a mirror and there scattering back into the main beam, or by hitting the corner mirror and being transmitted through it, along with the main beam, onto the photodiode. We shall consider these two recombination processes in turn.

Scattering back into main beam. When a scattered photon impinges on the mirror at an angle θ to the incoming main beam, it has a probability $P_{sc}(\theta) = \alpha/\theta^2$ of being scattered into a unit solid angle in the direction of the reflected main beam. In order to actually rejoin the main beam rather than going into some other mode of the Fabry-Perot cavity, the photon must scatter into a solid angle which is equal to that subtended by the main beam's spot on the distant mirror, $\Delta\Omega = \lambda L/L^2 = \lambda/L$. Correspondingly, the probability for the photon to recombine with the main beam via scattering is

$$P_{rec}(\theta) = \frac{\alpha}{\theta^2}\left(\frac{\lambda}{L}\right) . \tag{3.5}$$

Transmission through mirror onto photodiode. Scattered light, arriving at the photodiode from a direction that makes an angle θ with the main beam, has its phase fronts at an angle θ relative to those of the main beam. Since the phase fronts of the main beam and the beam from the other arm of the detector have been made to agree at the photodiode, so far as possible, this means that the scattered light's phase fronts disagree in angle by θ with those of the light from the other arm. The result is a pattern of interference fringes on the photodiode with wavelength λ/θ. In these fringes the scattered light alternately increases, then decreases the light intensity and hence the photocurrent. If the photodiode's efficiency η for converting light intensity into photocurrent were spatially uniform, these fringes would give a net averaged contribution to the photocurrent that is the Fourier transform (at wave number $\theta k = 2\pi\theta/\lambda$) of the Gaussian shape (4.1) of the main beam — and because Fourier transforms of Gaussians are Gaussians and are notoriously small out on the wings where we are operating ($\sim e^{-(\pi/4)[\theta/\sqrt{\lambda/L}]^2}$), the effects of the scattered light would be totally negligible. Unfortunately, the efficiency η will be spatially variable due to imperfections in the photodiode; and correspondingly the effect of the scattered light on the photocurrent will be proportional to a spatial Fourier transform of the photodiode efficiency.

More specifically (see Sec. IV.A.3), if the scattered light's effect is described by the probability $P_{rec}(\theta)$ for each scattered photon to recombine at the photodiode with the main beam in such a way as to act like a main-beam photon, then that $P_{rec}(\theta)$ will be essentially the square of the spatial Fourier transform of η. A conservative estimate of the magnitude of that Fourier transform (more likely an overestimate than an underestimate; see Sec. IV.A.3 for details) gives

$$P_{rec} = \frac{(1-\bar{\eta})^2}{2\theta}\left(\frac{\lambda}{L}\right)^{1/2} . \tag{3.6}$$

Here $\bar{\eta}$ is the spatially averaged efficiency of the photodiode, which in numerical estimates we shall take equal to 0.9.

Comparison of recombination via scattering with recombination via transmission to photodiode. The probability (3.6) for recombination via transmission to the photodiode is always (for all relevant θ) several orders of magnitude larger than the probability (3.5) for recombination via scattering. Fortunately, if recombination via transmission becomes a serious noise source, it can be suppressed by putting a mode cleaner on the output of the interferometer. Thus, with a mode cleaner recombination by scattering dominates; without a mode cleaner recombination by transmission to the photodiode dominates.

3. Scattering Noise Expressed as $\tilde{h}(f)$

When scattered light recombines with the main beam, it produces a change of phase $\delta\Phi_{mb}$ in the main beam. This $\delta\Phi_{mb}$ fluctuates slightly due to fluctuations in the phase of the scattered light — fluctuations put onto the scattered light by interactions with the pipe walls and baffles. The square root of the spectral density of those phase fluctuations is given by

$$\tilde{\Phi}_{mb}(f) = \cos(\Phi_{sc} - \Phi_{mb}) \left(\frac{dE_{sc}/dtdf}{I} \right)^{1/2} .$$

Here $\Phi_{sc} - \Phi_{mb}$ is the phase difference between the scattered light and the main beam, I is the power of the main beam and $dE_{sc}/dtdf$ is the power spectral density carried into the main beam by the scattered light. We can express $dE_{sc}/dtdf$ as

$$\frac{dE_{sc}}{dtdf} = \int_{\text{mirror}} P_{rec} \frac{dE_{sc}}{dtdAd\Omega df} (\lambda L) d\Omega ,$$

where $dE_{sc}/dtdAd\Omega df$ is the specific intensity of scattered light arriving at the receiving mirror, and λL is the area of the main beam on that mirror. Since we make no attempt to compute the relative phase $\Phi_{sc} - \Phi_{mb}$ of the scattered light and the main beam, we shall simply replace the cosine of that relative phase by its rms value, $1/\sqrt{2}$. By doing so and by using the standard relation

$$\tilde{h}(f) = \frac{1}{2\pi BL} \tilde{\Phi}_{mb}(f)$$

between the gravitational-wave noise and the main-beam phase noise (a relation which is essentially the definition of B), and by multiplying by a factor of $\sqrt{2}$ to account for the fact that scattered light can originate at either of the two ends of the vacuum pipe, we obtain the following formula for the gravitational-wave noise due to scattered light:

$$\tilde{h}(f) = \frac{\lambda}{2\pi BL} \left(\int P_{rec}(\theta) \frac{dE_{sc}/dtdAd\Omega df}{I/\lambda L} d\Omega \right)^{1/2} . \tag{3.7}$$

Here and in the preceding equation the proportionality factor in front of the integral takes account of both arms: *The integral is to be performed at the corner mirror of only one arm, and the $\tilde{h}(f)$ which results is that for the entire detector.*

4. Reflection of Scattered Light off the Pipe Wall

In the remaining subsections of this section we shall discuss the interaction of scattered light with the wall of the vacuum pipe and with baffles, as it propagates from the scattering mirror to the receiving mirror. For each interaction process we shall describe the incoming light by the energy flux (power per unit area) $dE/dtdA$ of the unmodulated component of the incoming light, and by the specific energy flux (power per unit area per unit frequency) $dE/dtdAdf$ of the tiny portion that has been frequency-modulated by previous interactions with baffles and/or the wall. If the light has a well-defined propagation direction, then these energy fluxes carry all the relevant information. If there were a spread of propagation directions, then one would need to work with the intensity (flux per unit solid angle) $dE/dtdAd\Omega$ and specific intensity $dE/dtdAd\Omega df$.

Consider, first, the reflection of scattered light off the wall of the vacuum pipe. We shall treat such reflection as precisely specular (angle of reflection equals angle of incidence) and as lossless. (Losses are taken into account by placing the upper limit θ_o on angles of photons included in the analysis.)

Acoustical vibrations of the wall produce time variations (modulations) of the path length between the scattering and receiving mirrors, and corresponding modulations of the phase and thence frequency of the scattered light. In Sec. IV.A.4 [Eq. (4.19)] it is shown that the dominant contribution to the frequency modulation comes not from radial displacements ξ of the wall, but rather from changes in the longitudinal slope μ of the wall. The standard of reference for that slope is the line-of-site direction between two points: the point at which the reflecting light initiates its sequence of specular reflections, and the point at which it terminates them. Both of these points are baffles at which diffraction occurs, if both ends of the pipe are baffled; otherwise one or both points are the centers of interferometer mirrors. Correspondingly, if there are no baffles or only one end of the pipe is baffled, the modulations result from beam wiggle as well as from wall vibrations; but if both ends are baffled, wall vibrations are the sole source of the modulations.

The analysis of Sec. IV.A.4 shows that the frequency modulation put onto the light by the fluctuations in μ is described by

$$\Delta \frac{dE}{dtdAdf} = \frac{dE}{dtdA}\left[4\pi\tilde{\mu}\left(\frac{(L-2l)\theta + L\sqrt{L\theta/2R}\,\sigma_{\mu}}{\lambda}\right)\right]^2 . \tag{3.8}$$

Here $\tilde{\mu}^2(f)$ is the spectral density of μ, $dE/dtdA$ is the energy flux of the unmodulated component of scattered light, $\Delta(dE/dtdAdf)$ is the amount by which the tiny modulated component is augmented during the reflection, θ is the angle of the scattered light relative to the wall, $\bar{\mu}$ is the time-averaged slope of the wall, l is the distance of the reflection point from the scattering mirror, and in this version of the formula it is presumed that the scattering and receiving mirrors or diffracting baffles are at opposite ends of the pipe. For further discussion see Sec. IV.A.4.

5. *Diffraction of Scattered Light Off a Baffle*

Although the diffraction of light is a phenomenon that depends crucially on the phase differences of different propagation paths, in analyzing scattering we can embody the influence of those phase differences in a differential cross section $d\sigma/dxd\theta'$ and then use that cross section in a phase-ignorant intensity analysis. In this approach one must look carefully at phase when deriving formulas for $d\sigma/dxd\theta'$; but once that cross section has been derived, one can ignore phase when applying it.

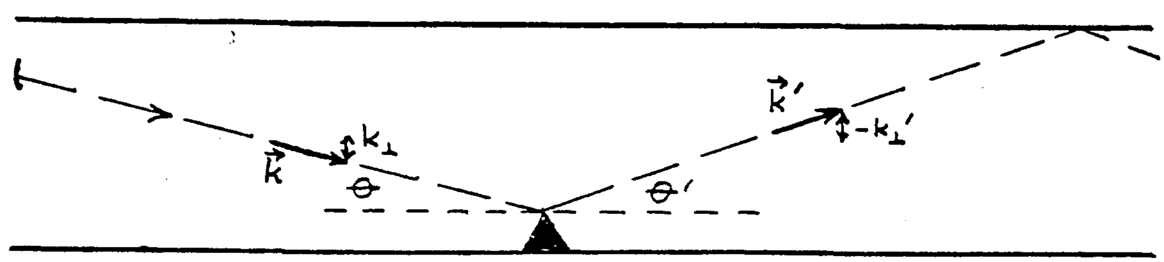

Fig. 3.1 Diffraction of light off a baffle. The long-dashed line is the path of a light ray. The projection of that path into the pipe wall cannot change in the diffraction: the angle θ relative to the wall changes into θ'.

The differential cross section $d\sigma/dxd\theta'$ is defined by the following formula in which one uses it: Suppose that light with wave vector $\vec{k}$ impinges on a baffle (Fig. 3.1). Then the diffracted light appears to emerge from the edge of the baffle. As for reflection off the wall, so also here, the component of the wave vector $\vec{k}_\parallel$ parallel to the wall is unchanged in the diffraction, while the component $k_\perp$ perpendicular can change. Define the angle of incidence by $\theta = k_\perp/k$ and the angle of diffraction by $\theta' = -k'_\perp/k$, where $\vec{k}'$ is the wave vector of the diffracted light. For wall reflection θ' must be equal to θ. For diffraction off a baffle θ' can have a range